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CURRENT AND CONSTITUENT QUARKS :
THEIR IMPLICATIONS FOR RESONANCE EXCITATIONS,
POLARIZED AND UNPOLARIZED INELASTIC STRUCTURE FUNCTIONS

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A B S T R A C T

The transformation between constituent and current quarks is discussed and applied to the calculation of matrix elements for nucleon to resonance transitions induced by arbitrary currents belonging to an octet. In particular, previous results on πN and γN transitions are subsumed while weak interactions ($\Delta Q = 0$ or 1) and longitudinal photon induced transitions are discussed for the first time. The implications of a non-trivial Melosh transformation upon previous calculations in quark parton models of the non-diffractive component of the deep inelastic structure functions are discussed. The magnitudes of the unpolarized structure functions are found to be unchanged but for the polarized structure functions significant corrections to previous calculations are discovered. In particular it is found that it is not necessary that the polarization asymmetry $A_{\gamma P}$ be positive in the deep inelastic region. Our approach is valid for all Q^2 , even for photoproduction, and is not restricted to deep inelastic scattering in contrast to earlier parton model calculations. The saturation of various current algebra sum rules is discussed.

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E R R A T U M

Page 16, line 7 "where $F_1^V(q^2)$ is..."

Page 17, at bottom $\mu_N = -\frac{1}{M}$ not $-\frac{1}{2M}$

Page 22 In the second equation was omitted
 $D^{223} = \frac{1}{\sqrt{3}} \Sigma^{*-}$

1. - INTRODUCTION

Resonance spectroscopy strongly suggests that the nucleon is well described as a state composed of three "constituent" quarks and that it belongs to a 56 dimensional representation of $SU(6)_W$ acting on this quark basis ¹⁾. Success has also been obtained in using quark parton models for interpretation of deep inelastic electron and neutrino scattering data, both as an "explanation" of the scale invariance phenomenon exhibited by the various structure functions measured there ²⁾⁻⁴⁾ and in relating their magnitudes ⁵⁾. Nevertheless, the nucleon appears less simple in these models requiring many quark-antiquark pairs ("sea") and some neutral glue in addition to the three "valence" quarks ^{3),5)-7)}.

The resolution between these apparently conflicting pictures is now elegantly suggested to lie in the distinction between constituent and current quarks ⁸⁾. The former are the components of the nucleon wave function under the classification symmetry $SU(6)_{W, \text{strong}}$ while the current quarks are those to which the photon or, for weak interactions, the W couple in the normal point-like fashion. The good charges (whose matrix elements do not vanish when taken between finite mass states at infinite momentum) constructed from current quarks also define an algebra $SU(6)_{W, \text{currents}}$ but the assignment of the nucleon in this basis is unknown until a link between the strong and current bases is obtained by assuming the existence of an explicit unitary transformation between them. In the absence of such a connection it is only possible to get various bounds such as for the virtual photoabsorption cross-sections in the scaling region ⁷⁾

$$\frac{1}{4} \leq \frac{\sigma^{\gamma N}}{\sigma^{\gamma p}} \leq 4 \quad (1.1)$$

which appears to be satisfied by the data ⁹⁾.

If the difference between constituent and current quarks were neglected, corresponding to the transformation being 1, and the nucleon had been in a $\underline{56}$ plet in the current quark basis (i.e., a quark structure ppn for proton, nnp for neutron) then exact ratios among differing structure functions would be predicted. For absorption cross-sections defined for

the electromagnetic current $\sigma^{\delta P}$, $\sigma^{\delta N}$ or the vector part of the $\Delta Q = 1$ weak current ^{*)} σ^{W^+P} , σ^{W^+N} (as measured with a ν beam) then in the scaling region ^{5),6),10)}

$$\sigma^{\delta P} : \sigma^{\delta N} : \sigma^{W^+P} : \sigma^{W^+N} = 1 : 2/3 : 1 : 2 \quad (1.2)$$

Further, using the spin information in the $SU(6)$ $\underline{56}$ plet wave function, with its concomitant neglect of all internal orbital angular momentum, then ^{6),10)}

$$A^{\delta P} = 5/9, \quad A^{\delta N} = 0, \quad A^{W^+P} = -1/3, \quad A^{W^+N} = 2/3 \quad (1.3)$$

where $A \equiv (\sigma_{1/2} - \sigma_{3/2}) / (\sigma_{1/2} + \sigma_{3/2})$ is the polarization asymmetry for transverse currents, $\sigma_{1/2}$, $\sigma_{3/2}$ are the absorption cross-sections for total spin component along the current direction of $1/2, 3/2$.

This picture has the virtue of making definite predictions and consequently has been used to constrain the non-diffractive (ND) component of deep inelastic structure functions. In simple quark parton models, following the ideas of two component duality ¹¹⁾, a diffractive contribution has then straightforwardly been obtained by adding a term representing interactions with the sea of $q\bar{q}$ pairs ⁵⁾. In these models the ratios (1.2) and (1.3) tend to be true at threshold $\omega \equiv 2M\nu/Q^2 \rightarrow 1$, while for $\omega \rightarrow \infty$ the ratios (1.2) all uniformly go to 1 and the asymmetries (1.3) go to zero. However, experimental data ⁹⁾, in which the lower bound (1.1) is approached as $\omega \rightarrow 1$, while confirming the existence of a substantial non-diffractive component, also show the deficiencies of these naive models.

It is clear that it would now be an improvement to take into account the existence of a non-trivial transformation ¹²⁾ between $SU(6)_{W, \text{strong}}$ and $SU(6)_{W, \text{currents}}$ and so include the crucial difference between constituent and current quarks. There has been little discussion in the literature ¹³⁾ which takes account of this distinction and the resultant modification of (1.2) and (1.3). It is our intention here to discuss both spin averaged and spin dependent inelastic scattering of currents on nucleons. We work in the constituent basis where the $SU(6)_W$ classification of the nucleon is by hypothesis known and we then use the most general form

*) We neglect the $\Delta S = 1$ part throughout.

for the external current consistent with it belonging to a 35 plet under $SU(6)_W$. The resulting algebraic form for the current interaction with the constituent quark is then identical with that obtained with the explicit Melosh transformation for free quarks ¹²⁾. The procedure is akin to the methods of Close, Gilman and Karliner ¹⁰⁾ in that within our model we calculate amplitudes for electromagnetic or weak currents to excite the nucleon to resonance states in any arbitrary 56 plet or 70 plet and then sum over these states to obtain predictions for the non-diffractive part of the inelastic current absorption cross-section.

In Section 2 we discuss the algebraic form of the electromagnetic or weak currents that are implied by a non-trivial transformation relating constituent and current quarks. We exhibit the transition amplitudes for both transverse and longitudinal currents between a nucleon and any excited 56 or 70 state classified under $SU(6) \otimes O(3)$. These include matrix elements for the $\Delta Q = 1$ weak currents and hence, via PCAC, resonance pionic decay amplitudes ¹⁴⁾. Also presented are the matrix elements for transverse and longitudinal electromagnetic transitions, the former including the results of Refs. 15), 16). To allow for conceivable extensions to neutral and also strangeness changing weak processes we tabulate results for arbitrary currents belonging to an octet.

In Section 3 the 56 and 70 states are summed over to obtain the relative contributions to the total cross-sections, or inelastic structure functions, from any 56 or 70 plet representation. We examine the constraints that arise when one demands that no exotics ^{*)} are exchanged in the t channel of the forward current-nucleon scattering amplitude, and the simplifications necessary to obtain previous results are considered.

Finally in Section 4 and also the conclusion we discuss what constraints current algebra sum rules might place on the various contributions and the possibility of learning about patterns of $SU(6)$ and exchange degeneracy breaking at large Q^2 . It is shown in particular how the Drell-Hearn-Gerasimov ¹⁷⁾ sum rule for real photons requires a negative asymmetry $A^{\gamma P}$ in contrast to the idealized result (1.3) which, within our framework, entails the definite presence of the additional terms in the electromagnetic current induced by the transformation from current to constituent quarks.

*) i.e., only 1 and 35 representations in the t channel.

2. - TRANSFORMATION TO THE CONSTITUENT BASIS

Here we consider the algebraic properties of the electromagnetic and weak currents entailed by the transformation to a constituent quark basis, i.e., the transition operators are decomposed into representations of the constituent $[SU(6)_W \times O(2)_{L_Z}]_{\text{strong}}$, the collinear classification symmetry for resonance states moving along the z direction. The explicit form of the Melosh transformation¹²⁾ is generated by an operator with the transformation properties under $SU(6)_W \otimes O(2)_{L_Z}$ of ^{*})

$$\{\underline{35}; W=1, W_z = \pm 1, L_z = \mp 1\}; \{SU(3), \underline{8}\}; Y=C=+ \quad (2.1)$$

and we derive the algebraic form of the current operators in the constituent basis by combining the known transformation representation in the current basis with the representation (2.1) as many times as necessary to get additional inequivalent extra contributions. Further, we restrict our attention to interactions of the current with a single constituent quark or equivalently assume that the current belongs to a $\{SU(3), \underline{8}\}$ part of a $\underline{35}$ plet. This assumption is in accord with results from the explicit free quark transformation¹²⁾ although should perhaps be relaxed in the future¹⁸⁾. This framework then allows us to write down the most general form of the currents under $[SU(6)_W \otimes O(2)_{L_Z}]_{\text{strong}}$.

For transverse currents with $\Delta J_z = 1$ then between collinear states with momentum along the z direction the possible transformation properties are

$$\begin{aligned} J_T^+ \sim & \{\underline{35}; W=0, W_z=0, L_z=1\} + \{\underline{35}; W=1, W_z=1, L_z=0\} \\ & + \{\underline{35}; W=1, W_z=0, L_z=1\} + \{\underline{35}; W=1, W_z=-1, L_z=2\} \end{aligned} \quad (2.2)$$

in arbitrary combinations. If the transformation is neglected only the second term is present. For purposes of calculation it is convenient to express (2.2) in terms of operators acting on the single constituent quark variables

^{*}) Y denotes the parity under reflection in the x - z plane. Use of Y allows the same consequences as parity but leaves the collinear z direction invariant.

$$J_T^+ = (a v^+ + i b \sigma^+ + i c v^+ \sigma^0 + i d v^+ v^+ \sigma^-) J \quad (2.3)$$

factoring into a spin-orbital part, in parentheses, and a $SU(3)$ octet operator J . Here v^+ is the $+$ component of a vector operator, as defined by commutation with L_z ^{*)}, and $\sigma^\pm, \sigma^0$ are the quark spin operators, decomposed according to commutation with $W_z = S_z$. Explicitly we assume

$$J_T^\pm = \frac{1}{\sqrt{2}} (\mp J_x - i J_y) \quad (2.4)$$

and then take

$$v^\pm = \frac{1}{\sqrt{2}} (\mp v_x - i v_y), \quad \sigma^\pm = \frac{1}{2} (\sigma_x \pm i \sigma_y), \quad \sigma^0 = \sigma_z \quad (2.5)$$

so that it follows that a, b, c, d are $SU(6)_W$ singlet operators which commute with L_z . In explicit quark models these depend on the masses and other quark dynamical variables ¹⁹⁾²⁰⁾, but until results following from the general algebraic structure (2.2) or (2.3) are confirmed it is not worth pursuing detailed attempts to determine these operators.

