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AN IMPACT PARAMETER DESCRIPTION OF SINGLE AND DOUBLE
DIFFRACTION DISSOCIATION

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A B S T R A C T

An s channel impact parameter model of inelastic diffraction dissociation is constructed which describes quantitatively the momentum transfer distributions of both pion and nucleon induced reactions. It is stressed that the impact parameter profile is peripheral and is approximately the same for all helicity amplitudes. From the fits to the data, the concept of a universal limiting strength (ULS) for each amplitude is deduced and its consequences for other diffractive processes, and the approximate TCHC hypothesis are discussed. Finally, detailed predictions are presented for the momentum transfer distributions in the double diffraction process $NN \rightarrow (N\pi)(N\pi)$, and compared with a simple t channel picture of diffraction dissociation.

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1. - INTRODUCTION

Over the past few years it has been usual to describe elastic diffractive reactions in terms of a simple t channel Regge pole parametrization, assuming the existence of a leading J plane pole singularity at $J = 1$ (the Pomeron). This description has been extended in the multi-Regge or Deck models to multiparticle diffractive amplitudes by the inclusion of other factorizable t channel exchanges. The hope has been that while such models clearly cannot account in the most detailed way for resonance production they could still provide a global description of multiparticle production phenomena. However, just as in elastic scattering, it is now becoming clear that this simple t channel picture cannot account, even in a qualitative way for certain features observed in the production data.

For example, it has been pointed out ^{1),2)} that the factorization hypothesis in the multi-Regge (Deck) model predicts the wrong sign for the $d\sigma/dt$ cross-over effect in the reactions $K^0 p \rightarrow Q^0 p$, $\bar{K}^0 p \rightarrow \bar{Q}^0 p$, where Q^0 , $\bar{Q}^0$ denote the $K^* \pi$, $\bar{K}^* \pi$ final state systems. Also there is now some evidence ³⁾ to suggest these Deck models cannot reproduce the slope-mass correlation effect in the momentum transfer dependence of diffractive reactions; i.e., that $d\sigma/dt \sim \exp[A(M^*)t]$, where $A(M^*)$ decreases monotonically to some limiting value $A(\infty)$ as the mass of the diffractively produced system, M^* , increases. It had been thought that this effect was a simple kinematical consequence of the double peripherality of the five-point amplitude, i.e., that $T \propto \exp[at_1 + bu_3]$ (where the definition of the invariants is given in Fig. 1). Then, since, at the M^* threshold, t_1 and u_3 are linearly related, it can be seen that for low M^* values $T \propto \exp[(a+b)t]$. On the other hand, as M^* increases the peripherality at one vertex can no longer be simply added to the peripherality at the other vertex, and hence the apparent slope of the momentum transfer distribution decreases. However, in a recent four-dimensional analysis it was pointed out that this kinematical description is not sufficient to describe the data, essentially because the data still seem to show some M^* dependence in a even when u_3 is kept fixed.

It could be argued that the data are not sufficiently good for the above analysis to be completely safe. Also it is possible that there may be additional diagrams which explain the Q , $\bar{Q}$ cross-over effect without destroying the correct cross-over predictions in other reactions. Nevertheless it would seem more likely, considering both results together, that there is

indeed something at fault in the simple multiperipheral description of production phenomena. Moreover, it appears that the error probably lies in the factorization property of the model.

This view is supported when one considers the prediction the factorization hypothesis makes for the momentum transfer slope of double diffractive dissociation processes. Phenomenologically the asymptotic value of the single diffractive slope $A(\omega)$ is found to be approximately half the size of the slope obtained in elastic cross-sections, i.e., $A(\omega) \approx 5.0 \pm 1.0$. Hence if one took this factorization assumption seriously one would expect the slope of double diffraction processes (e.g., $pp \rightarrow N^*N^*$) would be very small - if not zero. That is to say one would predict a fairly copious production of pions with very large transverse momenta coming from the decaying N^* 's.

In this paper therefore we would like to promote an alternative s channel description of diffraction dissociation phenomena in terms of the impact parameter representation of the scattering amplitude. Our aim is to show :

- a) this approach succeeds in explaining simply the slope-mass correlation effect, in a quantitative as well as qualitative way ; and
- b) the impact parameter description makes specific predictions for the slope of the double diffraction processes which are very different from the factorizable t channel predictions.

In achieving these goals we shall argue that inelastic diffractive interactions are predominantly peripheral. Moreover, we shall present evidence to suggest that the data are compatible with two universal features :

- i) the impact parameter profile is the same for all helicity amplitudes and depends only on the particles in the initial state (i.e., on how the colliding particles "see" each other) ; and
- ii) the relative strengths of the helicity amplitudes are the same for all diffractive interactions irrespective of the colliding particles, and depend only on the mass of the dissociated system.

