PION DISTRIBUTIONS IN INELASTIC NEUTRINO INTERACTIONS

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A B S T R A C T

Using data on semi-inclusive pion electroproduction as input we calculate in the framework of the quark parton model the distribution of pions in the current fragmentation region for inelastic neutrino interactions. Results for the π^+/π^- asymmetry are presented for both charged and neutral currents. These provide stringent tests of the parton fragmentation hypothesis and the Weinberg-Salam gauge model.

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1. INTRODUCTION

The parton model gives a distinctive feature to the current fragmentation region : hadrons observed in this region of phase space should be the fragments of a single parton ^{1),2),3)}. This leads naturally to the prediction of Feynman scaling for inclusive hadron distributions in the variable z (fraction of the scattered parton's energy carried by the observed hadron in the Breit frame) and also to factorization of the x (Bjorken's scaling variable) and z dependences. Several authors ^{4),5),6)} have tried to test these predictions, specific to the parton model, by looking at inclusive pion electroproduction. Although the results are encouraging, a more striking test is the comparison of pion production in inelastic neutrino collisions with pion production in electron collisions. This comparison would provide a very clean experimental check for the factorization of the x and z dependence and thereby throw light on an important aspect of the parton model.

It is the purpose of the present paper to exploit fully our present knowledge about inclusive pion electroproduction to predict the shape and normalization of the distribution of pions produced in neutrino collisions for both charged and neutral currents. In this respect we go further than the recent analysis of Dakin and Feldman ⁵⁾ who predict the integrated π^+/π^- ratio for charged weak currents starting from electron data. We will also present expectations for the x dependence of pion multiplicities in the current fragmentation region. The π^+/π^- asymmetry is discussed for all types of neutrino induced interactions. The charged current reactions test the parton model predictions. If the model proves useful there, it will provide another severe test of the Weinberg-Salam scheme for the neutral current reactions.

Section 2 recapitulates some of the useful formulae. A fuller account can be found in Refs. 2),3) and 7). Section 3 is devoted to the presentation and discussion of our results. In Section 4 we discuss the relevance of recent experimental results of e^+e^- annihilation into hadrons, the importance of the target fragmentation region and the sensitivity of our results to the electroproduction data fits.

2. INCLUSIVE CROSS-SECTIONS

In units of G^2_{ME}/π , the inclusive cross-sections can be written as ⁷⁾:

$$\frac{d\sigma}{dx dz} (\nu + p \rightarrow \mu^- + H + \dots) = 2x d(x) D_u^H(z) \quad (1.a)$$

$$\frac{d\sigma}{dx dz} (\bar{\nu} + p \rightarrow \mu^+ + H + \dots) = \frac{2}{3}x u(x) D_d^H(z) \quad (1.b)$$

$$\begin{aligned} \frac{d\sigma}{dx dz} (\nu + p \rightarrow \nu + H + \dots) = 2x (u(x) D_u^H(z) F \\ + d(x) D_d^H(z) F') \end{aligned} \quad (1.c)$$

$$\begin{aligned} \frac{d\sigma}{dx dz} (\bar{\nu} + p \rightarrow \bar{\nu} + H + \dots) = 2x (u(x) D_u^H(z) \bar{F} \\ + d(x) D_d^H(z) \bar{F}') \end{aligned} \quad (1.d)$$

where x is the usual Bjorken scaling variable ($x = -q^2/2q.p$), z is the fraction of the parton's energy carried by the hadron in the Breit frame, (in the lab. frame $z = E_h/\nu$ with E_h the energy of the hadron and ν the energy transferred by the leptons to the hadrons). $u(x)$ and $d(x)$ denote the average number of p and n type quarks within the proton carrying a fraction x of the proton momentum in the infinite momentum frame and $D_u^H(z)$ is a function expressing the probability for a parton u to fragment into a hadron H plus anything else. To obtain the corresponding expressions for neutron targets it suffices to interchange $u(x)$ and $d(x)$ (but not D_u^H and D_d^H).