Under reflection for a vector current

$$\gamma J_T^\pm \gamma^{-1} = -J_T^\mp \quad (2.6)$$

and taking

$$\gamma v^\pm \gamma^{-1} = -v^\mp, \quad \gamma \sigma^\pm \gamma^{-1} = -\sigma^\mp, \quad \gamma \sigma^0 \gamma^{-1} = -\sigma^0 \quad (2.7)$$

with a, b, c, d scalars under Y we obtain

$$J_T^- = (a v^- + i b \sigma^- - i c v^- \sigma^0 + i d v^- v^- \sigma^+) J \quad (2.8)$$

*) The v^+ in the separate terms of (2.3) are not necessarily the same operator, of course.

Taking the Hermitian conjugate

$$(\mathcal{J}_T^+)^{\dagger} = -\bar{\mathcal{J}}_T^- , \text{ for } \mathcal{J}^{\dagger} = \bar{\mathcal{J}} \quad (2.9)$$

so that with $(v^+)^{\dagger} = -v^-$, by comparing (2.3) and (2.8), it is clear that a, b, c, d are Hermitian. Of course, for an axial transverse current the over-all sign in (2.8) is reversed, implying that a, b, c, d are each anti-Hermitian.

The transverse current $\mathcal{J}_T(\pm)$ is a bad operator in the sense that its matrix elements are not invariant under boosts along the z direction. Usually the transformation between current and constituent quarks is expected to be well defined when applied to good operators. There is some motivation in that these operators when expressed in terms of the quark fields have the same form for an interacting theory as for the free case ^{8),12)} and so the transformation may be abstracted from the free case where it has been explicitly constructed. Since we are not using any particular form of the transformation, save for the algebraic properties discussed above, we may hopefully circumvent the requirements of direct calculation of its action and employ the bad operators $\mathcal{J}_T(\pm)$ in the form (2.3) or (2.8) without further trouble. For real photons, $Q^2 = 0$, another approach is to attempt to relate $\mathcal{J}_T(\pm)$ to good operators by current conservation ^{12),15),16)} but this procedure is not adequate for our purposes where we also discuss $Q^2 \neq 0$.

These difficulties do not arise for the good longitudinal currents. Furthermore the number of independent terms allowed generally is reduced to two by reflection invariance under Y . We then find that for a vector current

$$\mathcal{J}_L \sim \left\{ \underline{3S}; W=0, W_z=0, L_z=0 \right\} + \left[\left\{ \underline{3S}; W=1, W_z=-1, L_z=1 \right\} - \left\{ \underline{3S}; W=1, W_z=1, L_z=-1 \right\} \right] \quad (2.10)$$

or

$$\mathcal{J}_L \sim \left(s + it \frac{1}{\sqrt{2}} (\underline{\sigma} \times \underline{v})_z \right) \mathcal{J} = \left(s + it (\sigma^- v^+ + \sigma^+ v^-) \right) \mathcal{J} \quad (2.11)$$

On the other hand for an axial current

$$\tilde{J}_L \sim \{3S; W=1, W_z=0, L_z=0\} + \left[\{3S; W=1, W_z=1, L_z=-1\} + \{3S; W=1, W_z=-1, L_z=1\} \right] \quad (2.12)$$

or

$$\tilde{J}_L = \left(\tilde{s} \sigma_z - \tilde{t} \frac{1}{\sqrt{2}} \sigma_x \cdot \hat{v} \right) J = \left[\tilde{s} \sigma^0 + \tilde{t} (-\sigma^+ v^- + \sigma^- v^+) \right] J \quad (2.13)$$

The expressions (2.11) and (2.13) have been constructed to satisfy

$$Y J_L Y^{-1} = J_L, \quad Y \tilde{J}_L Y^{-1} = -\tilde{J}_L, \quad J_L^\dagger = \overline{J}_L, \quad \tilde{J}_L^\dagger = \tilde{\overline{J}}_L \quad (2.14)$$

with $s, t, \tilde{s}, \tilde{t}$ Hermitian scalars under Y .

In both (2.10) and (2.12), or (2.11) and (2.13), the second term is generated from the first by the assumption of a non-trivial transformation. The form (2.12) or (2.13), with J the isovector part of an octet, is that which by PCAC describes the pion vertex and which was employed in Refs. 14). Although somewhat similar to Lipkin¹⁹⁾ we believe that the general discussion above of the transformation properties of these currents between collinear states is substantially new.

Employing the algebraic structures (2.3), (2.11) and (2.13) it is a tedious, though straightforward, task to calculate the various helicity amplitudes for excitation of any resonance. We define these for collinear initial baryon B and final resonance R states as

$$\begin{aligned} A_{3/2} &= \langle R^{3/2} | J_T^+ | B^{1/2} \rangle, \quad A_{1/2} = \langle R^{1/2} | J_T^+ | B^{-1/2} \rangle = \gamma_R (-)^{J_R - 1/2} \langle R^{-1/2} | J_T^- | B^{1/2} \rangle, \\ A_L &= \langle R^{1/2} | J_L | B^{1/2} \rangle = \gamma_R (-)^{J_R - 1/2} \langle R^{-1/2} | J_L | B^{-1/2} \rangle, \\ \tilde{A}_L &= \langle R^{1/2} | \tilde{J}_L | B^{1/2} \rangle = -\gamma_R (-)^{J_R - 1/2} \langle R^{-1/2} | \tilde{J}_L | B^{-1/2} \rangle \end{aligned} \quad (2.15)$$

where η_R is the parity and J_R the spin of the resonance R . To utilize the knowledge of the transformation properties of the various currents requires the detailed $[SU(6)_W \otimes O(2)_{L_Z}]_{\text{strong}}$ classification of the particular resonance. In order to relate states with differing values of L_Z we use the standard $SU(6) \otimes O(3)$ wave functions with given angular momentum L , and a consequent well-defined parity throughout the $SU(6) \otimes O(3)$ multiplet. The details are sketched in an Appendix.

Transition matrix elements for transverse octet currents to excite members of $\underline{56}$ and $\underline{70}$ plet representations for arbitrary L are given in Table I, including the specialized cases of electromagnetic and weak $\Delta Q = 1$ currents for proton or neutron targets. This table also covers transverse axial currents although the detailed coefficients will, of course, be different. The corresponding matrix elements for longitudinal vector and axial currents are given in Tables II and III. For the special cases of $L = 1$ $\underline{70}$ plets and $L = 2$ $\underline{56}$ plets, of most practical interest, the results for transverse photons and for pions (longitudinal axial) reduce to those of Refs. 14)-16), save for matters of sign convention which are discussed in the Appendix. The generalizations to arbitrary L , and for arbitrary octet currents, using a consistent uniform sign convention in all cases, are new. It is simple from the tables to obtain predictions for strangeness changing or arbitrary neutral octet currents.

3. - NON-DIFFRACTIVE CONTRIBUTION TO INELASTIC STRUCTURE FUNCTIONS

The transition matrix elements listed in Tables I-III contain the full implications of $[SU(6)_W \otimes O(2)_{L_Z}]_{\text{strong}}$ for resonance excitations by electromagnetic or weak currents acting on a nucleon. For each and every $\underline{56}$ ($\underline{70}$) plet, and particular orbital angular momentum L , the coefficients A, B, C, D and $\alpha, \beta, \tilde{\alpha}, \tilde{\beta}$ (A', B', C', D' and $\alpha', \beta', \tilde{\alpha}', \tilde{\beta}'$) will differ, depending on the details of the dynamics. With explicit models, such as the harmonic oscillator quark model, then some of the coefficients can be calculated²⁰⁾ in terms of a few parameters, for example the quark mass. Such models contain further restrictive assumptions and do not at present reproduce the full generality of the structures (2.3), (2.11) and (2.13). Only if successful confrontation with data obtains for the algebraic structures used here it is worth while placing any emphasis on the construction of explicit models.

The contribution to the inelastic structure functions, for electromagnetic or weak currents, arising from any given $SU(6) \otimes O(3)$ supermultiplet, with orbital angular momentum L , can easily be obtained by summing over the individual resonance contributions, given in Tables 1 - 3, using the orthogonality relations for Clebsch-Gordan coefficients. We define

$$\begin{aligned} \sigma_{1/2} &= \sum A_{1/2}^2, \quad \sigma_{3/2} = \sum A_{3/2}^2, \quad \sigma_T = \sigma_{1/2} + \sigma_{3/2}, \\ \sigma_L &= \sum A_L^2, \quad \tilde{\sigma}_L = \sum \tilde{A}_L^2, \quad \sigma_{TL} = \sum A_L A_{1/2}, \end{aligned} \quad (3.1)$$

and the results for $J = J_{e.m.}$ and $J_{wk.}$ for proton and neutron targets are given in Tables 4 and 5.

We are assuming $SU(6) \otimes O(3)$ to be an exact classification symmetry and that each supermultiplet contributes at some definite mass. Each $\underline{56}$ or $\underline{70}$ plet contribution to the cross-section defined in (3.1) has separately a common form, independent of L , and so the total contributions for all $\underline{56}$'s, or separately all $\underline{70}$'s, weighted appropriately by their masses are, for electromagnetic currents, given by ^{*})

$$\begin{aligned} \sigma_T^{\gamma P} &= 2A^2 + \frac{17}{9}(2C^2 + B^2 + D^2) & ; & \quad \sigma_T^{\gamma N} = \frac{4}{3}(2C^2 + B^2 + D^2) \\ \sigma_{1/2}^{\gamma P} - \sigma_{3/2}^{\gamma P} &= -4AC + \frac{5}{9}(B^2 - D^2) & ; & \quad \sigma_{1/2}^{\gamma N} - \sigma_{3/2}^{\gamma N} = 0 \\ \sigma_L^{\gamma P} &= \alpha^2 + \frac{17}{9}\beta^2 & ; & \quad \sigma_L^{\gamma N} = \frac{4}{3}\beta^2 \\ \sigma_{TL}^{\gamma P} &= \alpha B + \beta(A - \frac{5}{9}C) & ; & \quad \sigma_{TL}^{\gamma N} = 0 \end{aligned} \quad (3.2)$$

for any combination of $\underline{56}$ plets, while for $\underline{70}$ plets

$$\begin{aligned} \sigma_T^{\gamma P} &= 4A'^2 + \frac{10}{9}(2C'^2 + B'^2 + D'^2) & ; & \quad \sigma_T^{\gamma N} = 4A'^2 + \frac{2}{3}(2C'^2 + B'^2 + D'^2) \\ \sigma_{1/2}^{\gamma P} - \sigma_{3/2}^{\gamma P} &= -\frac{8}{3}A'C' + \frac{10}{9}(B'^2 - D'^2) & ; & \quad \sigma_{1/2}^{\gamma N} - \sigma_{3/2}^{\gamma N} = 0 \\ \sigma_L^{\gamma P} &= 2\alpha'^2 + \frac{10}{9}\beta'^2 & ; & \quad \sigma_L^{\gamma N} = 2\alpha'^2 + \frac{2}{3}\beta'^2 \\ \sigma_{TL}^{\gamma P} &= \frac{2}{3}\alpha'B' + \frac{2}{3}\beta'(A' - \frac{5}{3}C') & ; & \quad \sigma_{TL}^{\gamma N} = 0. \end{aligned} \quad (3.3)$$

^{*}) Here $A^2, B^2, \dots$ are the weighted sums of the individual $A^2, B^2, \dots$ for each $\underline{56}$ plet. Similarly for $A'^2, B'^2, \dots$ and the $\underline{70}$ plets.

