CM-P00060543

Ref.TH.1833-CERN

POSSIBLE REPRESENTATIONS OF LOCAL CURRENT ALGEBRA

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A B S T R A C T

Mass operators which lead
to possible representations of local
(vector) current algebra are discussed.

Ref.TH.1833-CERN

12 March 1974

Some time ago, Dashen and Gell-Mann ¹⁾ derived equations, for the matrix elements of the vector and axial vector charge densities at $P_z = \infty$ ²⁾, which follow from the requirements of local current algebra and relativistic covariance (the so-called angular conditions). The problem of representation of local current algebra at infinite momentum is then whether one can find an infinite set of single particle hadronic states, including (hopefully) the known meson and baryon resonances, which obey these equations exactly. If such representations can be constructed, they provide an approximate description of the hadron spectrum (i.e., masses and spins of the mesons and baryons) as well as of all weak and electromagnetic transitions between any two levels of the spectrum ³⁾.

Let us examine the problem in the meson case and, for simplicity, restrict our discussion to vector currents only. The equations one then has to solve are the following :

a) the local SU(3) algebra

$$[F_i(\vec{k}_\perp), F_j(\vec{k}'_\perp)] = i f_{ijk} F_k(\vec{k}_\perp + \vec{k}'_\perp) \quad (1)$$

b) the angular condition (for $\vec{k}_\perp$ in the x direction)

$$\langle N', h', P_x = -\frac{k}{2}, P_y = 0, P_z = \infty | e^{i\gamma_y \varphi} F_i(k) e^{i\gamma_y \varphi'} | N, h, P_x = \frac{k}{2}, P_y = 0, P_z = \infty \rangle$$

has

$$\Delta h = 0, \pm 1 \quad (2)$$

In this last equation, $|N, h, \vec{P}_\perp; P_z = \infty\rangle$ is an arbitrary one-meson state with mass m , helicity h and transverse momentum $\vec{P}_\perp$; J_y is a component of the "intrinsic angular momentum operator" which, at $P_z = \infty$ acts only on the helicity indices of the state $|N, h, \vec{P}_\perp; P_z = \infty\rangle$ and, finally, the angles φ and φ' are given by ^{1), 3)}

$$\varphi = \arctan \frac{2 m' k}{m^2 - m'^2 + k^2}$$

$$\varphi' = \arctan \frac{2 m k}{m'^2 - m^2 + k^2}$$

To satisfy these equations, we follow the suggestion of Dashen and Gell-Mann ¹⁾ and represent the algebra by

$$F_i(\vec{k}_\perp) = U_1 \frac{\lambda_i}{2} e^{i\vec{k}_\perp \cdot \vec{x}_\perp} U_1^{-1} + U_2 \frac{\bar{\lambda}_i}{2} e^{-i\vec{k}_\perp \cdot \vec{x}_\perp} U_2^{-1}$$

where $\vec{x}$ can be viewed as a relative position operator of a quark and an antiquark inside a meson and where $\lambda_i/2$ and $\bar{\lambda}_i/2$ are the usual SU(3) matrices for triplets and antitriplets. The unitary operators U_1 and U_2 are to be chosen in such a way that Eqs. (1) and (2) are satisfied. In the exact SU(3) limit which we consider, these operators commute with the λ matrices.

Define

$$\begin{aligned} U_1 \vec{x}_\perp U_1^{-1} &= \vec{h}_\perp && \text{with components } h_x \text{ and } h_y \\ U_2(-\vec{x}_\perp) U_2^{-1} &= \vec{h}'_\perp && \text{with components } h'_x \text{ and } h'_y ; \end{aligned}$$

Equation (1) then implies

$$\begin{aligned} [h_x, h_y] &= 0 \\ [h_x, h'_x] &= 0 \quad \text{etc...} \end{aligned} \quad (3)$$

Expanding Eq. (2) in powers of k , one finds in first order

$$[j_z, h_x] = i h_y \quad [j_z, h_y] = -i h_x \quad (4)$$

and similar relations for h'_x and h'_y . Higher orders in k lead to a series of conditions of which we need only the following ⁴⁾

$$\begin{aligned} [2 M j_+, h_+] &= \frac{i}{2} [M^2, h_+]^2 && [2 M j_+, h'_+] = \frac{i}{2} [M^2, h'^2_+] \\ [h_+, [h_+, [h_+, M^2]]] &= 0 && [h'_+, [h'_+, [h'_+, M^2]]] = 0 \end{aligned} \quad (5)$$

Here $j_+ = j_x + i j_y$; $h_+ = h_x + i h_y$; $h'_+ = h'_x + i h'_y$ and M is the mass operator.

The problem to solve is now well defined ³⁾ : one has to construct operators $\vec{J}$, M , $\vec{h}_\perp$ and $\vec{h}'_\perp$ such that Eqs. (3)-(5) are satisfied. A solution to the problem defines a meson spectrum with masses and spins given by the eigenvalues of M and $\vec{J}$ and determines all weak and electromagnetic transitions between any two states of the spectrum.

