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ANALYTIC PARAMETRIZATION OF HIGH ENERGY pp AND $p\bar{p}$
FORWARD SCATTERING AMPLITUDE

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A B S T R A C T

The pp and $p\bar{p}$ forward scattering amplitude $F(E)$ is represented in a parametric form which is analytic in the cut E plane and which gives the asymptotic behaviour of the total cross-sections $\sigma^{pp}, \sigma^{p\bar{p}} \sim (\log \frac{E}{E_0})^\gamma$, E being the laboratory energy and γ being real. Fits to high energy data favourize a $(\log \frac{E}{E_0})^2$ increase of $\sigma^{p\bar{p}}$ for $E \rightarrow \infty$, but admit also other growth rates. The predicted real part of the forward amplitude becomes positive at E about 300 GeV and is equal to 6-10 % of the imaginary part at 2000 GeV.

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1. Recent measurements of total proton-proton cross-section σ^{pp} performed at the CERN Intersecting Storage Rings have yielded values of σ^{pp} in the energy range from 290 up to 1480 GeV/c of the equivalent laboratory momentum p and indicate a significant increase of σ^{pp} up to 43 mb in this region [Refs. 1)-3), see also Refs. 4) and 5)]. Moreover, values of the ratio ρ of the real to the imaginary part of the forward proton-proton elastic scattering amplitude have been obtained at $p = 290$ and 500 GeV/c¹⁾. These results suggest the possibility of an asymptotic increase of σ^{pp} and raise the question of the corresponding very high energy behaviour of the real part.

It was pointed out by Khuri and Kinoshita⁶⁾ that even the general frame of local field theory implies a strong correlation between the sign of the real part and the behaviour of the total cross-section at high energies. Indeed, if the cross-section diverges, the real part cannot stay negative for all large energies. Thus, if we make the usually adopted "physical" assumption that $\lim_{E \rightarrow \infty} \rho(E) = 0$, where $\rho(E) = \text{Re } F(E) / \text{Im } F(E)$, this limit should be approached from above.

Our aim is to use the high energy proton-proton and proton-antiproton forward scattering data for obtaining information about : i) the rate of this increase; ii) the corresponding behaviour of ρ at high energies. Our main assumptions are :

- i) the forward scattering amplitude is a real analytic function in the cut E plane^{*)} and is crossing symmetric;
- ii) optical theorem;
- iii) σ^{pp} and $\sigma^{p\bar{p}} \rightarrow (\log E/E_0)^\gamma$ for $E \rightarrow \infty$;
- iv) $\sigma^{p\bar{p}} - \sigma^{pp} \rightarrow 0$ for $E \rightarrow \infty$;
- v) $F(E)$ does not oscillate at infinity and $\rho \rightarrow 0$ for $E \rightarrow \infty$;
- vi) the term dominating in σ^{pp} and in $\sigma^{p\bar{p}}$ at the ISR energies dominates at all higher energies too.

*) Concerning assumption i), it should be noted that analyticity of the proton-proton scattering amplitude in the cut energy plane has not been proved from first principles. Omitting all details in this short note, we take up here the standpoint usually adopted in papers dealing with applications of dispersion theory and assume the validity of the dispersion relation merely on the basis of analogy with other processes, where it has been exactly proved.

2. As an alternative method to the conventional dispersion relation approach to this problem, we represent the spin independent proton-proton forward scattering amplitude $F(E)$ in a parametric form, which exhibits all the analytic properties of $F(E)$ in the cut E plane ^{*}) required by dispersion theory. This is mathematically equivalent to writing down dispersion relations [see, e.g., Ref. 7) for details]. The procedure of inserting the measured p - p and p - $\bar{p}$ total cross-sections as an input into dispersion integrals is replaced here by fitting the parameters so as to reproduce the high energy imaginary parts. The real part is then obtained, up to a term of the form $c_6 + c_7 E$, from the analyticity requirement. We find a good fit by putting $c_7 = 0$.