The analogous expressions for the weak cross-sections, keeping only the vector part of the transverse current, are for $\underline{56}$ plets

$$\begin{aligned}
 \sigma_T^{W^+P} &= \frac{8}{3}(2C^2+B^2+D^2) & ; & \quad \tilde{\sigma}_T^{W^+N} = 2A^2 + \frac{11}{3}(2C^2+B^2+D^2) \\
 \sigma_{\frac{1}{2}}^{W^+P} - \sigma_{\frac{3}{2}}^{W^+P} &= -\frac{4}{3}(B^2-D^2) & ; & \quad \sigma_{\frac{1}{2}}^{W^+N} - \sigma_{\frac{3}{2}}^{W^+N} = -\frac{20}{3}AC + \frac{7}{3}(B^2-D^2) \\
 \sigma_L^{W^+P} &= \frac{8}{3}\beta^2 & ; & \quad \sigma_L^{W^+N} = \alpha^2 + \frac{11}{3}\beta^2 \\
 \tilde{\sigma}_L^{W^+P} &= \frac{8}{3}(\tilde{\alpha}^2 + \tilde{\beta}^2) & ; & \quad \tilde{\sigma}_L^{W^+N} = \frac{11}{3}(\tilde{\alpha}^2 + \tilde{\beta}^2)
 \end{aligned} \tag{3.4}$$

and for $\underline{70}$ plets,

$$\begin{aligned}
 \sigma_T^{W^+P} &= 6A'^2 + \frac{1}{3}(2C'^2+B'^2+D'^2) & ; & \quad \sigma_T^{W^+N} = 10A'^2 + \frac{7}{3}(2C'^2+B'^2+D'^2) \\
 \sigma_{\frac{1}{2}}^{W^+P} - \sigma_{\frac{3}{2}}^{W^+P} &= 4A'C' + \frac{1}{3}(B'^2-D'^2) & ; & \quad \sigma_{\frac{1}{2}}^{W^+N} - \sigma_{\frac{3}{2}}^{W^+N} = -\frac{28}{3}A'C' + \frac{5}{3}(B'^2-D'^2) \\
 \sigma_L^{W^+P} &= 3\alpha'^2 + \frac{1}{3}\beta'^2 & ; & \quad \sigma_L^{W^+N} = 5\alpha'^2 + \frac{7}{3}\beta'^2 \\
 \tilde{\sigma}_L^{W^+P} &= \frac{1}{3}(\tilde{\alpha}'^2 + \tilde{\beta}'^2) & ; & \quad \tilde{\sigma}_L^{W^+N} = \frac{7}{3}(\tilde{\alpha}'^2 + \tilde{\beta}'^2).
 \end{aligned} \tag{3.5}$$

The most immediate result following from the above is that, independent of the relative abundances and couplings of $\underline{56}$ or $\underline{70}$'s

$$A^{\delta N} \equiv (\sigma_{\frac{1}{2}}^{\delta N} - \sigma_{\frac{3}{2}}^{\delta N}) / \sigma_T^{\delta N} = 0 ; \quad \sigma_{TL}^{\delta N} = 0, \tag{3.6}$$

since these relations are true for each $\underline{56}$ or $\underline{70}$ alone. Within our framework a non-zero value for $A^{\delta N}$, or $\sigma_{TL}^{\delta N}$ will be direct evidence for violation of the SU(6) symmetry assumption. It will be interesting to see if $A^{\delta N} = 0$ as $\omega \rightarrow 1$ or whether $A^{\delta N} \rightarrow 1$, as suggested by some models (21), (22). This is a difficult measurement but hopefully some empirical information will eventually be forthcoming.

With these two exceptions, the expressions for the cross-sections in (3.2)-(3.5) arising from $\underline{56}$ and $\underline{70}$ plet final states differ. Hence in the absence of any information on the relative importance of $\underline{56}$ and $\underline{70}$'s it is only possible to obtain bounds for the various cross-section ratios, rather than make absolute predictions. We here explore only the ratio $\sigma^{\delta N} / \sigma^{\delta P}$ but others can be similarly examined in a trivial fashion.

If only $\underline{56}$'s were present then, for either transverse or longitudinal electromagnetic currents,

$$0 \leq \frac{\sigma^{\gamma N}}{\sigma^{\gamma P}} \leq \frac{12}{17} \quad ; \quad \underline{56} \quad (3.7)$$

where the lower bound results from A or α dominant (i.e., the spin zero terms) and the upper bound obtains if B, C, D or β are dominant (i.e., the spin one terms). On the other hand, for only $\underline{70}$'s present, one finds

$$\frac{3}{5} \leq \frac{\sigma^{\gamma N}}{\sigma^{\gamma P}} \leq 1 \quad ; \quad \underline{70} \quad (3.8)$$

the upper bound here arising from the spin zero terms and the lower from the spin one parts of the current. From just SU(6) symmetry assumptions it is then only possible to assert

$$0 \leq \frac{\sigma^{\gamma N}}{\sigma^{\gamma P}} \leq 1 \quad ; \quad \underline{56} + \underline{70} \quad (3.9)$$

which is, of course, satisfied by the data.

The lower bound of zero in (3.9) is perhaps surprising in that the quark parton model is usually supposed to imply a lower limit of 1/4 for this ratio. However, the derivation of this result relies upon the incoherent impulse approximation in addition to quark quantum numbers. The possible lower limit of zero is for $\underline{56}$ plets alone, which then requires the necessity of exotic exchanges in the t channel, in particular states containing $q\bar{q}q\bar{q}$ would be present and these correspond to non-incoherent impulse contributions, see Fig. 1.

It is interesting to consider the constraints on the relative couplings of $\underline{56}$ and $\underline{70}$ if only non-exotic representations are exchanged in the t channel, i.e., the two currents in these collinear current scattering processes couple only to $\underline{1}$'s and $\underline{35}$'s of $SU(6)_W$. These requirements are just those of exchange degeneracy of the $\underline{56}$ and $\underline{70}$ contributions. From the Appendix Eqs. (A.5) and (A.6), we find

$$\sum_{J'M'B'} \left| \langle B'_{J'M'}^{56,L} | J_T^+ | B_{JM}^{56,0} \rangle \right|^2 = 3U_+ + 6E_+$$

$$\sum_{J'M'B'} \left| \langle B'_{J'M'}^{70,L} | J_T^+ | B_{JM}^{56,0} \rangle \right|^2 = 6U'_+ - 6E'_+ \quad (3.10)$$

where

$$U_+ = (\bar{B}_{JM}^{56,0})_{(\alpha a)(\beta b)(\gamma c)} (B_{JM}^{56,0})^{(\alpha' a')(\beta' b')(\gamma' c')} (\bar{J}J)_{\alpha'}^{\alpha} \left\{ (A^2 + C^2 + \frac{1}{2}B^2 + \frac{1}{2}D^2) + (2AC - \frac{1}{2}B^2 + \frac{1}{2}D^2)\sigma^0 \right\}_{\alpha'}^a ;$$

$$E_+ = (\bar{B}_{JM}^{56,0})_{(\alpha a)(\beta b)(\gamma c)} (B_{JM}^{56,0})^{(\alpha' a')(\beta' b')(\gamma' c')} \bar{J}_{\alpha'}^{\alpha} J_{\beta'}^{\beta} \left\{ (A + C\sigma^0)_{\alpha'}^a, (A + C\sigma^0)_{\beta'}^b, + B^2 \sigma_{\alpha'}^{-a}, \sigma_{\beta'}^{+b}, + D^2 \sigma_{\alpha'}^{+a}, \sigma_{\beta'}^{-b} \right\} \quad (3.11)$$

and U'_+, E'_+ are obtained for $A \rightarrow A'$, etc. The terms E_+, E'_+ by their tensor structure correspond in a quark picture to the currents landing on different quarks (Fig. 1b). Such contributions include $SU(3)$ exotic $\underline{27}^*$ and $\underline{10} + \underline{10}^*$ representations. Cancellation obviously requires $E_+ = E'_+$ which is true if and only if

$$(A \pm C)^2 = (A' \pm C')^2, \quad B^2 = B'^2, \quad D^2 = D'^2 \quad (3.12)$$

which are the exchange degeneracy constraints on the coupling of transverse vector currents to $\underline{56}$ and $\underline{70}$ states. Similarly by considering $\sigma_L, \tilde{\sigma}_L$ and σ_{TL} the following constraints obtain for elimination of t channel exotics

$$\alpha^2 = \alpha'^2, \quad \beta^2 = \beta'^2, \quad \tilde{\alpha}^2 = \tilde{\alpha}'^2, \quad \tilde{\beta}^2 = \tilde{\beta}'^2$$

$$\beta(A \pm C) = \beta'(A' \pm C'), \quad \alpha B = \alpha' B' \quad (3.13)$$

*) One cannot naively apply $SU(6)_W$ crossing as the t channel centre-of-mass amplitudes at $t = 0$ are not collinear. One can demand that no $SU(3)$ exotics are exchanged and this may be computed by demanding that the amplitudes for exotic processes like $\gamma^+ \Sigma^- \rightarrow \gamma^- \Sigma^+$ with Σ in $\underline{8}$ or $\underline{10}$ of the $\underline{56}$ vanish in all helicity modes. The resulting constraints are found to be identical to those obtained here by removing $q\bar{q}q\bar{q}$ t channel states.

These results, (3.12) and (3.13), inserted into (3.2)-(3.5) now give for the total $\underline{56}$ and $\underline{70}$ contributions to the cross-sections, with no t channel exotics :

$$\sigma^{\gamma P} : \sigma^{\gamma N} : \sigma^{W^+ P} : \sigma^{W^+ N} = 1 : 2/3 : 1 : 2 \quad (3.14)$$

for either transverse or longitudinal vector currents. These are precisely the results found in the old simplistic quark-parton models, when the transformation between current and constituent quarks was neglected, Eq. (1.2). Hence the inclusion of a non-trivial transformation does not affect these results.

The results for the polarization asymmetries are, however, more interesting. On summing over both $\underline{56}$ and $\underline{70}$'s, with their relative magnitude fixed by the exchange degeneracy constraint, then

$$\begin{aligned} A^{\gamma P} &= \frac{5}{9} X, & A^{\gamma N} &= 0 \\ A^{W^+ P} &= -\frac{1}{3} X, & A^{W^+ N} &= \frac{2}{3} X \end{aligned} \quad (3.15)$$

where

$$X \equiv \frac{-4AC + B^2 - D^2}{2(A^2 + C^2) + B^2 + D^2}, \quad -1 \leq X \leq 1. \quad (3.16)$$

These are therefore generalizations of the old results (1.3) when effectively only the term B was kept and so $X = 1$ ^{*)}. Unfortunately the asymmetries $A^{W^+ P}$, $A^{W^+ N}$ are unlikely to be measured in the near future so as to test (3.16) viz. that for each and every ω we predict

$$A^{\gamma N} : A^{\gamma P} : A^{W^+ P} : A^{W^+ N} = 0 : 5 : -3 : 2 \quad (3.17)$$

independent of X . The extremes $X = 1$ in (3.16) come from $A = -C$ and $D = 0$ while $X = -1$ comes from $B = 0$ and $A = C$. It is clear that, unlike the naïve quark parton models, we are unable even to predict the sign of the polarization asymmetries although the magnitudes are bounded.