In equation (1) we have neglected contributions from antiquarks and from strange quarks. If we believe current CERN-Gargamelle data ⁸⁾, this approximation seems to be extremely good in the region $0.3 < x < 1$. For x approaching zero this approximation is not as good but the corrections are probably small for quantities of interest. In case one assumes the Weinberg-Salam theory to describe neutral currents ⁹⁾, F , F' , $\bar{F}$ and $\bar{F}'$ can be expressed in terms of the parameter $\sin^2 \theta_W$:

$$F = \frac{1}{4} - \frac{2}{3} \sin^2 \theta_W + \frac{16}{27} \sin^4 \theta_W \quad (2.a)$$

$$F' = \frac{1}{4} - \frac{1}{3} \sin^2 \theta_W + \frac{4}{27} \sin^4 \theta_W \quad (2.b)$$

$$\bar{F} = \frac{1}{12} - \frac{2}{9} \sin^2 \theta_W + \frac{16}{27} \sin^4 \theta_W \quad (2.c)$$

$$\bar{F}' = \frac{1}{12} - \frac{1}{9} \sin^2 \theta_W + \frac{4}{27} \sin^4 \theta_W \quad (2.d)$$

The general consequences following from relations (1) have been given in Ref. 7); here we want to use specific information from inclusive pion electroproduction ¹⁰⁾ to fix $D_u^{\pi^+}$ and $D_d^{\pi^+}$. This allows us to calculate the inclusive pion cross-sections completely in the current fragmentation region.

3. PREDICTIONS FOR WEAK PION PRODUCTION

The functions $u(x)$ and $d(x)$ are extracted in the following manner ¹¹⁾: since neutrino interactions show that antiquark contributions are consistent with zero ⁸⁾ in the interval 0.3 to 1 we can safely neglect them there. For $x = 0.2$ antiquarks make a contribution of the order of 10 % compared to quarks. At the same time the model suggests that if antiquarks are negligible in a certain region so will be strange quarks. This means that $u(x)$ and $d(x)$ can be extracted directly from electron data, so long as we avoid values of x close to 0, according to the equations :

$$u(x) = \frac{3}{5x} (4 F_2^{ep}(x) - F_2^{en}(x)) \quad (3.a)$$

$$d(x) = \frac{3}{5x} (4 F_2^{en}(x) - F_2^{ep}(x)) \quad (3.b)$$

We used the following forms for the electroproduction structure functions :

$$F_2^{ep}(x) = 0.645 (1-x)^3 + 1.902 (1-x)^4 - 2.343 (1-x)^5$$

taken from the x' fit of Miller et al. ¹²⁾ and

$$F_2^{ep}(x) + F_2^{en}(x) = 0.800 (1-x)^3 + 2.777 (1-x)^4 - 1.676 (1-x)^5 - 1.512 (1-x)^6$$

taken from a straight line fit to the neutron to proton ratio ¹³⁾.

The easiest way to extract $D_u^{\pi^+}(z)$ and $D_d^{\pi^+}(z)$ is to consider inclusive pion production data from Bebek et al. ¹⁰⁾. If one excludes single pion production and ρ production (which both decrease rapidly with Q^2), these data are consistent with scaling although it should be kept in mind clearly that they do not prove it. The advantage of these data is that since they were taken for a fixed value of x ($x = 0.204$), we can then use our estimates of $u(x)$ and $d(x)$. The neglect of antiquarks and of strange quarks presumably introduces an error of at most 10 % here.

The data of Bebek et al. ¹⁰⁾, concern the following quantity

$$F^\pi(x', p_\perp^2) = \frac{1}{\sigma_{tot}} \frac{1}{\pi} \frac{E}{[P_{MAX}^2 - p_\perp^2]^{1/2}} \frac{d\sigma}{dx' dp_\perp^2} (\bar{e} + p \rightarrow \bar{e} + \pi + \dots) \quad (4)$$

where

$$x' = \frac{p_{||}}{[P_{MAX}^2 - p_\perp^2]^{1/2}}$$

and all quantities refer to the current proton centre-of-mass system. Because of the smallness of $p_\perp^2$ ($< 0.02 \text{ GeV}^2$) we have neglected the difference between x' and z (given by $p_{||}/P_{MAX}$ in the centre-of-mass system). The v and Q^2 dependence is implicitly understood in (4).

To perform the necessary integration over transverse momentum, we have used the fit $e^{-B p_\perp^2}$ taking B constant equal to 6. This introduces some uncertainty because a simple exponential is not always the best fit to the data, see Ref. 14) for more details on the transverse momentum dependence. This value for B averages the values quoted by Bebek et al. ¹⁰⁾. [Because of this choice of B the functions $D_u^{\pi^+}(z)$ and $D_d^{\pi^+}(z)$ below are slightly higher than those given in Ref. 6].