2. - THE MODEL FOR SINGLE DIFFRACTION DISSOCIATION

Let us start by writing the s channel helicity amplitudes for the process $A_a + N_b \rightarrow A_1 + N_2 + \pi_3$ as a sum over all possible angular momentum states of the diffractively produced $N_2 \pi_3$ system, i.e.,

$$F_{\lambda_a \lambda_b, \mu_1 \mu_2} = \sum_j \sum_{m=-j}^j \sqrt{2j+1} d_{\nu \lambda}^j(\theta_1) e^{i\nu\phi_1} t_{\lambda \nu}^j(s, t', M^*) \quad (1)$$

where the projectile A is either a proton or a meson, θ_1, ϕ_1 describe the $N_2 \pi_3$ final state decay; $\lambda = \lambda_a - \lambda_b$, $\nu = \mu_1 - m$, and $t' = t - t_{\min}$. Taking the absolute square of Eq. (1), summing over all values of j and integrating over θ_1, ϕ_1 we obtain the differential cross-section in terms of the quasi-two-body, $AN \rightarrow AN^*$ s channel helicity amplitudes $T_n(s, t', M^*)$ such that

$$\frac{d\sigma}{dt'} = \frac{1}{s^2} \sum_{n=0}^{n_{\max}} |T_n(s, t', M^*)|^2 \quad (2)$$

where the net helicity flip $n = |\lambda - \nu|$. Here we use " N^* " to represent the $N_2 \pi_3$ system of invariant mass M^* , not necessarily resonant, with angular momenta $j = 0, 1, 2$, etc. (We could, of course, obtain a similar expression for the production of any number of particles.)

Notice that since the helicity m of the N^* system is bounded by the spin of the system, such that $|m| \leq j$, the net helicity flip n is also bounded, i.e., $n \leq j + 3/2$; or, when A is a proton, if we assume that helicity is conserved at the unexcited vertex, $n \leq j + 1/2$. Hence for low M^* masses, where the spin of the excited object (the $N_2 \pi_3$ system) will be approximately the same as the spin of the initial particle there can only be a few values of n contributing to the sum in Eq. (2), typically $n = 0, 1, 2$. As the mass of the produced system increases, however, its spin is also likely to increase, and so there is a probability of obtaining significant contributions from several helicity amplitudes. We shall take it as a basic hypothesis of the model that the only M^* dependence in $T_n(s, t', M^*)$ comes from a multiplicative scale factor which determines the size of each helicity amplitude.

This assumption takes care of the M^* dependence of amplitudes. However, in order to calculate the differential cross-sections we must also know the t dependence of each of the amplitudes. We therefore make the second hypothesis that each helicity amplitude is described by the same universal impact parameter representation. That is to say, if we transform the s channel helicity amplitudes T_n in b space such that

$$H_n(s, M^*, b) = \int_0^\infty T_n(s, t', M^*) J_n(b\sqrt{-t'}) d(-t') \quad (3)$$

then $H_n(s, M^*, b)$ has the same b dependence for all values of n , i.e.,

$$H_n(s, M^*, b) = h_n(M^*) H(s, b) \quad (4)$$

where we have used our first hypothesis to factor out the M^* dependence. Assuming this universal profile in impact parameter space $H(s, b)$ is a delta function $\delta(b-b_0)$, i.e., that all scattering takes place peripherally at a radius $b = b_0$, then Eq. (3) can be immediately inverted to give

$$T_n = h_n J_n(b_0\sqrt{-t'}) \quad (5)$$

More generally for a roughly Gaussian distribution in b space again peaked around b_0 , the amplitudes have the form ^{*)}

$$T_n = h_n J_n(b_0\sqrt{-t'}) e^{at'} \quad (6)$$

where the width of the Gaussian is given approximately by $2\sqrt{a}$.

Notice that such an impact parameter profile predicts a very distinctive t' dependence for each helicity amplitude corresponding to the structure of the Bessel functions. For $n = 0$, $J_0(x) = 1$ at $x = 0$ and has a zero at $x = b_0\sqrt{-t'} \approx 2.4$; while for $n \neq 0$, $J_n(x) = O(x^n)$ near $x = 0$ and has its first zero at successively larger values of x .

^{*)} In fact Eq. (6) corresponds to a profile of the form $1/2a \exp[-(b+b_0)^2/4a] I_n(bb_0/2a)$ where I_n is the hyperbolic Bessel function ($I_n(x) = J_n(ix)$). However, since $I_n(x) \approx e^x/\sqrt{2\pi x}$ for large x this profile has the approximately Gaussian form $\exp[-(b-b_0)^2/4a]/\sqrt{4\pi bb_0 a}$ for $b > 2ab_0$. The approximation becomes better as n increases.