^{*)} See also Appendix B for a derivation of these results in the quark-parton model with incoherent impulse.

If the results (3.14) and (3.15)-(3.17) were violated, as is known to be the case for $\sigma^{\gamma N} / \sigma^{\gamma P}$ near $\omega \sim 1$, then knowledge of both $\sigma^{\gamma N} / \sigma^{\gamma P}$ and $A^{\gamma N}, A^{\gamma P}$ would allow opportunity of examining where the assumptions made have broken down. This will most probably be through a violation of the basic $SU(6)$ symmetry (for instance, the nucleon may have a component of its wave function in a $\underline{70}$ plet), or, perhaps less likely, there will be a breakdown of the no t channel exotics constraint. In that case, for example, if $A^{\gamma P} \rightarrow 1$, then it would require dominance of $\underline{70}$ plets in the transverse current cross-sections, with $A' = C' = D' = 0$, and so $\sigma^{\gamma N} / \sigma^{\gamma P} = 3/5$.

4. - CURRENT ALGEBRA AND SUM RULES

It is of considerable interest to understand the constraints and implications of the various current algebra sum rules when they are saturated with resonance states belonging to $\underline{56}$ and $\underline{70}$ representations of $SU(6)$, for arbitrary L , and the resonance current excitation matrix elements have the forms displayed in Tables 1-3. Here we make a partial attempt in this direction, restricting our attention only to those sum rules involving forward scattering amplitudes, so that collinear $SU(6)_W \otimes O(2)_{L_z}$ may sensibly be applied in simultaneously classifying both the initial and final baryon states. Also, unlike most of the discussion in the previous section, we confine ourselves to currents with $Q^2 = 0$, for which there may be a greater validity¹⁵⁾ for our assumptions on the transformation properties of J_T^+ in (2.2).

First we consider the generalized Adler-Weisberger sum rules obtained from the commutator of two axial charges

$$[F_i^5, F_j^5] = i f_{ijk} F_k \quad (4.1)$$

We may determine the matrix elements of the left-hand side of (4.1) between any states of the ground state $L = 0$ $\underline{56}$, summed over any intermediate $\underline{56}$ or $\underline{70}$ resonance multiplets, with arbitrary L , by using a slight extension of the results of the Appendix in Eqs. (A.5) and (A.6), where F_i^5 is represented by $\tilde{J}_{Li}$ [i.e., $J = 1/2$ λ_i in (2.15)]. The results are

$$\sum_{J_M' B'} \langle B_{J_M}^{56,0} | \tilde{J}_{L,i} | B_{J_M'}^{56,L} \rangle \langle B_{J_M'}^{56,L} | \tilde{J}_{L,j} | B_{J_M}^{56,0} \rangle - i \leftrightarrow j$$

$$= (\tilde{\alpha}^2 + \tilde{\beta}^2) i f_{ijk} V_k \quad (4.2)$$

where

$$V_k = 3 \left(\bar{B}_{J_M}^{56,0} \right)_{(\alpha a)(\beta b)(\gamma c)} \left(B_{J_M}^{56,0} \right)^{(\alpha' a')(\beta b)(\gamma c)} \left(\frac{1}{2} \lambda_k \right)_{\alpha'}^{\alpha} \delta_a^a \quad (4.3)$$

and for a $\underline{70}$ there is instead on the right-hand side of (4.2) a factor $2(\tilde{\alpha}'^2 + \tilde{\beta}'^2)$. Now in fact V_K is the matrix element of the vector charge F_K between any members of the ground state $L = 0$ $\underline{56}$ plet, so that the generalized Adler-Weisberger sum rules read

$$\sum_{\underline{56}} (\tilde{\alpha}^2 + \tilde{\beta}^2) + 2 \sum_{\underline{70}} (\tilde{\alpha}'^2 + \tilde{\beta}'^2) = 1. \quad (4.4)$$

This is certainly a strong constraint on the magnitudes of $\tilde{\alpha}, \tilde{\beta}$ or $\tilde{\alpha}', \tilde{\beta}'$ for any excited $\underline{56}$ or $\underline{70}$ super-multiplet. If the crudest possible approximation of including only the lowest $L = 0$ ground state $\underline{56}$ plet is made, for which $\tilde{\beta} = 0$, then (4.4) requires $\tilde{\alpha}^2 = 1$. From Table 3, $\tilde{A}_L^{W+N} = 5/3 \tilde{\alpha}$, so the old (bad) $SU(6)$ result of $G_A/G_V = 5/3$ is recovered. A non-trivial Melosh transformation requires the presence of the additional terms in (4.4) so necessarily $G_A/G_V < 5/3$. The virtue of this generalized form of the sum rule, as written in (4.2) and (4.3), is that it makes it clear that the super-convergence relations associated with the Adler-Weisberger sum rule, such as taking (4.1) between octet and decuplet states or taking the difference of (4.1) between $M = 3/2$ and $M = 1/2$ decuplet states, are automatically satisfied within each set of $\underline{56}$ or $\underline{70}$ resonance states. This was previously known in the absence of the explicit Melosh transformation, but the present argument is an extension in that it demonstrates that the existence of the $\tilde{\beta}, \tilde{\beta}'$ terms does not affect the result.

The commutator of the vector charges corresponding to (4.1) is trivially satisfied within each $SU(3)$ multiplet, in our approximation of perfect $SU(3)$ symmetry. For the zero momentum transfer sum rules involving the transverse vector current, we have, as yet, not been able to formulate

a general discussion similar to that above for the Adler-Weisberger sum rules. Consequently we confine ourselves to just considering isovector or isoscalar currents between nucleon states. The sum rules in question are the Cabibbo-Radicati ²³⁾ and the Drell-Hearn-Gerasimov ¹⁷⁾ decomposed according to isospin.

The Cabibbo-Radicati sum rule reads, in our notation

$$2 F_1^{\prime V}(0) = \mu_A^V - \frac{16}{9} \mu^2 + \sum_R \frac{2}{(M_R^2 - M^2)^2} \left(\sigma_T^{W^+ N}(R) - \sigma_T^{W^+ P}(R) \right) \quad (4.5)$$

where $F_1^{\prime V}(q^2)$ is the usual Dirac isovector form factor, $F_1^V(0) = 1$,

$$F_1^{\prime V}(0) > 0 \quad \text{and} \quad \mu_A^V + \frac{1}{2M} = \mu_P - \mu_N = \frac{5}{3} \mu$$

using $\mu_P = \mu$ and $\mu_P/\mu_N = -3/2$, which follows from our main assumption that $J_T^\pm$ is in a 35plet between collinear states. Also in (4.5) we have exhibited explicitly the Δ contribution in the limit of degeneracy with the nucleon and have employed the standard SU(6) result for its M1 coupling, as required by our assumption on $J_T^\pm$.

Further the corresponding isovector Drell-Hearn sum rule reads

$$(\mu_A^V)^2 = \frac{16}{9} \mu^2 + \sum_R \frac{2}{(M_R^2 - M^2)^2} \left(\sigma_{3/2}^{W^+ N}(R) + \sigma_{3/2}^{W^+ P}(R) - \sigma_{1/2}^{W^+ N}(R) - \sigma_{1/2}^{W^+ P}(R) \right) \quad (4.6)$$

For photon-neutron scattering the resulting Drell-Hearn sum rule is immediately satisfied with our results for $\sigma_{3/2}^{\gamma N} - \sigma_{1/2}^{\gamma N}$. From (3.2) and (3.3) each 56 or 70 by itself gives zero and so the sum rule reads

$$\mu_N^2 = \frac{4}{9} \mu^2$$

where only the Δ survives to make a contribution, and this is the normal SU(6) result for μ_N . We choose just to display the Drell-Hearn sum rule for isoscalar photons

$$(\mu_A^S)^2 = \sum_R \frac{4}{(M_R^2 - M^2)^2} \left(\sigma_{3/2}^{\gamma_S P}(R) - \sigma_{1/2}^{\gamma_S P}(R) \right) \quad (4.7)$$

where

$$\mu_A^S + \frac{1}{2M} = \mu_P + \mu_N = \frac{1}{3}\mu \quad \text{and} \quad \sigma_{S^P}^{\chi} = \sigma_{S^N}^{\chi} = \sigma^{\chi P} + \sigma^{\chi N} - \frac{1}{2}(\sigma^{W^+P} + \sigma^{W^+N})$$

may be easily obtained from our results.

Hence, from (4.5), (4.6) and (4.7) we find

$$2F_1'(0) = (\mu_A^V)^2 - \frac{16}{9}\mu^2 + \sum_{S_6} \frac{2}{(M_{S_6}^2 - M^2)^2} \left\{ 2(A^2 + C^2) + B^2 + D^2 \right\} \\ + \sum_{7_0} \frac{4}{(M_{7_0}^2 - M^2)^2} \left\{ 2(A'^2 + C'^2) + B'^2 + D'^2 \right\} \quad (4.8a)$$

$$(\mu_A^V)^2 = \frac{16}{9}\mu^2 + \sum_{S_6} \frac{2}{(M_{S_6}^2 - M^2)^2} \left(\frac{20}{3}AC - B^2 + D^2 \right) \\ + \sum_{7_0} \frac{4}{(M_{7_0}^2 - M^2)^2} \left(\frac{8}{3}A'C' - B'^2 + D'^2 \right) \quad (4.8b)$$

$$(\mu_A^S)^2 = \sum_{S_6} \frac{2}{(M_{S_6}^2 - M^2)^2} \left(\frac{4}{3}AC - \frac{1}{9}(B^2 - D^2) \right) - \sum_{7_0} \frac{4}{(M_{7_0}^2 - M^2)^2} \frac{1}{9}(B'^2 - D'^2) \quad (4.8c)$$

The crudest zero-order approximation to (4.8b) and (4.8c) in which all higher resonant states are neglected, requires

$$(\mu_A^V)^2 = \frac{16}{9}\mu^2 \quad ; \quad (\mu_A^S)^2 = 0$$

or

$$\mu_P = \frac{3}{2M} \quad ; \quad \mu_N = -\frac{1}{2M}$$

which is already fairly close to experiment. Even with an admixture of 7_0 , $L = 1$, (where $D' = 0$), it is clear that

$$(\mu_A^S)^2 = B'^2 = 0$$

is the only consistent solution, indicating that μ_A^S and B' for $\underline{70}$, $L = 1$ are likely to remain small. In general the main conclusions to be drawn from Eqs. (4.8) are that a consistent saturation requires the necessary presence of the terms C or D (C' or D') for some excited $\underline{56}$ ($\underline{70}$) plet. Further each set of excited super-multiplets contributes positively to $F_1^V(0)$ in accord with its known positive result.

To obtain additional constraints in this framework requires a discussion of the sum rules for decuplet targets, otherwise we will not have used all available information. A systematic investigation along these lines would certainly be worth while and will hopefully be returned to in the future.