The parton fragmentation functions can then be determined from the equations :

$$\begin{aligned} \frac{\pi}{Bz} F^{\pi^+}(z,0) &\simeq \frac{1}{\sigma_{tot}} \frac{d\sigma}{dz} (e^- + p \rightarrow e^- + \pi^+ + \dots) \Big|_{x=0.204} \\ &\simeq \frac{4u(x) D_u^{\pi^+}(z) + d(x) D_d^{\pi^+}(z)}{4u(x) + d(x)} \Big|_{x=0.204} \quad (5.a) \end{aligned}$$

$$\begin{aligned} \frac{\pi}{Bz} F^{\pi^-}(z,0) &\simeq \frac{1}{\sigma_{tot}} \frac{d\sigma}{dz} (e^- + p \rightarrow e^- + \pi^- + \dots) \Big|_{x=0.204} \\ &\simeq \frac{4u(x) D_d^{\pi^-}(z) + d(x) D_u^{\pi^-}(z)}{4u(x) + d(x)} \Big|_{x=0.204} \quad (5.b) \end{aligned}$$

$F^{\pi^+}(z,0)$ was fitted to $1.7 \exp(-3.15z)$ which provides a rough qualitative fit through most of the data. It does exclude the non-scaling part coming from single pion production which falls rapidly with increasing Q^2 and also the rapidly decreasing exclusive p production.

The π^- distribution being less well known than the π^+ distribution, we have taken for it $\frac{1}{2}$ times the π^+ distribution. We do not expect this approximation to be very good for z approaching zero since we anticipate that, in this region, corresponding to the so-called central region, the π^+ and π^- distributions will eventually become equal. At the same time for z approaching 1, we expect $D_u^{\pi^+} \gg D_d^{\pi^+}$. In the region we are considering however ($0.1 < z < 1.0$) this approximation describes fairly enough the experimental situation, as the contribution from the edge ($z > 0.8$) is negligible.

Having extracted $D_u^{\pi^+}(z)$ and $D_d^{\pi^+}(z)$ we can now return to the neutrino induced hadron production cross-sections. Denoting by : $d\sigma(\ell \rightarrow \ell' + H)/dx dz$ the distribution for the general reaction $\ell + N \rightarrow \ell' + H + \dots$ averaged over neutron and proton targets (in units of G_{ME}^2/π for $\ell = \nu$ or $\bar{\nu}$), we have for charged currents :

$$\frac{d\sigma(\nu \rightarrow \mu^- + \pi^\pm)}{dx dz} = x(u(x) + d(x)) D_u^{\pi^\pm}(z) \quad (6)$$

The resulting curves are shown in Fig. 1 for different fixed values of x . For z approaching zero the distribution goes like $1/z$ which leads to a logarithmic increase for pion multiplicities in the current fragmentation region. The corresponding distributions for antineutrinos can be obtained using the symmetry relation :

$$\frac{d\sigma(\nu \rightarrow \mu^- + \pi^\pm)}{dx dz} = 3 \frac{d\sigma(\bar{\nu} \rightarrow \mu^+ + \pi^\mp)}{dx dz} \quad (7)$$

Figure 2 shows the same distributions but integrated over all values of x . Also shown is the x integrated pion distribution expected from neutral currents for some typical values of the Weinberg angle :

$$\frac{d\sigma(\nu \rightarrow \nu + H)}{dx dz} = x(u(x) + d(x))(FD_u^H(z) + F'D_d^H(z)) \quad (8)$$

Figure 3 shows the corresponding distributions initiated by antineutrinos.

Of special interest are the ratios of these distributions when integrated over z

$$R(x) \equiv \frac{\pi^+}{\pi^-} \equiv \frac{\int_{0.1}^1 dz \frac{d\sigma}{dx dz} (\ell + p \rightarrow \ell' + \pi^+ + \dots)}{\int_{0.1}^1 dz \frac{d\sigma}{dx dz} (\ell + p \rightarrow \ell' + \pi^- + \dots)} \quad (9)$$

where we have cut off the lower part of the z integration since our estimate of the D functions is not reliable there. These ratios are unaffected by the uncertainties introduced by B , the transverse momentum dependence parameter, since they cancel out here.