We can now see very easily how the slope-mass correlation of inelastic diffraction processes follows qualitatively from this simple model, i.e., from Eq. (4), and the assumption of a peripheral interaction ¹⁾. At low M^* values when only a few helicity amplitudes contribute there is a good chance that the $n = 0$ amplitude dominates. Hence we should expect the differential cross-section, $d\sigma/dt'$ to reflect this dominance by a steep t' dependence near $n = 0$ associated with a dip or break corresponding to the first zero of the J_0 function. An examination of the data (see Figs. 2,3,6) indicates that in the lowest mass bin there is indeed such a break in the momentum transfer slope near $t' \approx 0.22 \text{ GeV}^2$ suggesting a value of b_0 at about 1 Fermi. As the mass M^* increases, however, we should expect to obtain significant contributions from several helicity amplitudes, i.e., with $n = 1, 2, 3, \dots$ as well as $n = 0$. Since each of these contributions has the particular t' dependence of the $J_n(b\sqrt{-t'})$ Bessel function, their net effect will be to produce a rather structureless distribution in t' with a slope significantly less than that associated with the non-flip amplitude alone. Of course, if all values of n were allowed to contribute equally then one would eventually obtain a $d\sigma/dt'$ distribution which was flat in t' . Hence there must be some damping of the large n amplitudes.

If this simple picture of inelastic diffraction dissociation is correct it makes a number of testable predictions. For example, it seems likely that the momentum transfer distributions may flatten out, or even turn over very near to the forward direction for large produced masses. Also a consideration of the individual helicity amplitudes, through a study of the density matrix elements should reveal very significant and predictable structure.

Notice also that the model indicates the dominance of the $n = 0$ amplitude for low M^* values (which is a phenomenon of approximate s channel helicity conservation, SCHC). However, as M^* increases this gives way to a more complex situation in which several helicity amplitudes are populated. Depending on the mass and spin of the excited state, and the value of t' one is considering these amplitudes could conspire to give a result which suggests the process approximately satisfies t channel helicity conservation (TCHC). We shall investigate this possibility later. It may be that at some more fundamental level there is some dynamical mechanism which controls the size of each helicity amplitude in order to produce this approximate TCHC effect for all s, M^*, t' ⁴⁾. For the moment, however,

we shall not make any such demands on the strengths of the various helicity amplitudes, except to say that they must be sufficiently damped for large n in order to retain a limiting slope in t' .

Before discussing the fit to the data, it is worth making a few comments on the peripheral nature of the model. On fairly general grounds we might expect most scattering amplitudes to be peripheral since absorption tends to cut out the lower partial waves. This has been made a somewhat stronger statement by Harari ⁵⁾ who, using the dominance of certain peripheral resonances and duality arguments, suggested that the imaginary part of the scattering amplitude should be peripheral. While we may be able to assume that diffractive amplitudes are predominantly imaginary ^{*)}, it is clear that we cannot use duality to suggest the peripheral nature of the interaction. Moreover, it is well known that elastic diffractive scattering is not peripheral - the impact parameter profile being again roughly Gaussian in shape, but centred about $b = 0.0$, not 1.0 Fermi. Nevertheless as we shall show, the inelastic diffractive data have a very natural and satisfactory interpretation in terms of a peripheral interaction.

3. - FIT TO DATA

A) $\pi N \rightarrow \pi_1 (\pi_2 N_3)$

Having outlined the qualitative features of the model, let us now report on certain quantitative aspects. Besides the parameters b_0 and a describing the position and width of the impact parameter profile, the model also contains the functions $h_n(M^*)$ which control the strengths of the different helicity amplitudes. We take as one convenient parametrization the form

$$h_n(M^*) = 0, \quad M^* \leq M_R^{(n)} - \Delta^{(n)}$$

$$h_n(M^*) = h_n(M^{*2})^\gamma, \quad M^* \geq M_R^{(n)}$$

(7)

*) In fact we shall assume for simplicity that the amplitudes either are all imaginary, or that the real parts have the same impact parameter profile as the imaginary parts.

where $M_R^{(n)}$ and $\Delta^{(n)}$ are the mass and width of the lowest diffractively produced resonance with spin $n-1/2$, i.e.,

$$\begin{aligned} M_R^{(1)} &= 1470 \text{ MeV} , & \Delta^{(1)} &= 180 \text{ MeV} \\ M_R^{(2)} &= 1520 \text{ MeV} & \Delta^{(2)} &= 120 \text{ MeV} \\ M_R^{(3)} &= 1688 \text{ MeV} & \Delta^{(3)} &= 120 \text{ MeV} \\ M_R^{(4)} &= 2190 \text{ MeV} & \Delta^{(4)} &= 300 \text{ MeV} \end{aligned} \quad (8)$$

and $h_n(M^*)$ varies smoothly between these two limits from $M_R^{(n)} - \Delta^{(n)}$ to $M_R^{(n)}$. The factor $(M^{*2})^\gamma$ determines the over-all normalization of $d\sigma/dt'$ in each mass interval ^{*}). In this way we incorporate our basic assumption that as the mass of the dissociated system increases, the amplitudes with successively higher net helicity flip become important.