Due to the negligible rôle of low energy structures at very high energies, the dependences are smooth so that only few parameters are needed. Our approach is therefore suitable for high energy predictions.

Two different "analytic" parametrizations of $F(E)$ are considered in more detail :

$$F_a(E) = -a_1 (M^2 - E^2)^{\frac{1}{2}} (1 + a_2 \mathcal{L}_a)^\alpha - a_4 ((M-E)^{\frac{1}{2}} + (M+E)^{\frac{1}{2}}) - a_5 (E - E_1)^{\frac{1}{2}} + a_6 + 2\pi f^2 \frac{1}{E - E_p} \quad (1)$$

and

$$F_b(E) = -b_1 (M^2 - E^2)^{\frac{1}{2}} (1 + b_2 \mathcal{L}_b^2)^\beta - b_5 (E - E_1)^{\frac{1}{2}} + b_6 + 2\pi f^2 \frac{1}{E - E_p} \quad (2)$$

where

$$\begin{aligned} \mathcal{L}_a &= \log (1 + a_3 (M^2 - E^2)^{\frac{1}{2}}) \\ \mathcal{L}_b &= \log (b_3 + b_4 (M^2 - E^2)^{\frac{1}{2}}) \end{aligned} \quad (3)$$

$$E_1 = M - 2\mu^2/M, \quad E_p = M - \mu^2/2M, \quad f^2 = 0.08$$

μ being the pion mass.

^{*}) We denote by p and E the laboratory momentum and energy of the incident particle, respectively. We have

$$E = \sqrt{p^2 + M^2}, \quad E = \frac{1}{2} s/M - M$$

where s is the total centre-of-mass energy squared and M is the proton mass.

All parameters are real and, moreover, the following restrictions have been required

$$a_3 > 0 \quad , \quad b_4/b_3 > 0 \quad (4)$$

in order to avoid additional singularities on the physical sheet. The highest power admitted by the Froissart bound ⁸⁾ is obtained by putting

$$\alpha = 2 \quad \text{or} \quad \beta = 1$$

in (1) and (2), respectively.

The functions $(M-E)^{\frac{1}{2}}$, $(M+E)^{\frac{1}{2}}$ and $(E-E_1)^{\frac{1}{2}}$ in (1) and (2) produce the right-hand cut $E > M$ in the proton-proton channel and two (overlapping) left-hand cuts $E < -M$, $E < E_1$ in the proton-antiproton channel, respectively. Additional logarithmic singularities, corresponding to solutions of the equations

$$1 + a_3 (M^2 - E^2)^{\frac{1}{2}} = 0 \quad (5)$$

$$b_3 + b_4 (M^2 - E^2)^{\frac{1}{2}} = 0 \quad (6)$$

are situated on the second Riemannian sheet due to (4). Singularities corresponding to solutions of the equations

$$1 + a_2 \mathcal{L}_a = 0 \quad \text{and} \quad 1 + b_2 \mathcal{L}_b^2 = 0 \quad (7)$$

are on unphysical sheets provided that

$$a_2 > 0 \quad \text{and} \quad 0 < b_2 < 4/\pi^2 \quad (8)$$

respectively. Actual values of the parameters following from the fits to data (to be given below) prove that these conditions are fulfilled.

3. The total cross-sections are obtained from (1) and (2) by use of the optical theorem

$$\begin{aligned} \sigma^{pp}(E) &= \frac{\text{Im } F(E+i0)}{p} & E > M \\ \sigma^{p\bar{p}}(E) &= \frac{\text{Im } F(-E-i0)}{p} & E > M \end{aligned} \quad (9)$$