As noted by Feynman²⁾ and Gronau, Ravndal and Zarmi³⁾, in the case of charged currents, the x dependence cancels out and the ratio (9) becomes a constant. For neutral currents this happens only in the case of isoscalar

targets. The results are shown in Fig. 4, in the case of a proton target. The behaviour near $x = 1$ reflects the fact that in electroproduction the ratio of the structure functions of neutron and proton seems to approach the value $1/4$ as $x \rightarrow 1$ implying that u quarks dominate over d quarks. This has as a consequence that all π^+/π^- ratios (except in the case of $\bar{\nu} \rightarrow \mu^+$) tend to $\int D_{\mu}^{\pi^+}(z) dz / \int D_d^{\pi^+}(z) dz$ for $x \rightarrow 1$ (for $\bar{\nu} \rightarrow \mu^+$, the π^+/π^- ratio is the inverse of this quantity).

In Table 1 we compare the $\pi^{\pm}$ yield in charged current reactions

$$\frac{\text{neutral}}{\text{charged}} \equiv \frac{\int_{0.1}^1 dz \frac{d\sigma}{dx dz} (\nu \rightarrow \nu + \pi)}{\int_{0.1}^1 dz \frac{d\sigma}{dx dz} (\nu \rightarrow \mu^- + \pi)} \quad (10)$$

As the determination of x promises to be very hard (although possible in principle) we have integrated the above asymmetry over x and present the results in Table 1 for different values of the Weinberg angle. Table 2 gives the ratio of the added π^+ and π^- contributions. In the case of isoscalar targets it was already known that this ratio coincides with the prediction made by the parton model for the ratio of total cross-sections ⁷⁾ (e.g., for neutrinos 0.46 in the case of $\sin^2 \theta_W = 0.4$). It is, however, surprising that the separate π^+ and π^- yields deviate so much from the average value (Table 2). This makes it particularly easy to test the model in this case.

Table 3 presents the π^+/π^- ratio in the case of isoscalar targets integrated over x . A very striking difference is predicted for the π^+/π^- asymmetry when one compares neutrino and antineutrino induced charged current reactions ⁵⁾. Comparing these ratios in the case of neutrino neutral currents with antineutrino neutral currents one observes a noticeable dependence on the Weinberg angle which might be useful for providing a consistency check on the Weinberg-Salam theory ^{*)}.

*) A similarly strong dependence on $\sin^2 \theta_W$ was noted by one of us when a comparison is made of the neutrino and antineutrino neutral current y distributions, see Ref. 15).

Figure 5 shows our results for the quantity

$$S(z_{\text{MIN}}) = \frac{1}{d\sigma(\ell \rightarrow \ell')/dx} \int_{z_{\text{MIN}}}^1 \frac{d\sigma}{dz dx} (\ell \rightarrow \ell' + \pi) \quad (11)$$

where the target is taken as isoscalar. In this case the x dependence cancels out and we can study the variation of this quantity as a function of z_{MIN} . For $z_{\text{MIN}} < 0.8$ we note approximately exponential decrease for increasing z_{MIN} .

4. DISCUSSION

In view of the experimental results ^{16),17)} on the cross-section for e^+e^- annihilation into hadrons it appears that our present picture of hadrons, composed of quarks and neutral gluons, is incomplete. It is then appropriate to ask how many of the results mentioned in this paper should be expected to hold in the real world. This question is of course very hard to answer. In view of the spectacular confirmation of the quark parton model ⁸⁾ by deep inelastic neutrino-nucleon scattering it certainly still seems worth while to go one step further and analyze final state hadron distributions in the framework of the quark-parton model. It might well be possible that there exist some scale breaking interactions (e.g., due to charged vector particles having small momentum in the infinite momentum frame of the proton or quarks having substructure) which are strongly suppressed (at present x values) in lepton hadron collisions but give a constant cross-section for e^+e^- annihilation into hadrons. If this turns out to be the situation, we would still expect our results to hold since one can still imagine a dominant interaction kicking one parton out of the proton.