The data we consider initially come from an excellent analysis of several $\pi N \rightarrow \pi\pi N$ reactions at 8 and 16 GeV/c ⁶⁾. This analysis provides a determination of the $I = 0$ exchange contribution which shows little change ($\leq 10\%$) with the incident energy, and hence is probably mainly diffractive. As we see from Figs. 2, 3, these data show very nicely the decreasing slope in t' as the $\pi_2^{N_3}$ mass increases. Notice also that in the lowest mass region there is some evidence for dips at $t' \approx 0.25$ and $t' \approx 0.35$ in the 8 GeV/c and 16 GeV/c data, respectively. Taking the parameters of the $N^*(1470)$ resonance to be $M_R^{(1)} = 1470$ MeV, $\Delta^{(1)} = 180$ MeV, we see from the parametrization of Eq. (7) that only the $n = 0$ amplitude contributes in this low mass range. If this were true the Bessel function $J_0(b_0\sqrt{-t'})$ in Eq. (6) would give not only a dip but an absolute zero. In order to fill in this zero therefore we can either increase the size of $\Delta^{(1)}$ so that the amplitude with $n = 1$ also contributes, or alternatively we can add some small background effect, T_{BG} , where, for example

$$T_{BG} = g (M^{*2})^\delta e^{\beta t'} \quad (9)$$

^{*}) When there are prominent resonances in the mass distribution such a simple parametrization of the normalization must be modified. However, for our purposes, we find the present power behaviour quite adequate.

Taking initially this second possibility, the distributions at 8 GeV/c were fitted by varying the parameters b_0 , a , γ , h_n (as well as these background parameters g , δ , β) to minimize χ^2 for all M^* mass bins simultaneously. From our preceding remarks regarding the dip near $t' = 0.25 \text{ GeV}^2$ it is not surprising that the best fit was obtained with a value of $b_0 = 0.97 \text{ Fermi} = 4.84 \text{ GeV}^{-1}$. The values of all the parameters for this fit, which is shown in Fig. 2, are given in Table 1. Notice that the value of h_4 is very much smaller than the other coefficients ^{*)}; and in fact the individual contributions of the helicity amplitudes to the integrated cross-section, given in Table 2, are quite strongly damped as n increases. As we have already noted this behaviour is necessary if the momentum transfer distributions are to tend to some limiting distribution for large M^* , although we did not demand this damping behaviour in our fitting procedure.

It is also interesting to find that the parameter γ which determines the over-all normalization of the helicity amplitudes at each value of M^* is found to be close to -1.0 . This is just the value we would have expected from a simple double Regge parametrization with Pomeron and pion exchanges, i.e.,

$$T_n(s, t, M^*) \Big|_{t=0} \propto \left(\frac{s}{M^{*2}} \right)^{\alpha_P(0)} (M^{*2})^{\alpha_\pi(0)} \quad (10)$$

Hence, even though the t dependence is different in the s channel model, it would appear that we can still parametrize the s and M^* dependence in terms of simple exchange trajectories.

The fit to the 16 GeV/c data which we show in Fig. 3 has fairly similar values for the model parameters (see Table 1) except that the impact parameter profile parameters have now changed slightly. Nevertheless, as seen in Fig. 4, the resulting profiles are still approximately the same at the two energies. Moreover, parametrizing the distributions as

$$\frac{d\sigma}{dt'} = e^{B(M^*) t'} \quad (11)$$

^{*)} h_5 is even smaller, though the fit is not sensitive to this parameter.

in the range $0.05 \leq t' \leq 0.15$ we find from Fig. 5 that the model gives only small changes in the slope parameter $B(M^*)$ between the two energies.

It is worth mentioning that the background term, Eq. (9), used in these fits is very small, being only of the order of 7% even in the lowest M^* region at $t' = 0$. Also the large negative value of δ indicates that this term is completely negligible everywhere except in this lowest mass bin. In view of this it is tempting to suggest that this small central interaction term is a residual effect from the overlap function which describes elastic scattering as the shadow of inelastic processes. However, it is possible that there may be no central interaction at all. For example good fits to the data can also be obtained if we simply increase the values of $\Delta^{(n)}$ used in the parametrization of the dissociating system, i.e., in Eq. (7). In particular we show in Figs. 2, 3, the fit obtained by increasing $\Delta^{(n)}$ such that $h_1(M^*)$, on average, has 25% of its full value already in the first mass region, with similar increases in the size of the other $h_n(M^*)$ contributing to the higher mass regions. The interesting point of this fit is that the h_n parameters are still found to be very similar to those obtained previously, as indicated in Table 3. The only real difference in the two fits is the different shape of $d\sigma/dt'$ in the first mass region ; but in view of the errors on the data this difference is not statistically significant.