They have the following form

$$\sigma_a^{pp}(E) = a_1 \left[(1 + a_2 L_a)^2 + a_2^2 A_a^2 \right]^{\alpha/2} \cos \left(\alpha \operatorname{arctg} \frac{a_2 A_a}{1 + a_2 L_a} \right) + \frac{a_4}{(E + M)^{\frac{1}{2}}} \quad (10)$$

$$\sigma_a^{p\bar{p}}(E) = \sigma_a^{pp}(E) + a_5 \frac{(E + E_1)^{\frac{1}{2}}}{(E^2 - M^2)^{\frac{1}{2}}} \quad (11)$$

in the a-parametrization [Eq. (1)] and

$$\sigma_b^{pp}(E) = b_1 \left[(1 + b_2 (L_b^2 - A_b^2))^2 + 4 b_2^2 L_b^2 A_b^2 \right]^{\beta/2} \cos \left(\beta \operatorname{arctg} \frac{2 b_2 L_b A_b}{1 + b_2 (L_b^2 - A_b^2)} \right) \quad (12)$$

$$\sigma_b^{p\bar{p}}(E) = \sigma_b^{pp}(E) + b_5 \frac{(E + E_1)^{\frac{1}{2}}}{(E^2 - M^2)^{\frac{1}{2}}} \quad (13)$$

in the b-parametrization [Eq. (2)]. Here, the following abbreviations

$$L_a = \log |1 + i a_3 \sqrt{E^2 - M^2}|, \quad A_a = \operatorname{arctg} (a_3 \sqrt{E^2 - M^2}) \quad (14)$$

$$L_b = \log |b_3 + i b_4 \sqrt{E^2 - M^2}|, \quad A_b = \operatorname{arctg} \left(\frac{b_4 \sqrt{E^2 - M^2}}{b_3} \right)$$

have been introduced.

Asymptotic expressions for the total cross-sections are obtained from (10)-(13) by replacing (14) by

$$L_a = \log (a_3 E), \quad A_a = \frac{\pi}{2} \quad (15)$$

$$L_b = \log (|b_4| E), \quad A_b = \frac{\pi}{2}$$

and by neglecting the terms with a_4 , a_5 and b_5 . The ratio $\rho(E) = \operatorname{Re} F(E) / \operatorname{Im} F(E)$ behaves like

$$tg \left(\alpha \arctg \frac{\pi}{2 (1/a_2 + \log(a_3 E))} \right) \quad (16)$$

$$tg \left(\beta \arctg \frac{\pi \log(1/b_4/E)}{1/b_2 - \pi^2/4 + \log^2(1/b_4/E)} \right)$$

4. High energy experimental data on the total cross-section $\sigma^{pp}(E)$ were used to determine the parameters a_1, a_2, a_3, a_4 and, analogously, b_1, b_2, b_3, b_4 . The CERN MINUIT program was employed to compute their values as well as the corresponding χ^2 . Concerning the exponent α (or β), two different minimization procedures were performed :

- a) three five-parameter fits to the data on $\sigma^{pp}(E)$ were made in each parametrization to determine $a_1, a_2, a_3, a_4, \alpha$ in (10) and $b_1, b_2, b_3, b_4, \beta$ in (12). In either case, the three fits use data above $p_{\min} = 10$ GeV/c, 20 GeV/c and 40 GeV/c. Results collected in Table I show excellent χ^2 values and are very similar for all the three $p_{\min}$ values chosen;
- b) in order to see how compatible are the experimental data with other possible asymptotic growth rates, we made a series of four-parameter fits with α (or β) fixed at some given value. Thus, every value of α (or β) in (1) [or (2)] represents a specific ansatz. In this way, a continuous spectrum of asymptotic behaviours from $\sigma \rightarrow \text{const.}$ up to the Froissart bound $\sigma \rightarrow \log^2 s$ is covered by a single analytic expression (1) or (2), which has the correct analytic properties for all real values of the exponents. Some of the results are shown in Table II and show that the power of the $\log(E/E_0)$ term can be chosen rather freely between 0.8 and 2 without spoiling the fit to the data. The same can be seen, for example, by expanding the logarithmic term in (2) into a Taylor series,