As shown by Gell-Mann ³⁾, the free (spinless) quark model does lead to a representation of local current algebra with

$$M = \sqrt{m^2 + \vec{p}^2}$$

and

$$\vec{J} = \vec{r} \wedge \vec{p}$$

where m and $\vec{p}$ are the quark mass and (relative) momentum. Even in this trivial case, however, U_1 and U_2 are not unit operators.

A natural extension to a more "realistic" quark antiquark scheme for the mesons is to try a mass operator of the form

$$M = \sqrt{m^2 + \vec{p}^2} + V_0(r) \quad (6)$$

with $V_0(r)$ some (central) potential. It turns out, however, that with $\vec{J} = \vec{r} \wedge \vec{p}$ and M given by Eq. (6), it is impossible ⁵⁾ to solve Eqs. (3)-(5) for any (non-constant) potential $V_0(r)$. More precisely, formally expanding the basic equations in a power series in $1/m$ and defining

$$h_x = x + \frac{v_x}{m} + \frac{t_x}{m^2} + \dots$$

one finds that the angular conditions [Eqs. (4),(5)] do not restrict the form of the potential $V_0(r)$ but that the local algebra constraints [Eq. (3)] cannot be satisfied, i.e., $[\vec{h}_x, h'_x] \neq 0$ to order $1/m^2$ for any $V_0(r) \neq \text{const.}$

At this point, it is worth mentioning that the operators U_1 and U_2 are nothing but the so-called Melosh transformation ⁶⁾ between constituent and current quarks. Several phenomenological applications based on algebraic properties of these operators (abstracted from the free quark model where they can be explicitly constructed) have recently been made and have met with some successes ⁷⁾. This has led us to re-examine the problem of representations of local current algebra at infinite momentum and in this note we summarize the results obtained so far.

With mass operators of the form

$$M = \sqrt{m^2 + p^2} + \sum_{n=0}^2 \frac{V_n(r, p)}{m^n}$$

one finds that, to order $1/m^2$, the basic equations do have a solution with the following restrictions on the various "potentials" :

$$V_0(r, p) \equiv V_0(r)$$

and

$$\frac{1}{r} \frac{dV_0}{dr} = \frac{\alpha r}{3r + 4\beta}$$

where α and β are arbitrary constants ;

$$V_1(r, p) \equiv A_0(r) p^2 + p^2 A_0(r) + A_1(r)$$

with ⁸⁾

$$A_0(r) = \frac{1}{2} + \frac{\beta}{r}$$

and $A_1(r)$ arbitrary ; and finally

$$V_2(r, p) \equiv (\vec{p} \cdot \vec{r})^2 B_0(r) + B_0(r) (\vec{r} \cdot \vec{p})^2 + B_1(r)$$

with $B_1(r)$ arbitrary and

$$B_0(r) = \frac{\alpha}{4} \frac{3r + 4\beta}{r}$$

The explicit expression of h_x is rather long and will not be given here.

We conclude with some remarks.

1. - It is interesting that the simplest solution of the representation problem, i.e., $A_1(r) = 0$ and $\beta = 0$ leads to a mass spectrum which, to order $1/m$ is precisely that of an harmonic oscillator

$$M = m + \frac{3p^2}{2m} + \frac{\alpha r^2}{6} + O\left(\frac{1}{m^2}\right)$$

2. - The brute force approach adopted in this note is clearly of limited value : formal calculations of higher order terms in $1/m$ are very lengthy and, furthermore, no matter to which order a solution is found, it does not guarantee that a non-trivial solution exists at the next order. It should be mentioned, however, that similar formal expansions in the factorized case ⁴⁾ strongly suggest that if a solution can be found to second order in $1/m$, a complete solution (i.e., to all orders in $1/m$) does exist.

3. - A possible technique to avoid power series in $1/m$ is to use infinite component wave equations. In the factorized case, it has been shown ⁸⁾ that all infinite wave equations which could lead to solutions of local current algebra are physically unacceptable : they all admit space-like solutions (i.e., $m^2 < 0$) which, even at $P_z = \infty$, are coupled through the current to the time-like solutions. A way to avoid this problem might be to use, from the start, Foldy's ¹⁰⁾ approach to relativistic covariance.

4. - It is of interest to extend our calculation to axial currents (where the quark spins will play a rôle) as well as to the baryon case. If solutions can be found, say, to order $1/m^2$, one would have a class of models in which all weak and electromagnetic transitions between hadronic states are explicitly computable to that order.

Note that, in such models, the $SU(3) \otimes SU(3)$ algebraic properties of the currents are precisely those abstracted from the free quark Melosh transformation ^{6),7)}. Further extensions, such as to an $SU(6)_W$ local current algebra, would also be worth while ¹¹⁾.

We hope to discuss some of these problems in forthcoming publications.

ACKNOWLEDGEMENTS

The author wishes to thank S. Fubini and H. Rubinstein for conversations on the subject of this note.

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