5. - CONCLUSION

We have discussed the transformation properties of the Melosh transformation under $SU(6)_W \otimes O(2)_{L_z}$ and then applied this in computing the matrix elements for transitions between nucleons and resonance states in $\underline{56}$ or $\underline{70}$ dimensional representations of $SU(6)$ induced by arbitrary currents belonging to an octet. These results contain the matrix elements for transverse electromagnetic current induced transitions and, via PCAC, resonance pionic decay amplitudes $^{14}), 15), 16)$. Our results for longitudinal electromagnetic currents and for $\Delta Q = 1$ weak currents are new. Further, the tabulation of our results for arbitrary currents in an octet enables one to straightforwardly deduce the matrix elements for neutral and also strangeness changing weak processes.

We then summed over the $\underline{56}$ and $\underline{70}$ states, assuming $SU(6)$ to be an exact symmetry, in order to obtain the relative contribution to the total cross-sections or inelastic structure functions from any $\underline{56}$ or $\underline{70}$ plet representation. Upon imposing the constraint that no t channel exotics are exchanged then for the unpolarized cross-sections we found that the consistent treatment of the difference between current and constituent quarks did not alter the results

$$\sigma^{\gamma P} : \sigma^{\gamma N} : \sigma^{W^+ P} : \sigma^{W^+ N} = 1 : 2/3 : 1 : 2 \quad (3.14)$$

found previously in naïve quark parton models. Hence the observation that as $\omega \rightarrow 1$ the ratio

$$\frac{\sigma^{\gamma N}}{\sigma^{\gamma P}} \rightarrow \frac{1}{4}$$

suggests that either t channel exotics are present in the deep inelastic region as $\omega \rightarrow 1$ or that the $SU(6)$ symmetry assumption is violated ^{3),13),22)}.

In the case of polarized neutrino and electroproduction we find the significant result that the results (1.3) of the naïve quark-parton model, which did not distinguish between current and constituent quarks, are not true in general in the presence of a non-trivial Melosh transformation. For the polarization asymmetry

$$A \equiv \frac{\sigma_{1/2} - \sigma_{3/2}}{\sigma_{1/2} + \sigma_{3/2}}$$

we found

$$A^{\gamma N} : A^{\gamma P} : A^{W^+P} : A^{W^+N} = 0 : 5 : -3 : 2 \quad (3.17)$$

exactly as in the old model, but now

$$A^{\gamma P} = \frac{5}{9} X \quad \text{etc.} \quad (3.15)$$

where X is bounded by ± 1 whereas in the absence of the Melosh transformation its value was 1.

This result is interesting for several reasons. First, in our approach we are at no point restricted to large Q^2 and in principle our results are valid even for $Q^2 = 0$. In photoproduction the Drell-Hearn-Gerasimov sum rule ¹⁷⁾ requires that the asymmetry on protons be negative over a substantial region and there is some empirical evidence to support this ²⁴⁾. For large Q^2 on the other hand a sum rule of Bjorken ²⁵⁾ suggests that $A^{\gamma P} > 0$ (if $A^{\gamma N} = 0$) and so it seems probable that the sign of the polarization asymmetry will show an interesting Q^2 variation and hence determine $X(Q^2)$.

It is expected on the basis of naïve parton models, or light cone analyses, that $X(Q^2)$ will be scale (be Q^2 independent) for large Q^2 (6),10),26). Now, in the resonance region it seems that if $X(Q^2)$ is changing sign at all as Q^2 goes from zero to space-like then it is doing so very slowly with increasing Q^2 . Contrast this with the unpolarized structure functions where, even in the resonance region, scaling obtains on the average down to very small Q^2 (10),27). It therefore seems possible that either the asymmetry remains negative in the deep inelastic region and that the Bjorken sum rule fails, or that scaling obtains only at very large Q^2 for the polarized structure functions as compared to the unpolarized case ^{*)}. There can be no doubt that measurement of the polarized structure functions is pregnant with exciting possibilities.

Finally we note that in this approach of summing over the final baryon states we at no point offer any explanation of scaling ; on the other hand we are not troubled by the failure to observe quarks in the final state, for us the final state consists of the familiar decay products of baryon states. Our results depend upon the underlying quark quantum numbers, or $SU(6)$ symmetry, and do not depend upon the presence or absence of scaling. Should a violation of scaling be observed at some value of Q^2 or ω the numerical results of this approach would still apply [modulo the $SU(6)$ assumptions] and indeed the philosophy of getting away from explicit assumptions such as the incoherent impulse approximation would be in part strengthened. In any event we have shown that some results of the quark parton model are independent of the incoherent impulse approximation.

In this approach we are unable to set an over-all scale for the structure functions unlike the naïve parton approach. However, if we appeal to the current algebra constraints, as in Section 4, then starting from the Adler sum rule for the vector currents

$$\int_1^\infty \frac{d\omega}{\omega} (F_2^{\nu N}(\omega) - F_2^{\nu P}(\omega)) = 1$$

one can invoke the results (3.14) and hence obtain the other sum rules familiar in the quark-parton model

$$\int_1^\infty \frac{d\omega}{\omega} (F_2^{\delta P}(\omega) - F_2^{\delta N}(\omega)) = 1/3$$

^{*)} A non-scaling contribution in $\sigma_{1/2}$ and $\sigma_{3/2}$ of roughly the same magnitude but of opposite sign could cause apparent early scaling in $\nu W_2 \sim \sigma_{1/2} + \sigma_{3/2}$ but show itself in $A \sim \sigma_{1/2} - \sigma_{3/2}$.

and if the vector-axial interference is maximal and negative (as predicted in the naïve model) then we obtain

$$\int_1^{\infty} \frac{d\omega}{\omega^2} \left(F_3^{\nu p}(\omega) + F_3^{\nu n}(\omega) \right) = -6$$

These few remarks high-light some of the ways that the two approaches, viz. the naïve quark-parton model and the present paper, are similar but different. For the reader enamoured of the quark-parton model with incoherent impulse at large Q^2 , we have in Appendix B shown how one can derive the asymmetries (3.15), (3.16) in such an approach while taking consistent account of the non-trivial Melosh transformation.

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APPENDIX A

This appendix outlines the $SU(6) \otimes O(3)_L$ wave functions²⁸⁾ for the $\underline{56}$ and $\underline{70}$ representations of baryon states with arbitrary orbital angular momentum L . We employ $\alpha, \beta, \gamma \dots = 1, 2, 3$, as $SU(3)$ indices, $a, b, c, \dots = 1, 2$ for $SU(2)$ spin indices which are combined $A = (\alpha a)$, $B = (\beta b), \dots$ to display the properties under $SU(6)$. Octet, $\underline{8}$, baryon states are described by a matrix

$$B_{\beta}^{\alpha} = \begin{pmatrix} \frac{1}{\sqrt{6}} \Lambda + \frac{1}{\sqrt{2}} \Sigma^0 & -\Sigma^+ & P \\ Z^- & \frac{1}{\sqrt{6}} \Lambda - \frac{1}{\sqrt{2}} \Sigma^0 & N \\ -\Xi^- & \Xi^0 & -\frac{2}{\sqrt{6}} \Lambda \end{pmatrix}$$

while decuplets, $\underline{10}$, are represented by a completely symmetric tensor $D^{\alpha\beta\gamma}$,

$$D^{111} = \Delta^{++}, D^{112} = \frac{1}{\sqrt{3}} \Delta^+, D^{122} = \frac{1}{\sqrt{3}} \Delta^0, D^{222} = \Delta^-, D^{113} = \frac{1}{\sqrt{3}} \Sigma^{*+}$$

$$D^{123} = \frac{1}{\sqrt{6}} \Sigma^{*0}, D^{133} = \frac{1}{\sqrt{3}} \Xi^{*0}, D^{233} = \frac{1}{\sqrt{3}} \Xi^{*-}, D^{333} = \Omega^-$$

Spin $1/2$, $2S+1=2$, states are given in terms of the usual spinors

$$\chi_{1/2}^a = a \uparrow \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad \chi_{1/2}^a = a \downarrow \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

while for spin $3/2$, $2S+1=4$,

$$\xi_m^a = \sum_{rs} \left\langle \frac{3}{2} m \middle| \frac{1}{2} r, 1 s \right\rangle \chi_r^a \xi_m^{(s)}, \quad \xi_m^{(\pm 1)} = \frac{1}{\sqrt{2}} (\mp \hat{x} - i \hat{y}), \quad \xi_m^{(0)} = \hat{z}$$

$$(\underline{\sigma})_b^a \cdot \xi_m^b = 0,$$

with $\underline{\sigma}$ the conventional Pauli matrices. We utilize $\epsilon^{\alpha\beta\gamma} = \epsilon_{\alpha\beta\gamma}$ as the totally antisymmetric $SU(3)$ singlet tensor and $C^{ab} = C_{ab}$ for the corresponding tensor in $SU(2)$, having the properties

$$C^{12} = -C^{21} = 1, (\underline{\sigma} C)^{ab} = (\underline{\sigma} C)^{ba}, (\underline{\sigma} C)^{ab*} = -(\underline{\sigma} C)_{ab}, C^{ab} C_{bc} = -\delta_c^a$$

With this notation the completely symmetric $\underline{56}$ plet, for total angular momentum J and z component M , is decomposed into $\underline{28}$ and $\underline{40}$ representations under $SU(3) \otimes SU(2)_S$ according to

$$\begin{aligned}
 & |B_{JM}^{56,L} \rangle^{(\alpha a)(\beta b)(\gamma c)} \\
 &= \frac{1}{3\sqrt{2}} \sum_{l_3, m} \left\{ \epsilon^{\alpha\beta\sigma} B_{\sigma}^{\gamma} C^{ab} \chi_m^c + \epsilon^{\beta\delta\sigma} B_{\sigma}^{\alpha} C^{bc} \chi_m^a + \epsilon^{\gamma\delta\sigma} B_{\sigma}^{\beta} C^{ca} \chi_m^b \right\} \\
 &\quad * |L l_3 \rangle \langle JM | L l_3, \frac{1}{2} m \rangle \\
 &+ \frac{1}{\sqrt{2}} \sum_{l_3, m} D^{\alpha\beta\sigma} (\sigma C)^{ab} \chi_m^c |L l_3 \rangle \langle JM | L l_3, \frac{3}{2} m \rangle
 \end{aligned} \tag{A.1}$$

