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GENERALIZED VECTOR DOMINANCE AND THE PION FORM FACTOR

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A B S T R A C T

We have calculated the pion form factor using the bound state Bethe-Salpeter amplitudes and the quark form factor of a relativistic quark model. We obtain a generalized vector dominance structure and an asymptotic behaviour in the space-like region $F_{\pi}(Q^2) \sim 0.33 \text{ GeV}^2/Q^2$.

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At present there is sufficient experimental information on the pion form factor in the time-like ¹⁾ and space-like ²⁾ region to test dynamical models ³⁾. We consider the pion form factor to be dominated by the vector mesons [generalized VDM ^{4),5),6)}] and to depend on the wave function describing a composite structure of the pion. It is the purpose of this paper to discuss the interplay of generalized vector meson dominance and bound state structure in a relativistic quark model ⁷⁾. In this model the meson mass spectrum and wave functions were obtained by solving the bound state Bethe-Salpeter equation ⁸⁾. We now use the BS amplitude of the pion $\chi^\pi(k, P)$ ^{*} and the quark form factor $\Gamma_\mu(k, Q)$ to determine the pion form factor with help of the triangle diagram

(1)

$$F_\mu(Q^2) = (2\pi)^{-9/2} i \int d^4k \text{Tr} \Gamma_\mu(k, Q) \chi^\pi(k + \frac{1}{2}P_2, P_1) \cdot (k - \frac{P_1}{2} + \frac{P_2}{2} - m) \chi^\pi(k - \frac{P_2}{2}, P_2)$$

The quark form factor is given by the integral equation

$$\Gamma_\mu(k, Q) = 2\gamma_\mu + i \int dk' K(k, k') [\frac{1}{2}Q + k' - m]^{-1} \cdot \Gamma_\mu(k', Q) [\frac{1}{2}Q - k' + m]^{-1} \quad (2)$$

The BS kernel K describes the quark-antiquark interaction, and $2\gamma_\mu$ results from the ansatz $j_\mu^{el} = 2[\bar{\psi}(x), \gamma_\mu \psi(x)]$ for the electromagnetic current of the quarks. This inhomogeneous BS equation can be solved in good approximation for $Q^2 \ll 4m^2$ (where m is the mass of the heavy quarks) by using the completeness of the BS amplitudes. We obtain in the rest frame of the photon [$Q = (Q_0, \vec{Q})$] ^{*}.

$$\Gamma_\mu(k, Q) = 2(2\pi)^{-4} i \sum_N \frac{\text{Tr} \gamma_\mu \int dk' \chi_N(k', M_N) (\gamma_0 \frac{M_N}{2} + k - m) \chi_N(k, M_N) (\gamma_0 \frac{M_N}{2} - k + m)}{Q^2 - M_N^2}$$

$$= \sum_{r, S_3} \frac{\epsilon_{\mu}^{S_3} \cdot g_{Nrs} (\gamma_0 \frac{M_r}{2} + k - m) \bar{\chi}_r^{S_3}(k, M_r) (\gamma_0 \frac{M_r}{2} - k + m)}{Q^2 - M_r^2} \quad (3)$$

^{*}) N is a shorthand for the quantum numbers of the mesons, r denotes the radial quantum number.

This expression shows a generalized vector dominance structure for the quark form factor. Because of the local current $j_\mu^d(x)$ only the radially excited ⁶⁾ vector mesons contribute ; the residues factorize into BS amplitude and vector meson-photon coupling constant $g_{V\gamma}$.

From the mathematical point of view a function is determined by its poles and residues up to an additive entire function only. We assume, following duality arguments, that the unsubtracted Mittag-Leffler expansion (3) may be used, i.e., that the poles without a background term build up the quark form factor correctly.