So far as the target fragmentation region is concerned, it is more appropriate to use a standard Regge-Mueller analysis since this region does not have a very specific interpretation in the parton model. We will consider here only the case where Pomeron exchange dominates; the role played by secondary Regge poles in deep inelastic lepton-hadron scattering is not very clear especially in view of the recent data ¹³⁾ on electron-neutron and electron-proton inelastic cross-sections. While the approximation of Pomeron dominance should only be made in the region of x close to zero, nevertheless, it gives

us some intuition for a rough first order guess of the importance of the target fragmentation region.

With Pomeron dominance, we expect the target fragmentation region to be independent of the type of projectile. This leads to :

$$\frac{1}{\sigma(\ell+T \rightarrow \ell'+\dots)} \frac{d\sigma}{dz} (\ell+T \rightarrow \ell'+H+\dots) \approx \frac{1}{\sigma(m+T \rightarrow m'+\dots)} \frac{d\sigma}{dz} (m+T \rightarrow m'+H+\dots) \quad (12)$$

where ℓ, ℓ', m and m' are leptons and T stands for the target. As a consequence one has :

$$\frac{d\sigma(\ell+T \rightarrow \ell'+H+\dots)}{d\sigma(m+T \rightarrow m'+H+\dots)} \cong \frac{\sigma(\ell+T \rightarrow \ell'+\dots)}{\sigma(m+T \rightarrow m'+\dots)} \quad (13)$$

This relation becomes interesting in special cases, for it confirms the guess that, e.g., on isoscalar targets, the ratio

$$\frac{\sigma(\nu \rightarrow \nu + \pi^+) + \sigma(\nu \rightarrow \nu + \pi^-)}{\sigma(\nu \rightarrow \mu^- + \pi^+) + \sigma(\nu \rightarrow \mu^- + \pi^-)}$$

should be the same in the target fragmentation region as in the current fragmentation region. Our estimate of 0.22 for this ratio (corresponding to $\sin^2 \theta_W = 0.4$) could thus well be a valid estimate for the whole region of phase space. [A relation similar to (12) holds also in the central region in the case of Pomeron dominance]. For antineutrinos our estimate of the corresponding ratio is 0.46 (again for $\sin^2 \theta_W = 0.4$). Similar conclusions also hold for proton targets.

Recapitulating, the results we present hold strictly in the current fragmentation region, i.e., pions moving in the direction of the current in the current target centre-of-mass frame. However, as a rough first order guess, it may well be that the results presented in Table 2, referring to the average π^+ , π^- distributions, are valid over the whole region of phase space. This requires, however, an extra dynamical assumption (Pomeron dominance in the target fragmentation region) not inherent in the parton model.

In conclusion we discuss the sensitivity of our results to the electroproduction data fits. Figure 1 depends rather crucially on the assumed z dependence for F^π in Eq. (4) and also on $u(x)$ and $d(x)$. Figures 2, 3 and 5, depend only on the integrated value $\int_0^1 x [u(x) + d(x)] dx$ which is rather well known and only uncertainties in the z dependence come in. Figure 4 is sensitive to the ratio of the integrated D functions, uncertainties originating in the transverse momentum dependence cancel out here. For the charged currents the x dependence cancels out while the neutral current results are sensitive only to the ratio $d(x)/u(x)$. Tables 1, 2, and 3 depend on the x integrated values of $xu(x)$ and $xd(x)$ for proton targets only and the ratio of the integrated D functions. The tables are undoubtedly the least sensitive to future changes in the electroproduction data whereas Figure 1 for example is much more model dependent.

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FIGURE CAPTIONS

Figure 1 : π^+ and π^- distributions for charged current neutrino interactions for different values of x .

Figure 2 : π^+ and π^- distributions integrated over x for charged and neutral neutrino interactions corresponding to different values of the Weinberg angle.

Figure 3 : π^+ and π^- distributions integrated over x for charged and neutral antineutrino interactions for different values of the Weinberg angle.

Figure 4 : x dependence of the π^+/π^- asymmetry for charged and neutral currents on a proton target.

Figure 5 : z_{MIN} dependence of quantity (11).