B) $NN \rightarrow N_1(N_2\pi_3)$

The data we consider for this reaction come from the Scandinavian $pp \rightarrow p\pi^+n$ experiment at 19 GeV/c¹⁾. Because of the fairly high energy of this reaction we assume that the process (at least at small t') is purely diffractive. As seen in Fig. 6, these data also display the slope mass correlation effect, and for the lowest mass bin have a pronounced dip in $d\sigma/dt'$ at $t' = 0.18$ GeV². For this nucleon induced reaction we make the extra (though reasonable) assumption that s channel helicity is conserved at the unexcited vertex. Then using the same fitting procedure as before, with and without the background term, Eq. (9), and $h_n(M^*)$ given by Eq. (7) the best fits were found with a value of $b_0 = 1.14$ Fermi. These are shown in Fig. 6 and the values of the parameters given in Table 1, 2, 3. The impact parameter profile in this case extends to somewhat higher values of b as can be seen in Fig. 4. The effect of this is to increase the size of the slope parameter $B(M^*)$ of Eq. (11) by a few units over the $\pi N \rightarrow \pi\pi N$ case as indicated in Fig. 5.

It must be stressed that the difference in $B(M^*)$ is only because of the different impact parameter profiles, while the helicity amplitudes contribute in very much the same way as previously. See Tables 1, 2, 3. Indeed as shown in Fig. 6 a very reasonable description of the $NN \rightarrow NN\pi$ data can be achieved using the same strength parameters h_n which we found from the pion induced reaction.

In view of this, and considering the uncertainties in the present data, it would seem very reasonable to suggest that the s channel inelastic diffractive amplitudes have the same universal limiting strengths (ULS) irrespective of the particles involved. If this is valid then we have a very simple picture of diffraction dissociation indeed. The only variables are :

- (a) the peripheral impact parameter profiles which depend on the particles in the initial state (i.e., how they "see" each other at any given energy) and perhaps on the number of particles produced, though by assumption not on their invariant mass ; and
- (b) the way the dissociated system approaches its limiting helicity values ; i.e., on the M^* dependence of $h_n(M^*)$, which is purely a function of the dissociating pion, kaon, or nucleon system, etc.

In order to test this picture let us now look briefly at some other diffractive reactions.

C) $\pi N \rightarrow \pi(\pi\pi N)$

In this case, since we have a πN initial state we might expect the impact parameter profile to be the same as for the $\pi N \rightarrow \pi\pi N$ reaction ^{*)}. Hence the maximum value of $B(M^*)$ should also be the same. In fact from Fig. 7 we see that $B_{\max} = 11.8 \pm 1.2 \text{ GeV}^{-2}$ for the process $\pi^+ p \rightarrow \pi_f^+ (\pi_s^+ \pi^- p)$ at $16 \text{ GeV}/c$ ⁷⁾ compared with $12.1 \pm 0.9 \text{ GeV}^{-2}$ for $\pi N \rightarrow \pi(\pi N)$ at the same energy. Also if the ULS hypothesis is true we should expect $B(M^*)$ to have the same limiting value for large M^* in the two reactions. As we can see from Fig. 5 and Fig. 7, this is quite compatible with the present data. The solid curve in Fig. 7 is a straight transposition of the model fit to the $\pi N \rightarrow \pi\pi N$ data. Notice that if we change

^{*)} Although the impact parameter profile may change with the number of particles in the final state, it seems unreasonable to believe it will change greatly just with the production of one extra pion.

the parametrization of the strengths of the helicity amplitudes $h_n(M^*)$ in the present case so that the higher helicity states do not contribute until approximately 0.2 GeV higher than before then we again obtain a very good description of the slope-mass correlation (see the broken line in Fig. 7). This change in parametrization, of course, is quite consistent with the fact that low mass resonances have a relatively smaller branching ratio into many-particle systems.

D) $\pi N \rightarrow (\pi\pi\pi)N$

So far we have discussed nucleon dissociation. As an example of pion dissociation, we also show in Fig. 7 the slope parameter $B(M^*)$ for the process $\pi^+ p \rightarrow (\pi_f^+ \pi_s^- \pi_s^+) p$ at 16 GeV/c⁷⁾. Again the maximum value of $11.3 \pm 0.5 \text{ GeV}^{-2}$ is quite consistent with the value obtained in the other πN diffractive channels. In order to describe the variation of B with the mass of the three pion system in our model we should have to replace the strength parameter $h_n(M^*)$ of Eqs. (7) and (8) by a new set defined in terms of the resonant and non-resonant effects of this three-pion system. However, without doing this ULS would suggest that $B(M^*)$ still tends to the same limit as before, since we believe the impact parameter profile is the same for all πN channels. From Fig. 7 we see that the data are in fact apparently tending to a limit somewhat higher in this reaction than in the nucleon dissociation channels. This may be because the three-pion system has not yet reached its limiting helicity behaviour. However, a comparison of the two reactions $\pi^+ p \rightarrow (\pi_f^+ \pi_s^- \pi_s^+) p$ and $\pi^- p \rightarrow (\pi_f^- \pi_s^- \pi_s^+) p$ shows that B differs by two to three units between the two reactions. Hence a more likely explanation is that there are important non-diffractive contributions to these reactions which apparently are not present in the nucleon dissociation channel. We should not therefore compare our model predictions directly to the $\pi^+ p \rightarrow (\pi^+ \pi^- \pi^+) p$ data, at least at this energy.