Now we use the quark form factor, Eq. (3) to evaluate the pion form factor with help of Eq. (1)

$$\begin{aligned} F_\pi(Q^2) &= (2\pi)^{-9/2} i \sum_{r,s_3} \frac{g_{V\gamma} \cdot \epsilon_{\mu^3}^{s_3}}{Q^2 - M_V^2} \cdot \int dk \text{Tr} \bar{\chi}_r^s(k, M_N) \cdot \\ &\quad \cdot (k - \frac{i}{2} \gamma_0 M_N - m) \chi_{(k+\frac{p_1}{2}, l_1)}^{\pi} (k - \frac{1}{2} p_1 + \frac{1}{2} p_2 - m) \cdot \\ &\quad \cdot \chi_{(k-\frac{p_1}{2}, l_2)}^{\pi} (k + \frac{1}{2} \gamma_0 M_N - m) \\ &= \sum_{r,s_3} \frac{g_{V\gamma} \epsilon_{\mu^3}^{s_3} \cdot \mathcal{M}_{V\gamma \rightarrow 2\pi}^{s_3}}{Q^2 - M_V^2} \end{aligned} \quad (4)$$

The poles of the quark form factor also show up in the pion form factor. The residues are determined by the BS amplitudes of the pion and the heavy vector mesons.

The explicit result depends on the form and spin structure of the BS kernel. In Ref. 9) we found that a smooth kernel, approximated by a harmonic interaction and with a $\gamma_5 \chi \chi_5$ Dirac part yields a singlet-triplet structure for the spectrum with the mass formula

$$M_N^2 = M_0^2 + 2x\sqrt{\beta} \cdot N \quad ; \quad N = m + 2r; \quad m, r = 0, 1, 2, \dots \quad (5)$$

and describes the strong decays of the meson resonances reasonably well for $x = 3$ and $\sqrt{\beta} = 0.2 \text{ GeV}^2$. We use the Wick-rotated BS amplitudes of this model

$$\chi_{(k,p)}^{\pi^+} = \frac{2\pi\sqrt{\beta}}{\sqrt{\beta}} \cdot \left[\gamma_5 \frac{p}{M_\pi} \left(1 - \frac{p}{2m}\right) + \frac{i}{2} \gamma_5 \left[\frac{k}{m}, \frac{p}{M_\pi} \right] \right] \cdot e^{-k^2/2\sqrt{\beta}} \cdot |p\bar{n}\rangle$$

$$\chi_r^2(k, P) = \frac{2\pi\sqrt{3}}{\sqrt{\beta}} \left[\phi\left(1 - \frac{\beta}{2m}\right) - i \cdot \frac{ek}{m} \right] \cdot e^{-k^2/2\sqrt{\beta}} \cdot \frac{1}{\sqrt{r+1}} \cdot L_r^1(k^2/\sqrt{\beta}) \cdot \frac{1}{\sqrt{2}} (|p\bar{p}\rangle - |n\bar{n}\rangle). \quad (6)$$

for the evaluation of the integrals in Eqs. (3) and (4) and obtain

$$g_{\rho^0\gamma} = \frac{M_\rho^2}{2\gamma_{\rho^0}} = (-1)^r \cdot 2 \cdot \frac{2\sqrt{3}}{\pi} \cdot \sqrt{r+1} \cdot \sqrt{\beta} \cdot \frac{1}{\sqrt{2}}, \quad (7)$$

$$F_\pi(Q^2) = g_{\rho^0\gamma} \cdot g_{\rho^0\pi\pi} \cdot \sum_{r=0}^{\infty} \frac{(-1)^r e^{r/2} L_r^1(r+c)}{M_{\rho^0}^2 + 12r\sqrt{\beta} - Q^2}, \quad (8)$$

$$c = \frac{M_{\rho^0}^2 - 4M_\pi^2}{12\sqrt{\beta}} = 0,21.$$

For comparison with experiment in the time like region one has to take into account the finite widths of the vector mesons. We have used Breit Wigner forms with $\Gamma_\rho = 150$ MeV and $\Gamma_{\rho^0} = 500$ MeV and choose the only free parameter, the renormalization constant Z such that $F_\pi(0) = 1$. The results are given in Fig. 1.

The sum of the residues in Eq. (7) converges, we therefore obtain for large space-like values of Q^2

$$F_\pi(Q^2) \approx - \frac{0,33 \text{ GeV}^2}{Q^2}. \quad (9)$$

We find asymptotically a simple pole behaviour. This result differs substantially from calculations which take into account only the pion wave function and not the quark form factor, where one would obtain $F(Q^2) = e^{Q^2/\text{const.}}$, reflecting the Gaussians of the wave functions. Our result, Eqs. (8) and (9), is a consequence of the duality assumption that the pion form factor can be built up by resonances without a background term and the spectrum and the coupling constants of the relativistic quark model.

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