$$(1 + b_2 \mathcal{L}_b^2)^\beta = 1 + \beta b_2 \mathcal{L}_b^2 + \dots \quad (17)$$

where notation (3) is used. If $|b_2 \mathcal{L}_b^2|$ is sufficiently small, one should get an equally good fit to the data by fitting the left-hand side or by fitting the truncated expansion on the right-hand side of (17). Actual numerical values prove this to be the case, since βb_2 has almost the same value for all β between $\beta = 1$ ($\beta b_2 = 0.0115$) and $\beta = 0.4$ ($\beta b_2 = 0.0110$). Thus, the left-hand side of (17) is

indistinguishable from the first two terms on the right-hand side in the energy region measured. These two expressions, on the other hand, have a completely different asymptotic behaviour for any $\beta \neq 1$.

The proton-antiproton total cross-section $\sigma^{p\bar{p}}$ is then fit by one extra parameter only, a_5 in (11) and b_5 in (13), using all the data above 6 GeV/c. One of the best fits (b parametrization with $\beta = 1$ and with $p_{\min} = 10$ GeV/c for σ^{pp}) is shown in Fig. 1.

Finally, the real part is obtained uniquely from (1) or (2) by adjusting the subtraction constant, a_6 or b_6 , respectively. We used data on ρ^{pp} above $p = 5$ GeV/c. The corresponding curves of $\rho^{pp}(E)$ and $\rho^{p\bar{p}}(E)$ are displayed in Fig. 2. It follows, in particular, that ρ^{pp} becomes positive near $E = 300$ GeV. For $E = 2000$ GeV, ρ^{pp} takes a value about 0.06 in the a parametrization and around 0.10 in the b parametrization (see Table III for details).

5. A more complete analysis of the data, as well as discussion of questions of stability of our results, will be published elsewhere. We summarize now our conclusions.

- i) Both parametrizations (1) and (2) describe very well all experimental data on pp and $p\bar{p}$ scattering above $p = 10$ GeV/c and 6 GeV/c, respectively. If the pp data set is restricted to $p > 20$ GeV/c or $p > 40$ GeV/c the fits remain essentially unchanged.
- ii) The best χ^2 fits to the data suggest that σ^{pp} and $\sigma^{p\bar{p}}$ behave asymptotically like $(\log E/E_0)^\gamma$ where γ is close to 1 in the a-parametrization and close to 2 in the b parametrization. However, values of γ between 0.6 and 2 are also compatible in both parametrizations.
- iii) Experimental data above the Serpukhov energies are very incomplete. In addition, there is a gap in the data between 60 GeV and 290 GeV. This unbalanced distribution of measured points produces a bias in the sense that the high energy region has too small a weight in the χ^2 fit and is suppressed. Thus, to draw less ambiguous conclusions concerning the growth rate of total cross-sections, systematic measurements above the Serpukhov energy region are necessary.

- iv) Cosmic ray data ^{9),10)} favourize strongly a fast asymptotic growth of σ^{pp} . A systematic analysis of cosmic ray data would be desirable.
- v) Since σ^{pp} and $\sigma^{p\bar{p}}$ increase asymptotically, $\rho^{pp}(E)$ becomes positive, in both our parametrizations, around 300 GeV and, after reaching a maximum for E around 5×10^5 GeV, it tends to zero from above, with an asymptotic behaviour given by (16). This is an illustration of the general result obtained by Khuri and Kinoshita ⁶⁾.
- vi) Measurements of ρ^{pp} in the ISR energy region will yield more information about the actual behaviour of σ^{pp} at higher energy. Values of ρ^{pp} around or above 10% at $E = 2000$ GeV would favourize saturation of the Froissart bound (concerning the power only), whereas values below 6% would suggest a much lower power of the asymptotic growth.

After completion of the manuscript, we were informed by P. Kroll about his dispersion relation treatment of corrections to ρ (preprint TKP 6/73, Karlsruhe, March 1973). Although obtained by a different method, his result is compatible with our values of ρ .