4. - TWO QUESTIONS

A) Is the impact parameter profile the same for all n ?

This problem is particularly important because if the model is to have a strong predictive power we should like to be able to use the same profile for all helicity amplitudes. In fact since in Eq. (6) we used the simple parametrization $e^{at'} J_n(b_0 \sqrt{-t'})$ for each amplitude there is already a slight difference in each corresponding impact parameter profile. That is to say, we have actually used the form

$$H(s, b) = \frac{1}{2a} \exp(-(b+b_0)^2/4a) I_n(bb_0/2a)$$

which has a small n dependence, rather than the Gaussian form $\exp(-(b-b_0)^2/4a)/\sqrt{4\pi bb_0 a}$. However, from Fig. 4, where the exact profiles for $n = 0, 1$ are shown, we see that there is little difference at the larger b values which control the small t' behaviour.

Aside from this slight n dependence we believe there can be little significant change in the profiles unless for some unknown reason the profile has a very strong dependence on M^* . As we have stressed, the low M^* data appear to be dominated by the $n = 0$ amplitude. The marked change of slope, and even a dip between $t' = 0.2$, and 0.3 GeV^2 is then readily explained by the peripheral profile. If we make the very reasonable assumption that progressively higher T_n amplitudes begin to contribute as M^* increases (up to some maximum value of n) then the profile for each amplitude is already roughly determined by the peripherality of T_0 .

For example if in the second mass range in Fig. 2, i.e., $1.32 \leq M^*_N < 1.56 \text{ GeV}$, we have contributions from T_0 and T_1 , then T_1 cannot be too central. If it were, we would obtain a rather flatter slope in $d\sigma/dt'$ near $t' \approx 0.2$ (say) unless T_1 was very small. But in that case the slope near $t' = 0.0$ would be given entirely by T_0 and would be as large as in the first mass region. Only if one allowed some strong variation of the impact parameter with the mass of the produced system which reduces the slope of T_0 as M^* increases could one reproduce the observed t distributions. However, there seems no reason why this should happen, and, as we have seen there is a very simple explanation of the $d\sigma/dt'$ distributions without this b - M^* dependence.

B) Is t channel helicity conserved?

In our fits to the data we have made fairly specific predictions concerning the size of the different s channel helicity amplitudes. It is therefore interesting to see if these predictions satisfy the approximate TCHC condition observed in so much of the inelastic diffraction data ⁸⁾. However, to do this it is necessary to make additional assumptions concerning the spin, or spins, of the diffractively produced system in any given M* region. For example the t channel helicity amplitudes are obtained from the s channel amplitudes by the transformation ⁹⁾

$$T_{\lambda_c \lambda_d, \lambda_a \lambda_b}^{(t)}(s, t', M^*) = \sum_{\substack{\mu_a \mu_b \\ \mu_c \mu_d}} d_{\mu_a \lambda_a}^{s_a} d_{\mu_b \lambda_b}^{s_b} d_{\mu_c \lambda_c}^{s_c} d_{\mu_d \lambda_d}^{s_d} T_{\mu_c \mu_d, \mu_a \mu_b}^{(s)}(s, t', M^*) \quad (12)$$

where χ_i are the Trueman-Wick crossing angles and the crossing matrix depends on the spins s_i of all of the particles. If TCHC is exact only those t channel amplitudes with $(\lambda_c - \lambda_a) - (\lambda_d - \lambda_b) = 0$ will be non-zero. This reduces to only one amplitude if we assume the Gribov-Morrison rule ^{7), 8)} in the case of $\pi N \rightarrow \pi N^*$, plus SCHC at the unexcited vertex in the $NN \rightarrow NN^*$ case. Then since

$$\frac{d\sigma}{dt'} \propto \sum_{\text{all } \lambda_i} |T_{\lambda_i}^{(t)}|^2 = \sum_{\text{all } \mu_i} |T_{\mu_i}^{(s)}|^2 \quad (13)$$

the ratio

$$R(s, t', M^*) = \frac{|T_{n=0}^{(t)}|^2}{\sum_{\text{all } \mu_i} |T_{\mu_i}^{(s)}|^2} \quad (14)$$

should be unity in the limit of exact TCHC. Therefore if we knew the spin (spins) of the diffractively produced πN system we could obtain an estimate of TCHC by constructing $T_{n=0}^{(t)}$ from the s channel amplitudes via Eq. (13) and calculate the ratio R given in Eq. (14).