Here $|L l_3 \rangle$ are the orbital angular momentum L , z component l_3 , states of the three-quark system of the appropriate symmetry. For the mixed symmetry 70 plot the decomposition into $\underline{2}_8$, $\underline{4}_8$, $\underline{2}_{10}$ and $\underline{2}_1$ representations is, respectively

$$\begin{aligned}
 & |B_{JM}^{70,L} \rangle^{[(\alpha a)(\beta b)](\gamma c)} \\
 &= \frac{1}{\sqrt{6}} \sum_{l_3, m} \left\{ \epsilon^{\beta\delta\sigma} B_{\frac{1}{2}\sigma}^{\alpha} C^{bc} \chi_m^a - \epsilon^{\alpha\delta\sigma} B_{\frac{1}{2}\sigma}^{\beta} C^{ac} \chi_m^b \right\} |L l_3 \rangle \langle JM | L l_3, \frac{1}{2} m \rangle \\
 &+ \frac{1}{2} \sum_{l_3, m} \epsilon^{\alpha\beta\sigma} B_{\frac{3}{2}\sigma}^{\gamma} (\sigma C)^{ab} \chi_m^c |L l_3 \rangle \langle JM | L l_3, \frac{3}{2} m \rangle \\
 &+ \frac{1}{\sqrt{2}} \sum_{l_3, m} D^{\alpha\beta\sigma} C^{ab} \chi_m^c |L l_3 \rangle \langle JM | L l_3, \frac{1}{2} m \rangle \\
 &+ \frac{1}{6} \sum_{l_3, m} \epsilon^{\alpha\beta\sigma} Y (C^{ac} \chi_m^b + C^{bc} \chi_m^a) |L l_3 \rangle \langle JM | L l_3, \frac{1}{2} m \rangle
 \end{aligned} \tag{A.2}$$

where Y represents the $SU(3)$ singlet baryon. In principle there are ambiguities of sign in relating quark spin $S = 3/2$ to $S = 1/2$ states, decuplets to octets and singlets, and also in the choice of LS or SL coupling ; we consistently use the signs in (A.1) and (A.2). For observable cross-sections the sign conventions are of course immaterial, but they are crucial at intermediary stages of calculation. The wave functions (A.1) and (A.2) are normalized so that

$$\begin{aligned}
 & \langle B_{J'M'}^{56,L} |_{ABC} | B_{JM}^{56,L} \rangle^{ABC} = \delta_{JJ'} \delta_{M'M} \left\{ \text{Tr}(\bar{B}B) + \bar{D}_{\alpha\beta\sigma} D^{\alpha\beta\sigma} \right\}, \\
 & \langle B_{J'M'}^{70,L} |_{[AB]C} | B_{JM}^{70,L} \rangle^{[AB]C} = \delta_{JJ'} \delta_{M'M} \left\{ \text{Tr}(\bar{B}_{\frac{1}{2}} B_{\frac{1}{2}}) + \text{Tr}(\bar{B}_{\frac{3}{2}} B_{\frac{3}{2}}) \right. \\
 &\quad \left. + \bar{D}_{\alpha\beta\sigma} D^{\alpha\beta\sigma} + \bar{Y}Y \right\}
 \end{aligned} \tag{A.3}$$

The completeness properties are

$$\sum_{JM} |B_{JM}^{S_6, L}\rangle \langle B_{JM}^{S_6, L}|_{A'B'C'} = \sum_{l_3} |L l_3\rangle \langle L l_3| * \frac{1}{6} \left\{ \delta_{A'}^A \delta_{B'}^B \delta_{C'}^C + \delta_{A'}^B \delta_{B'}^C \delta_{C'}^A + \delta_{A'}^C \delta_{B'}^A \delta_{C'}^B + \delta_{A'}^B \delta_{B'}^A \delta_{C'}^C + \delta_{A'}^C \delta_{B'}^B \delta_{C'}^A + \delta_{A'}^A \delta_{B'}^C \delta_{C'}^B \right\}$$

$$\sum_{JM, \frac{1}{2}, \frac{1}{2}} |B_{JM}^{70, L}\rangle \langle B_{JM}^{70, L}|_{[AB]C'}^{[AB]C} = \sum_{l_3} |L l_3\rangle \langle L l_3| * \frac{1}{6} \left\{ 2\delta_{A'}^A \delta_{B'}^B \delta_{C'}^C - \delta_{A'}^B \delta_{B'}^C \delta_{C'}^A - \delta_{A'}^C \delta_{B'}^A \delta_{C'}^B - 2\delta_{A'}^B \delta_{B'}^A \delta_{C'}^C + \delta_{A'}^C \delta_{B'}^B \delta_{C'}^A + \delta_{A'}^A \delta_{B'}^C \delta_{C'}^B \right\} \quad (A.4)$$

The usefulness of (A.4) for us is that it may be employed to obtain

$$\sum_{J'M'B'D'} \left| \langle B_{J'M'}^{S_6, L} |_{A'BC} J_A^{A'} | B_{JM}^{S_6, 0} \rangle^{ABC} \right|^2 = \frac{1}{3} U + \frac{2}{3} E$$

$$\sum_{J'M'B'_1 B'_2 D'Y'} \left| \langle B_{J'M'}^{70, L} |_{[A'B]C} J_A^{A'} | B_{JM}^{S_6, 0} \rangle^{ABC} \right|^2 = \frac{1}{2} U - \frac{1}{2} E \quad (A.5)$$

where

$$U = \sum_{l_3} (\bar{B}_{JM}^{S_6, 0})_{ABC} (B_{JM}^{S_6, 0})^{A'BC} (J_{l_3}^* J_{l_3})_{A'}^A$$

$$E = \sum_{l_3} (\bar{B}_{JM}^{S_6, 0})_{ABC} (B_{JM}^{S_6, 0})^{A'B'C} (J_{l_3}^*)_{A'}^A (J_{l_3})_{B'}^B \quad (A.6)$$

where $(B_{JM}^{56,0})^{ABC}$ represents just the ground state $SU(6)$ 56 wave function and

$$(J_{l_3}^B)_{B'}^B = \langle L l_3 | J_{B'}^B | 0 \rangle$$

$|0\rangle$ being the $L = 0$ ground state orbital wave function. The term E in (A.5) contains the contributions of the only exotic t channel 405 representation (and in a quark picture corresponds to the currents coupling to different quarks as in Fig. 1b).

To calculate the couplings of the $L = 0$ octet baryons to excited 56 and 70 states under the action of a current $J^{(\alpha a)}(\beta b) = J^\alpha_\beta j^a_b$, belonging to a 35, the main step is to evaluate

$$\begin{aligned} & \langle B_{JM}^{56,L} | J_{ABC}^A | B_{J'M'}^{56,0} \rangle^{A'BC} \\ &= \sum_{l_3 m} \langle JM | L l_3, \frac{1}{2} m \rangle \frac{1}{9} \langle L l_3 | \left\{ \bar{\chi}_m j \chi_m, (\text{Tr}(\bar{B} B J) + 5 \text{Tr}(B \bar{B} J)) \right. \\ & \quad \left. - j_a^a \bar{\chi}_m \chi_m, (2 \text{Tr}(\bar{B} B J) + \text{Tr}(B \bar{B} J)) \right\} | 0 \rangle \end{aligned}$$

$$+ \sum_{l_3 m} \langle JM | L l_3, \frac{3}{2} m \rangle \frac{1}{3} \langle L l_3 | \left\{ \bar{\chi}_m \cdot j \chi_m, \bar{D}_{\alpha\beta\gamma} J_\alpha^a, \epsilon^{\alpha'\beta\gamma} B_{\sigma}^{\alpha'} \right\} | 0 \rangle \quad (A.7)$$

and

$$\begin{aligned} & + \dots \\ & \langle B_{JM}^{70,L} | J_{[AB]C}^A | B_{J'M'}^{56,0} \rangle^{A'BC} = \sum_{l_3 m} \langle JM | L l_3, \frac{1}{2} m \rangle \frac{1}{6\sqrt{3}} \langle L l_3 | \left\{ \bar{\chi}_m j \chi_m, (J_\alpha^a (\bar{B}_{\frac{1}{2}} B) \right. \\ & \quad \left. - \text{Tr}(\bar{B}_{\frac{1}{2}} B J) + 4 \text{Tr}(B \bar{B}_{\frac{1}{2}} J) \right) - j_a^a \bar{\chi}_m \chi_m, (2 J_\alpha^a \text{Tr}(\bar{B}_{\frac{1}{2}} B) - 2 \text{Tr}(\bar{B}_{\frac{1}{2}} B J) - \text{Tr}(B \bar{B}_{\frac{1}{2}} J)) \right\} | 0 \rangle \\ & + \sum_{l_3 m} \langle JM | L l_3, \frac{3}{2} m \rangle \frac{1}{6\sqrt{2}} \langle L l_3 | \left\{ \bar{\chi}_m \cdot j \chi_m, \left\{ 2 J_\alpha^a \text{Tr}(\bar{B}_{\frac{1}{2}} B) - 2 \text{Tr}(\bar{B}_{\frac{1}{2}} B J) - \text{Tr}(B \bar{B}_{\frac{1}{2}} J) \right\} \right\} | 0 \rangle \\ & + \sum_{l_3 m} \langle JM | L l_3, \frac{1}{2} m \rangle \frac{1}{6} \langle L l_3 | (2 j_a^a \bar{\chi}_m \chi_m - \bar{\chi}_m j \chi_m) \bar{D}_{\alpha\beta\gamma} J_\alpha^a, \epsilon^{\alpha'\beta\gamma} B_{\sigma}^{\alpha'} | 0 \rangle \quad (A.8) \\ & + \sum_{l_3 m} \langle JM | L l_3, \frac{1}{2} m \rangle \frac{1}{2\sqrt{2}} \langle L l_3 | \bar{\chi}_m j \chi_m, \bar{Y} \text{Tr}(JB) | 0 \rangle + \dots \end{aligned}$$

where the discarded terms are irrelevant for initial octet states.

For the transverse current from (2.3) we have

$$(j_T^+)_b^a = a v^+ \delta_b^a + i b (\sigma^+)_b^a + i c v^+ (\sigma^0)_b^a + i d v^{++} (\sigma^-)_b^a$$

with $\sigma^\pm, \sigma^0$ represented by the usual spin matrices and it is now easy to obtain the results of Table I where

$$\begin{aligned} A &= \frac{1}{3} \langle L_1 | a v^+ | 0 \rangle = \frac{n_L}{3} \langle L-1 | a v^- | 0 \rangle, \\ B &= \frac{i}{3} \langle L_0 | b | 0 \rangle = n_L \frac{i}{3} \langle L_0 | b | 0 \rangle, \\ C &= \frac{i}{3} \langle L_1 | c v^+ | 0 \rangle = n_L \frac{i}{3} \langle L-1 | c v^- | 0 \rangle, \\ D &= \frac{i}{3} \langle L_2 | d v^+ v^+ | 0 \rangle = n_L \frac{i}{3} \langle L-2 | d v^- v^- | 0 \rangle. \end{aligned} \tag{A.9}$$

We have used in (A.9)

$$Y | L l_3 \rangle = n_L (-)^{l_3} | L, -l_3 \rangle$$

n_L is the orbital normality, the parity being $(-)^{L} n_L$. For A', B', C', D' the results are identical save that the constant of proportionality in (A.9) is $1/(2\sqrt{3})$ instead of $1/3$. States with $n_L = -$, when $B = 0$, have not as yet been identified among low-lying resonances, but there is no general reason for their non-existence in highly excited SU(6) multiplets.

For the longitudinal vector and axial currents then from (2.13) and (2.15)

$$\begin{aligned} (j_L)_b^a &= s \delta_b^a + i t (\sigma^+ v^- + \sigma^- v^+) _b^a \\ (\tilde{j}_L)_b^a &= \tilde{s} (\sigma^0)_b^a + \tilde{t} (-\sigma^+ v^- + \sigma^- v^+) _b^a \end{aligned}$$

so that

$$\begin{aligned}
 \alpha &= \frac{1}{3} \langle L_0 | s | 0 \rangle, \quad \tilde{\alpha} = \frac{1}{3} \langle L_0 | \hat{s} | 0 \rangle, \\
 \beta &= \frac{i}{3} \langle L_1 | t_{v^+} | 0 \rangle = n_L \frac{i}{3} \langle L_{-1} | t_{v^-} | 0 \rangle, \\
 \tilde{\beta} &= \frac{1}{3} \langle L_1 | \tilde{t}_{v^+} | 0 \rangle = n_L \frac{1}{3} \langle L_{-1} | \tilde{t}_{v^-} | 0 \rangle
 \end{aligned}
 \tag{A.10}$$

where for $n_L = -$, $\alpha = \tilde{\alpha} = 0$ and in the same way from (2.18) $\alpha, \tilde{\alpha}$,
 $\beta, \tilde{\beta}$ can be shown to be real. For $\alpha', \tilde{\alpha}', \beta', \tilde{\beta}'$ as above
 $1/3 \rightarrow 1/(2\sqrt{3})$.