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TABLE I : Results of the five-parameter fits to σ^{pp} in the a and b parametrization,
 p_{min} being the lowest laboratory momentum used for the fit.

p_{min} (GeV/c)	α	error of α	a_1 (mb)	a_2	a_3 (GeV ⁻¹)	a_4 (mb.Gev ^{1/2})	χ^2 no. of points
10	0.88	0.03	26.8	0.10	0.40	33.0	24.8/40
20	1.00	0.02	30.0	0.08	0.09	33.5	14.0/27
40	1.16	0.08	29.7	0.08	0.05	42.1	6.2/15

p_{min} (GeV/c)	2β	error of 2β	b_1 (mb)	b_2	b_3	b_4 (GeV ⁻¹)	χ^2 no. of points
10	2.06	0.76	39.6	0.011	$8 \cdot 10^{-6}$	0.016	12.2/40
20	2.09	0.36	39.6	0.012	$3 \cdot 10^{-4}$	0.015	7.8/27
40	2.11	0.25	39.7	0.013	10^{-4}	0.014	2.6/15

TABLE II : Results of the four-parameter fits (with α or 2β fixed) to σ_{pp} in the a and b parametrization.

p_{min} (GeV/c)	α	a_1 (mb)	a_2	a_3 (GeV $^{-1}$)	a_4 (mb · GeV 2)	$\frac{x^2}{no. of points}$
10	2.0	26.9	0.035	0.815	30.6	22.6/40
10	0.8	26.7	0.117	0.373	33.6	25.2/40
20	2.0	27.6	0.038	0.297	34.9	11.9/27
20	0.8	24.1	0.154	0.397	40.9	12.7/27
40	2.0	26.2	0.047	0.186	46.8	4.8/15
40	0.8	24.6	0.181	0.103	55.6	5.6/15

p_{min} (GeV/c)	2β	b_1 (mb)	b_2	b_3	b_4 (GeV $^{-1}$)	$\frac{x^2}{no. of points}$
10	2.0	39.5	0.011	0.015	0.016	12.5/40
10	0.8	39.5	0.027	0.009	0.017	13.4/40
20	2.0	39.6	0.012	10^{-13}	0.015	7.8/27
20	0.8	39.5	0.029	10^{-9}	0.016	8.6/27
40	2.0	39.7	0.013	10^{-5}	0.014	2.6/15
40	0.8	39.5	0.031	0.073	0.015	3.2/15

TABLE III : Values of $\rho^{pp}(E)$ at $E=2000$ GeV for different values of α or 2β . The lowest laboratory momentum used for the fits is $p_{\min}$, 6 GeV/c and 5 GeV/c for σ^{pp} , $\sigma^{p\bar{p}}$ and ρ , respectively.

$\alpha \backslash p_{\min}(\text{GeV}/c)$	10	20	40
2.0	0.05	0.058	0.072
1.5	0.049	0.057	0.068
1.0	0.046	0.050	0.069
0.5	0.042	0.040	0.041

$2\beta \backslash p_{\min}(\text{GeV}/c)$	10	20	40
2.0	0.094	0.100	0.106
1.5	0.090	0.094	0.100
1.0	0.079	0.084	0.091
0.5	0.059	0.061	0.069

R E F E R E N C E S

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FIGURE CAPTIONS

Figure 1 : Total pp and $p\bar{p}$ cross-sections in the b parametrization, with $\beta = 1$ and $b_5 = 39 \text{ mb GeV}^{\frac{1}{2}}$ (full lines). The dashed line corresponds to σ^{pp} with $\beta = 0.25$. Data taken from Refs. 1)-5), 11). For comparison, two cosmic ray points taken from Ref. 9) are indicated.

Figure 2 : Energy dependence of ρ^{pp} and $\rho^{p\bar{p}}$ following from the fit shown in Fig. 1, with $b_6 = 18 \text{ mb GeV}$. Data taken from Refs. 1), 12)-15).

Fig I

Fig II