In Fig. 8 we show the values of $R(s, t', M^*)$ for the process $\pi N \rightarrow \pi \pi N$ at 16 GeV/c using the parameters of the fit IIb, and assuming certain values of the spin s^* of the N^* system. Notice that in the low mass region the ratio is rather small for both $s^* = 1/2$ and $3/2$. This is because only the $T_{n=0}^{(s)}$ and $T_{n=1}^{(s)}$ amplitudes contribute significantly here and $T_{n=1}^{(s)} = 0$ near $t' = -0.8 \text{ GeV}^2$ leaving only the s channel conserving amplitude $T_0^{(s)}$. However, as M^* increases it is possible to find a value of s^* for which the ratio R is fairly large over a wide range of t' . In reality, of course, each mass region contains many πN states with different values of angular momentum. Therefore before concluding anything about TCHC from our model we should first have to know the way in which these different spin states occur at each value of M^* . Nevertheless if we interpret s^* as the effective spin of the system then we see that TCHC is approximately satisfied (i.e., $|T_0^{(t)}|^2$ represents $\gtrsim 75\%$ of the cross-section) provided $s^* = 3/2$ for $1.56 \leq M^* < 1.80$, $-t' < 0.4$ and for $1.80 \leq M^* < 2.28$, $-t' < 0.8$, or alternatively if $s^* = 5/2$ for $1.80 \leq M^* < 2.28$, $-t' < 0.2$.

It would clearly be interesting to have a detailed partial wave analysis of the diffractively produced πN system and a careful study of TCHC as a function of t' and M^* . In any case we would conclude from Fig. 8 that the present s channel picture of inelastic diffraction is certainly non incompatible with approximate t channel helicity conservation.

5. - DOUBLE DIFFRACTION DISSOCIATION

As we pointed out in the Introduction, the factorizability assumption of the t channel models for diffraction dissociation has serious consequences when applied to double diffractive processes, i.e., to reactions where both initial particles become excited systems in the final state. As shown diagrammatically in Fig. 9, factorization allows us to write

$$\frac{d\sigma}{dt' dM_1^* dM_2^*} = \frac{d\sigma}{dt' dM_1^*} \cdot \frac{d\sigma}{dt' dM_2^*} / \frac{d\sigma}{dt_{\text{elastic}}} \quad (15)$$

which since

$$\frac{d^2\sigma}{dt'dM_i^*} \approx e^{\frac{1}{2}B_{el}t'}, \quad M_i^* \gg (m+\mu)^2, \quad i=1,2$$

and

$$\frac{d\sigma}{dt}_{elastic} \approx e^{B_{el}t}$$

where B_{el} is the slope of the elastic diffractive peak, would lead to the result

$$\frac{d^3\sigma}{dt'dM_1^*dM_2^*} \approx \text{roughly independent of } t' \text{ for } M_i^* > (m+\mu)^2$$

On the other hand the s channel model of diffraction dissociation which we have described in the previous sections predicts a very different result for these double diffractive reactions. We have observed in single diffraction dissociation that the number of s channel helicity amplitudes must be strongly restricted, if the single diffractive slope is to tend to some non-zero limit. There appears to be no reason why this same phenomenon does not also occur in double diffractive reactions ; and in particular the ULS hypothesis implies the momentum transfer distributions should in fact have the same limiting values as the single diffraction processes.

To illustrate this we have calculated the distributions for the double diffractive reaction $NN \rightarrow (N\pi)(N\pi)$ and compared them in Fig. 10 with the factorizable t channel model given by Eq. (15). The impact parameter representation of the s channel helicity amplitudes for this case is written in the form analogous to Eq. (4), i.e.,

$$H_n(s, M_1^*, M_2^*, b) = h_{n_1}(M_1^*) h_{n_2}(M_2^*) H(s, b) \frac{h_n}{h_{n_1} h_{n_2}} \quad (16)$$

where n_1 and n_2 are now the net s channel helicity flip at each vertex such that $n_1 = |\lambda_a - \lambda_{a*}|$ and $n_2 = |\lambda_b - \lambda_{b*}|$, and as before $n = |(\lambda_a - \lambda_{a*}) - (\lambda_c - \lambda_{c*})|$. The impact parameter profile $H(s, b)$ is the same as for the single diffractive process shown in Fig. 4, and the functions

$h_{ni}(M_i^*)$ are described in Eq. (8). Thus the model for the double diffractive process is given entirely by the parameters of the single diffractive process, as is the t channel model of Eq. (15). However, in Fig. 10, we see a very big difference in the predictions of the diffractive slope in the two models at the larger values of M_1^*, M_2^* despite the large errors on the single diffraction data which we used in Eq. (15) to evaluate the double diffractive process. Parametrizing the slope in the same way as previously, i.e.,

$$\frac{d^3\sigma}{dt'dM_1^*dM_2^*} = \exp [B_D(M_1^*, M_2^*) t'] \quad 0.05 \leq t' \leq 0.15 \quad (17)$$

we find from Eq. (15) that

$$B_D(M_1^*, M_2^*) = B_s(M_1^*) + B_s(M_2^*) - B_{elastic} \quad (18)$$

compared with

$$B_D(M_1^*, M_2^*) = B_s(M_0 + \sqrt{(M_1^* - M_0)^2 + (M_2^* - M_0)^2}) \quad (19)$$

which is, to a good approximation, the behaviour we find in the present model. Here B_s is the single diffraction slope and M_0 is the πN threshold.