APPENDIX B

To clarify the possibly surprising result that the polarization asymmetry for virtual photon-proton scattering could be negative we present the following simplified argument. From the proton's $SU(6)$ wave function the probabilities to find p or n quarks with spin along $\uparrow$ or opposite $\downarrow$ to the proton spin, taken along its direction of motion are

$$P_{p\uparrow} = \frac{5}{9}, \quad P_{p\downarrow} = \frac{1}{9}, \quad P_{n\uparrow} = \frac{1}{9}, \quad P_{n\downarrow} = \frac{2}{9}$$

The cross-sections for virtual photon scattering on quarks with spins parallel, antiparallel to the photon spin are $Q^2 y, Q^2 z$ where Q is the quark charge, and from Section 2

$$y = 3(A+C)^2 + 3D^2, \quad z = 3(A-C)^2 + 3B^2$$

Hence the cross-section for virtual photon spin parallel to the proton spin is

$$\sigma_{\frac{3}{2}}^{\gamma P} = 3 \left(\frac{5}{9} \cdot \frac{4}{9} + \frac{1}{9} \cdot \frac{1}{9} \right) y + 3 \left(\frac{1}{9} \cdot \frac{4}{9} + \frac{2}{9} \cdot \frac{1}{9} \right) z = \frac{7}{9} y + \frac{2}{9} z$$

the over-all factor 3 measuring the three quarks. Similarly

$$\sigma_{\frac{1}{2}}^{\gamma P} = \frac{7}{9} z + \frac{2}{9} y$$

so that $\sigma_T^{\gamma P} = y+z$ and

$$A^{\gamma P} = \frac{5}{9} \frac{z-y}{z+y}$$

This calculation ignores interference effects between different quarks but these are conventionally supposed absent in the deep inelastic scaling region.

In naïve quark models, without the Melosh transformation,
 $y = 0$ leading to $A \gamma^P = 5/9$.

A similar discussion for σ^{γ^N} leads directly to
 $\sigma^{\gamma^N} / \sigma^{\gamma^P} = 2/3$ and $A \gamma^N = 0$.

TABLE 1 : Transition matrix elements for transverse currents from initial baryon octet states to arbitrary states in $\underline{56}$ or $\underline{70}$ plet representations of $SU(6)$. $A_{\frac{3}{2}}$, $A_{\frac{1}{2}}$ are defined in (2.15), and $f = \text{tr}([B, \bar{B}]J)$, $d = \text{tr}(\{B, \bar{B}\}J)$, $\Delta = -\bar{D}_{\alpha\beta\gamma} J^\alpha_{\alpha'} \epsilon^{\alpha'\beta\sigma} B^\gamma_\sigma$, $x = \text{tr}(JB)$ are the $SU(3)$ invariant couplings between two octets, between an octet and a decuplet, and between an octet and a singlet under the action of an octet current J . If $n_L = (-)^L \eta_R = -$ then $B=0$. For $J_{e.m.} = \frac{1}{2}(\lambda_3 + 1/\sqrt{3} \lambda_8)$ on a proton $f=1$, $d=\frac{1}{3}$, $\Delta=-1/\sqrt{3}$, $x=0$ and on a neutron $f=0$, $d=-\frac{2}{3}$, $\Delta=-1/\sqrt{3}$, $x=0$. For $J_{wk} = \frac{1}{2}(\lambda_1 + i\lambda_2)$ on a proton $f=d=0$, $\Delta=1$, $x=0$ and on a neutron $f=d=1$, $\Delta=1/\sqrt{3}$, $x=0$.

$SU(6); {}^{2S+1}[SU(3)]$		
$\underline{56} ; {}^2_8$	$A_{\frac{3}{2}}$ $A_{\frac{1}{2}}$	$A \langle J_{\frac{3}{2}} L1, \frac{1}{2}\frac{1}{2} \rangle f + (C \langle J_{\frac{3}{2}} L1, \frac{1}{2}\frac{1}{2} \rangle + D \langle J_{\frac{3}{2}} L2, \frac{1}{2}-\frac{1}{2} \rangle) \frac{1}{3} (2f+3d)$ $A \langle J_{\frac{1}{2}} L1, \frac{1}{2}-\frac{1}{2} \rangle f + (B \langle J_{\frac{1}{2}} L0, \frac{1}{2}\frac{1}{2} \rangle - C \langle J_{\frac{1}{2}} L1, \frac{1}{2}-\frac{1}{2} \rangle) \frac{1}{3} (2f+3d)$
$\underline{56} ; {}^4_{10}$	$A_{\frac{3}{2}}$ $A_{\frac{1}{2}}$	$\sqrt{2/3} (\sqrt{3} B \langle J_{\frac{3}{2}} L0, \frac{3}{2}\frac{3}{2} \rangle - 2C \langle J_{\frac{3}{2}} L1, \frac{3}{2}\frac{1}{2} \rangle - D \langle J_{\frac{3}{2}} L2, \frac{3}{2}-\frac{1}{2} \rangle) \Delta$ $\sqrt{2/3} (B \langle J_{\frac{1}{2}} L0, \frac{3}{2}\frac{1}{2} \rangle - 2C \langle J_{\frac{1}{2}} L1, \frac{3}{2}-\frac{1}{2} \rangle - \sqrt{3} D \langle J_{\frac{1}{2}} L2, \frac{3}{2}-\frac{3}{2} \rangle) \Delta$
$\underline{70} ; {}^2_8$	$A_{\frac{3}{2}}$ $A_{\frac{1}{2}}$	$A' \langle J_{\frac{3}{2}} L1, \frac{1}{2}\frac{1}{2} \rangle \frac{1}{2} (f+3d) + (C' \langle J_{\frac{3}{2}} L1, \frac{1}{2}\frac{1}{2} \rangle + D' \langle J_{\frac{3}{2}} L2, \frac{1}{2}-\frac{1}{2} \rangle) \frac{1}{6} (5f+3d)$ $A' \langle J_{\frac{1}{2}} L1, \frac{1}{2}-\frac{1}{2} \rangle \frac{1}{2} (f+3d) + (B' \langle J_{\frac{1}{2}} J0, \frac{1}{2}\frac{1}{2} \rangle - C' \langle J_{\frac{1}{2}} L1, \frac{1}{2}-\frac{1}{2} \rangle) \frac{1}{6} (5f+3d)$
$\underline{70} ; {}^4_8$	$A_{\frac{3}{2}}$ $A_{\frac{1}{2}}$	$(\sqrt{3} B' \langle J_{\frac{3}{2}} L0, \frac{3}{2}\frac{3}{2} \rangle - 2C' \langle J_{\frac{3}{2}} L1, \frac{3}{2}\frac{1}{2} \rangle - D' \langle J_{\frac{3}{2}} L2, \frac{3}{2}-\frac{1}{2} \rangle) \frac{1}{6} (-f+3d)$ $(B' \langle J_{\frac{1}{2}} L0, \frac{3}{2}\frac{1}{2} \rangle - 2C' \langle J_{\frac{1}{2}} L1, \frac{3}{2}-\frac{1}{2} \rangle - \sqrt{3} D' \langle J_{\frac{1}{2}} L2, \frac{3}{2}-\frac{3}{2} \rangle) \frac{1}{6} (-f+3d)$
$\underline{70} ; {}^2_{10}$	$A_{\frac{3}{2}}$ $A_{\frac{1}{2}}$	$-\sqrt{3} ((A' - \frac{1}{3}C') \langle J_{\frac{3}{2}} L1, \frac{1}{2}\frac{1}{2} \rangle - \frac{1}{3}D' \langle J_{\frac{3}{2}} L2, \frac{1}{2}-\frac{1}{2} \rangle) \Delta$ $-\sqrt{3} ((A' + \frac{1}{3}C') \langle J_{\frac{1}{2}} L1, \frac{1}{2}-\frac{1}{2} \rangle - \frac{1}{3}B' \langle J_{\frac{1}{2}} L0, \frac{1}{2}\frac{1}{2} \rangle) \Delta$
$\underline{70} ; {}^2_1$	$A_{\frac{3}{2}}$ $A_{\frac{1}{2}}$	$\sqrt{\frac{3}{2}} ((A' + C') \langle J_{\frac{3}{2}} L1, \frac{1}{2}\frac{1}{2} \rangle + D' \langle J_{\frac{3}{2}} L2, \frac{1}{2}-\frac{1}{2} \rangle) x$ $\sqrt{\frac{3}{2}} ((A' - C') \langle J_{\frac{1}{2}} L1, \frac{1}{2}-\frac{1}{2} \rangle + B' \langle J_{\frac{1}{2}} L0, \frac{1}{2}\frac{1}{2} \rangle) x$

TABLE 2 : Transition matrix elements for longitudinal vector currents from initial baryon octet states to arbitrary states in $\underline{56}$ or $\underline{70}$ plet representation of $SU(6)$. A_L is defined in (2.15), otherwise see the caption to Table 1, $n_L = (-)^L \eta_R$ and if $n_L = -$, $\alpha = 0$.

$SU(6); {}^{2S+1}[SU(3)]$	A_L
$\underline{56} ; {}^2_8$	$\alpha \langle J_{\frac{1}{2}} L0, \frac{1}{2} \frac{1}{2} \rangle f + \beta \langle J_{\frac{1}{2}} L1, \frac{1}{2} -\frac{1}{2} \rangle \frac{1}{3} (2f+3d)$
$\underline{56} ; {}^4_{10}$	$-\sqrt{2/3} \beta (\langle J_{\frac{1}{2}} L1, \frac{3}{2} -\frac{1}{2} \rangle - n_L \sqrt{3} \langle J_{\frac{1}{2}} L-1, \frac{3}{2} \frac{3}{2} \rangle) \Delta$
$\underline{70} ; {}^2_8$	$\alpha' \langle J_{\frac{1}{2}} L0, \frac{1}{2} \frac{1}{2} \rangle \frac{1}{2} (f+3d) + \beta' \langle J_{\frac{1}{2}} L1, \frac{1}{2} -\frac{1}{2} \rangle \frac{1}{6} (5f+3d)$
$\underline{70} ; {}^4_8$	$-\beta' (\langle J_{\frac{1}{2}} L1, \frac{3}{2} -\frac{1}{2} \rangle - n_L \sqrt{3} \langle J_{\frac{1}{2}} L-1, \frac{3}{2} \frac{3}{2} \rangle) \frac{1}{6} (-f+3d)$
$\underline{70} ; {}^2_{10}$	$-\sqrt{3} (\alpha' \langle J_{\frac{1}{2}} L0, \frac{1}{2} \frac{1}{2} \rangle - \frac{1}{3} \beta' \langle J_{\frac{1}{2}} L1, \frac{1}{2} -\frac{1}{2} \rangle) \Delta$
$\underline{70} ; {}^2_1$	$\sqrt{\frac{3}{2}} (\alpha' \langle J_{\frac{1}{2}} L0, \frac{1}{2} \frac{1}{2} \rangle + \beta' \langle J_{\frac{1}{2}} L1, \frac{1}{2} -\frac{1}{2} \rangle) x$

TABLE 3 : Transition matrix elements for longitudinal axial currents from initial baryon octet states to arbitrary states in $\underline{56}$ or $\underline{70}$ plet representations of $SU(6)$, $\tilde{A}_L$ is defined in (2.15), otherwise see the caption to Table 1, $n_L = (-)^L \eta_R$ and if $n_L = -$, $\tilde{\alpha} = 0$.