	$\sin^2 \theta_W = 0.3$		$\sin^2 \theta_W = 0.4$		$\sin^2 \theta_W = 0.5$	
	π^+	π^-	π^+	π^-	π^+	π^-
$\frac{\sigma(\nu p \rightarrow \nu \pi x)}{\sigma(\nu p \rightarrow \mu^- \pi x)}$	0.24	0.62	0.19	0.51	0.16	0.44
$\frac{\sigma(\bar{\nu} N \rightarrow \bar{\nu} \pi x)}{\sigma(\bar{\nu} N \rightarrow \mu^+ \pi x)}$	0.16	0.54	0.13	0.45	0.11	0.39
$\frac{\sigma(\bar{\nu} p \rightarrow \bar{\nu} \pi x)}{\sigma(\bar{\nu} p \rightarrow \mu^+ \pi x)}$	0.67	0.19	0.83	0.21	1.08	0.25
$\frac{\sigma(\bar{\nu} N \rightarrow \bar{\nu} \pi x)}{\sigma(\bar{\nu} N \rightarrow \mu^+ \pi x)}$	0.75	0.27	0.91	0.29	1.17	0.33

Table 1 : Neutral current to charged current cross-section ratios for π^+ production and π^- production with selected values of $\sin^2 \theta_W$.

	$\sin^2 \theta_W = 0.3$	$\sin^2 \theta_W = 0.4$	$\sin^2 \theta_W = 0.5$
$\frac{\sigma(\nu p \rightarrow \nu \pi^+ x) + \sigma(\nu p \rightarrow \nu \pi^- x)}{\sigma(\nu p \rightarrow \mu^- \pi^+ x) + \sigma(\nu p \rightarrow \mu^- \pi^- x)}$	0.34	0.28	0.23
$\frac{\sigma(\nu N \rightarrow \nu \pi^+ x) + \sigma(\nu N \rightarrow \nu \pi^- x)}{\sigma(\nu N \rightarrow \mu^- \pi^+ x) + \sigma(\nu N \rightarrow \mu^- \pi^- x)}$	0.27	0.22	0.19
$\frac{\sigma(\bar{\nu} p \rightarrow \bar{\nu} \pi^+ x) + \sigma(\bar{\nu} p \rightarrow \bar{\nu} \pi^- x)}{\sigma(\bar{\nu} p \rightarrow \mu^+ \pi^+ x) + \sigma(\bar{\nu} p \rightarrow \mu^+ \pi^- x)}$	0.32	0.83	0.47
$\frac{\sigma(\bar{\nu} N \rightarrow \bar{\nu} \pi^+ x) + \sigma(\bar{\nu} N \rightarrow \bar{\nu} \pi^- x)}{\sigma(\bar{\nu} N \rightarrow \mu^+ \pi^+ x) + \sigma(\bar{\nu} N \rightarrow \mu^+ \pi^- x)}$	0.40	0.46	0.56

Table 2 : Neutral current to charged current cross-section ratios for π^+ or π^- production with selected values of $\sin^2 \theta_W$.

		$\sin^2 \theta_W = 0.3$	$\sin^2 \theta_W = 0.4$	$\sin^2 \theta_W = 0.5$
$\frac{\sigma[\nu(\bar{\nu}) \rightarrow \mu^+ \pi^- x]}{\sigma[\nu(\bar{\nu}) \rightarrow \mu^- \pi^+ x]}$	2.68			
$\frac{\sigma[\bar{\nu}(\nu) \rightarrow \mu^+ \pi^- x]}{\sigma[\bar{\nu}(\nu) \rightarrow \mu^- \pi^+ x]}$	0.37			
$\frac{\sigma(\nu p \rightarrow \nu \pi^+ x)}{\sigma(\nu p \rightarrow \nu \pi^- x)}$		1.05	0.99	0.97
$\frac{\sigma(\bar{\nu} N \rightarrow \bar{\nu} \pi^+ x)}{\sigma(\bar{\nu} N \rightarrow \bar{\nu} \pi^- x)}$		0.81	0.77	0.76
$\frac{\sigma(\bar{\nu} p \rightarrow \bar{\nu} \pi^+ x)}{\sigma(\bar{\nu} p \rightarrow \bar{\nu} \pi^- x)}$		1.34	1.49	1.64
$\frac{\sigma(\bar{\nu} N \rightarrow \bar{\nu} \pi^+ x)}{\sigma(\bar{\nu} N \rightarrow \bar{\nu} \pi^- x)}$		1.04	1.18	1.32

Table 3 : π^+ to π^- cross-section ratios for charged current and neutral current induced reactions.

FIG.1

FIG. 2

FIG. 3

FIG. 4

FIG. 5