Even though the cross-section for double diffraction is likely to be very small we believe this difference should be clearly identifiable in any high energy, high statistics study of these reactions at NAL or, using the split field magnet at the ISR.

Fit	Process	b_0/GeV	a/GeV^2	h_0	h_1	h_2	h_3	h_4	γ	β	δ	χ^2/NDF
I _a	$\pi\bar{N} \rightarrow \pi\pi\bar{N}$ 8 GeV/c	4.84	2.03	1.0	1.375	1.650	2.736	0.181	-1.06	1.04	-5.11	0.91
I _b	$\pi\bar{N} \rightarrow \pi\pi\bar{N}$ 16 GeV/c	4.30	2.25	1.0	1.450	2.035	2.478	0.203	-1.11	1.13	-3.59	1.41
I _c	$NN \rightarrow NN\pi$ 19 GeV/c	5.68	2.86	1.0	1.240	1.480	1.981	0.132	-0.96	0.94	-2.7	1.05

Table 1 : Parameters of fits to $\pi\bar{N}$ and NN single diffractive processes, assuming some background term given by Eq. (9). In each case h_0 has been normalized to unity.

$\int T_n ^2 dt$					
Fit	n = 0	1	2	3	4
I _a	1.0	0.678	0.230	0.182	0.007
I _b	1.0	0.675	0.295	0.180	0.008
I _c	1.0	0.560	0.210	0.135	0.006

Table 2 : The integrated contributions from the different helicity amplitudes given by the parameters in Table 1. In each case the n = 0 contribution is normalized to 0.1.

Fit	Process	b_0/GeV	a/GeV^2	h_0	h_1	h_2	h_3	h_4	γ	χ^2/NDF
II a	$\pi\bar{\pi} \rightarrow \pi\pi\pi$ 8 GeV/c	4.84	1.752	1.0	1.358	1.428	2.473	0.155	-1.11	0.96
II b	$\pi\bar{\pi} \rightarrow \pi\pi\pi$ 16 GeV/c	4.30	1.810	1.0	1.385	1.739	2.317	0.118	-1.13	1.52
II c	$\pi\bar{\pi} \rightarrow \pi\pi\pi$ 19 GeV/c	5.68	2.658	1.0	1.244	1.140	2.041	0.003	-1.03	0.90

Table 3 : Parameters of fits to $\pi\bar{\pi}$ and $\pi\pi$ single diffractive processes assuming no background term, but increasing the width parameters $\Delta^{(n)}$ as described in the text. In each case h_0 has been normalized to unity.

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FIGURE CAPTIONS

- Figure 1 Definition of invariants of single particle production reaction.
- Figure 2 $d\sigma/dt'$ in four mass regions for the process $\pi N \rightarrow \pi\pi N$ at 8 GeV/c. Solid and broken curves are given by the parameters of fits Ia, IIa, respectively.
- Figure 3 $d\sigma/dt'$ in four mass regions for $\pi N \rightarrow \pi\pi N$ at 16 GeV/c. Solid and broken curves represent fits Ib, IIb, respectively.
- Figure 4 Impact parameter profiles (in arbitrary units) corresponding to fits Ia, Ib, Ic, respectively. The solid lines correspond to the profiles for $n = 0$; the broken curves to $n = 1$. The reason for the difference between these $n = 0, 1$ profiles is described in the text.
- Figure 5 Slope parameter $B(M^*)$ given by Eq. (9). The curves correspond to the model fits using solutions Ia, Ib, Ic, respectively.
- Figure 6 $d\sigma/dt'$ in five mass regions for the process $NN \rightarrow NN\pi$ at 19 GeV/c. Solid curve is given by fit Ic, and broken curve represents the corresponding fit using the values of h_n given by the fit Ia to the 8 GeV/c πN data.
- Figure 7 Slope parameter for the reactions $\pi^+p \rightarrow \pi^+(\pi^+\pi^-p)$ and $\pi^+p \rightarrow (\pi^+\pi^-\pi^+)p$. The solid curve is taken directly from the fit to the $\pi N \rightarrow \pi\pi N$ process at 16 GeV/c (shown in Fig. 5) while the broken curve assumes a linear displacement of 0.2 GeV.
- Figure 8 The ratio $R(s,t',M^*)$ defined by Eq. (14).
- Figure 9 Diagrammatic representation of the t channel factorization hypothesis given by Eq. (15).
- Figure 10 The slope parameter $B_D(M_1^*M_2^*)$ defined by Eq. (17) for the double diffraction process for various ranges of the masses M_1^*, M_2^* . The solid curve represents the model predictions using Eq. (16) and the parameters Ic, while the shaded area is given by the factorization assumption, Eq. (15) and the single diffractive $NN \rightarrow NN\pi$ data shown in Fig. 6.

Fig. 1

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Fig. 8

Fig. 9

Fig. 10