$SU(6); {}^{2S+1}[SU(3)]$	$\tilde{A}_L$
$\underline{56} ; {}^2_8$	$(\tilde{\alpha} \langle J_{\frac{1}{2}} L0, \frac{1}{2} \frac{1}{2} \rangle + \tilde{\beta} \langle J_{\frac{1}{2}} L1, \frac{1}{2} -\frac{1}{2} \rangle) \frac{1}{3} (2f+3d)$
$\underline{56} ; {}^4_{10}$	$-\sqrt{2/3} (2\tilde{\alpha} \langle J_{\frac{1}{2}} L0, \frac{3}{2} \frac{1}{2} \rangle + \tilde{\beta} (\langle J_{\frac{1}{2}} L1, \frac{3}{2} -\frac{1}{2} \rangle + n_L \sqrt{3} \langle J_{\frac{1}{2}} L-1, \frac{3}{2} \frac{3}{2} \rangle)) \Delta$
$\underline{70} ; {}^2_8$	$(\tilde{\alpha}' \langle J_{\frac{1}{2}} L0, \frac{1}{2} \frac{1}{2} \rangle + \tilde{\beta}' \langle J_{\frac{1}{2}} L1, \frac{1}{2} -\frac{1}{2} \rangle) \frac{1}{6} (5f+3d)$
$\underline{70} ; {}^4_8$	$-(2\tilde{\alpha}' \langle J_{\frac{1}{2}} L0, \frac{3}{2} \frac{1}{2} \rangle + \tilde{\beta}' (\langle J_{\frac{1}{2}} L1, \frac{3}{2} -\frac{1}{2} \rangle + n_L \sqrt{3} \langle J_{\frac{1}{2}} L-1, \frac{3}{2} \frac{3}{2} \rangle)) \frac{1}{6} (-f+3d)$
$\underline{70} ; {}^2_{10}$	$1/\sqrt{3} (\tilde{\alpha}' \langle J_{\frac{1}{2}} L0, \frac{1}{2} \frac{1}{2} \rangle + \tilde{\beta}' \langle J_{\frac{1}{2}} L1, \frac{1}{2} -\frac{1}{2} \rangle) \Delta$
$\underline{70} ; {}^2_1$	$\sqrt{\frac{3}{2}} (\tilde{\alpha}' \langle J_{\frac{1}{2}} L0, \frac{1}{2} \frac{1}{2} \rangle + \tilde{\beta}' \langle J_{\frac{1}{2}} L1, \frac{1}{2} -\frac{1}{2} \rangle) x$

TABLE 4 : Contributions to the cross-sections, $\sigma_{\frac{1}{2}}$, $\sigma_{\frac{3}{2}}$, σ_L , σ_{TL} , defined in (3.1), for $J = J_{e.m.} = \frac{1}{2}(\lambda_3 + 1/\sqrt{3} \lambda_8)$ and for proton and neutron targets, σ^{YP} , σ^{YN} .

$SU(6); {}^{2S+1}[SU(3)]$	$\sigma_{\frac{1}{2}}^{YP}$	$\sigma_{\frac{3}{2}}^{YP}$	$\sigma_{\frac{1}{2}}^{YN}$	$\sigma_{\frac{3}{2}}^{YN}$
$\underline{56} ; \underline{2}_8$	$(A-C)^2 + B^2$	$(A+C)^2 + D^2$	$\frac{4}{9}(B^2+C^2)$	$\frac{4}{9}(C^2+D^2)$
$\underline{56} ; \underline{4}_{10}$	$\frac{2}{9}(B^2+4C^2+3D^2)$	$\frac{2}{9}(3B^2+4C^2+D^2)$	$\frac{2}{9}(B^2+4C^2+3D^2)$	$\frac{2}{9}(3B^2+4C^2+D^2)$
$\underline{70} ; \underline{2}_8$	$(A'-C')^2 + B'^2$	$(A'+C')^2 + D'^2$	$(A' - \frac{1}{3}C')^2 + \frac{1}{9}B'^2$	$(A' + \frac{1}{3}C')^2 + \frac{1}{9}D'^2$
$\underline{70} ; \underline{4}_8$	0	0	$\frac{1}{9}(B'^2+4C'^2+3D'^2)$	$\frac{1}{9}(3B'^2+4C'^2+D'^2)$
$\underline{70} ; \underline{2}_{10}$	$(A' + \frac{1}{3}C')^2 + \frac{1}{9}B'^2$	$(A' - \frac{1}{3}C')^2 + \frac{1}{9}D'^2$	$(A' + \frac{1}{3}C')^2 + \frac{1}{9}B'^2$	$(A' - \frac{1}{3}C')^2 + \frac{1}{9}D'^2$
$SU(6); {}^{2S+1}[SU(3)]$	σ_L^{YP}	σ_L^{YN}	σ_{TL}^{YP}	σ_{TL}^{YN}
$\underline{56} ; \underline{2}_8$	$\alpha^2 + \beta^2$	$\frac{4}{9}\beta^2$	$\alpha B + \beta(A-C)$	$-\frac{4}{9}\beta C$
$\underline{56} ; \underline{4}_{10}$	$\frac{8}{9}\beta^2$	$\frac{8}{9}\beta^2$	$\frac{4}{9}\beta C$	$\frac{4}{9}\beta C$
$\underline{70} ; \underline{2}_8$	$\alpha'^2 + \beta'^2$	$\alpha'^2 + \frac{1}{9}\beta'^2$	$\alpha' B' + \beta'(A'-C')$	$\frac{1}{3}\alpha' B' + \frac{1}{3}\beta'(A' - \frac{1}{3}C')$
$\underline{70} ; \underline{4}_8$	0	$\frac{4}{9}\beta'^2$	0	$\frac{2}{9}\beta' C'$
$\underline{70} ; \underline{2}_{10}$	$\alpha'^2 + \frac{1}{9}\beta'^2$	$\alpha'^2 + \frac{1}{9}\beta'^2$	$-\frac{1}{3}\alpha' B' - \frac{1}{3}\beta'(A' + \frac{1}{3}C')$	$-\frac{1}{3}\alpha' B' - \frac{1}{3}\beta'(A' + \frac{1}{3}C')$

TABLE 5 : Contributions to the cross-sections, $\sigma_{\frac{1}{2}}$, $\sigma_{\frac{3}{2}}$, σ_L , $\tilde{\sigma}_L$, defined in (3.1), for $J = J_{W^+N^k} = \frac{1}{2}(\lambda_1 + i\lambda_2)$ and for proton and neutron targets, σ^{W^+P} , $\sigma^{W^+N^k}$, $\sigma^{W^-N} = \sigma^{W^+P}$.

$SU(6); {}^{2S+1}[SU(3)]$	$\begin{matrix} W^+P \\ \sigma_{\frac{1}{2}} \end{matrix}$	$\begin{matrix} W^+P \\ \sigma_{\frac{3}{2}} \end{matrix}$	$\begin{matrix} W^+N \\ \sigma_{\frac{1}{2}} \end{matrix}$	$\begin{matrix} W^+N \\ \sigma_{\frac{3}{2}} \end{matrix}$
$\underline{56} ; \quad {}^2_8$	0	0	$(A - \frac{5}{3}C)^2 + \frac{25}{9}B^2$	$(A + \frac{5}{3}C)^2 + \frac{25}{9}D^2$
$\underline{56} ; \quad {}^4_{10}$	$\frac{2}{3}(B^2 + 4C^2 + 3D^2)$	$\frac{2}{3}(3B^2 + 4C^2 + D^2)$	$\frac{2}{9}(B^2 + 4C^2 + 3D^2)$	$\frac{2}{9}(3B^2 + 4C^2 + D^2)$
$\underline{70} ; \quad {}^2_8$	0	0	$4(A - \frac{2}{3}C')^2 + \frac{16}{9}B'^2$	$4(A + \frac{2}{3}C')^2 + \frac{16}{9}D'^2$
$\underline{70} ; \quad {}^4_8$	0	0	$\frac{1}{9}(B'^2 + 4C'^2 + 3D'^2)$	$\frac{1}{9}(3B'^2 + 4C'^2 + D'^2)$
$\underline{70} ; \quad {}^2_{10}$	$3(A' + \frac{1}{3}C')^2 + \frac{1}{3}B'^2$	$3(A' - \frac{1}{3}C')^2 + \frac{1}{3}D'^2$	$(A' + \frac{1}{3}C')^2 + \frac{1}{9}B'^2$	$(A' - \frac{1}{3}C')^2 + \frac{1}{9}D'^2$
$SU(6); {}^{2S+1}[SU(3)]$	$\begin{matrix} W^+P \\ \sigma_L \end{matrix}$	$\begin{matrix} W^+N \\ \sigma_L \end{matrix}$	$\begin{matrix} W^+P \\ \tilde{\sigma}_L \end{matrix}$	$\begin{matrix} W^+N \\ \tilde{\sigma}_L \end{matrix}$
$\underline{56} ; \quad {}^2_8$	0	$\alpha'^2 + \frac{25}{9}\beta'^2$	0	$\frac{25}{9}(\tilde{\alpha}'^2 + \tilde{\beta}'^2)$
$\underline{56} ; \quad {}^4_{10}$	$\frac{8}{3}\beta'^2$	$\frac{8}{9}\beta'^2$	$\frac{8}{3}(\tilde{\alpha}'^2 + \tilde{\beta}'^2)$	$\frac{8}{9}(\tilde{\alpha}'^2 + \tilde{\beta}'^2)$
$\underline{70} ; \quad {}^2_8$	0	$4(\alpha'^2 + \frac{4}{9}\beta'^2)$	0	$\frac{16}{9}(\tilde{\alpha}'^2 + \tilde{\beta}'^2)$
$\underline{70} ; \quad {}^4_8$	0	$\frac{4}{9}\beta'^2$	0	$\frac{4}{9}(\tilde{\alpha}'^2 + \tilde{\beta}'^2)$
$\underline{70} ; \quad {}^2_{10}$	$3\alpha'^2 + \frac{1}{3}\beta'^2$	$\alpha'^2 + \frac{1}{9}\beta'^2$	$\frac{1}{3}(\tilde{\alpha}'^2 + \tilde{\beta}'^2)$	$\frac{1}{9}(\tilde{\alpha}'^2 + \tilde{\beta}'^2)$

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FIGURE CAPTION

Figure 1a Incoherent impulse parton model diagram.

Figure 1b Example of non-incoherent impulse contribution with exotic
 $q\bar{q}q\bar{q}$ states in the t channel.

(a)

(b)

FIG 1