CM-P00060550

Archives

Ref.TH.1840-CERN

EQUIVALENCE BETWEEN THE UNIFIED DUAL MODEL
AND POMERON-REGGEON AMPLITUDES

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A B S T R A C T

We show that a recently proposed unified dual model for interacting open and closed strings actually describes Pomeron-Reggeon amplitudes of the conventional Veneziano model. We prove, in particular, that the Pomeron amplitudes are given by the Shapiro-Virasoro model.

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It is not clear whether the existing dual models ¹⁾ could be modified to describe the real world of hadrons, or whether they will ultimately be supplanted by completely different models. However, it is a matter of fact that the internal consistency of these models is so accurate that it may compete with field theory. A beautiful example of this consistency is the way in which the second order perturbative unitarity is restored at the critical value of the space-time dimension. Indeed it is well known that at non-critical value of the dimension the simple non-planar orientable loop shows, besides the unitarity cuts, new unitarity violating singularities ²⁾ in the channel with vacuum quantum numbers. At the critical value of the space-time dimension these singularities become a series of poles ^{3),4)} located on a linear Regge trajectory with twice the intercept and half the slope of the input trajectory ; they have been identified with the Pomeron poles.

Since these poles are factorizable ^{5),6)}, they represent a new set of resonances which are required by unitarity. Then one is urged to answer the following questions :

- i) what is the spectrum of the Pomeron resonances ?
- ii) what are their scattering amplitudes ?
- iii) what are their couplings with the ordinary resonances ?

The first question has been recently answered by Olive and Scherk ⁷⁾, who showed that the Pomeron sector of the conventional Veneziano model coincides at the critical dimension with the spectrum of the Shapiro-Virasoro (SV) model ⁸⁾.

In this paper we answer the latter two questions by showing that :

- a) the scattering amplitude of on-shell Pomeron states coincides with the corresponding SV amplitude, in agreement with a previous conjecture ⁹⁾ ;
- b) the mixed amplitudes among Pomeron states and ordinary resonances coincide with the ones we have constructed in the framework of a unified version [Ref. 10) hereafter quoted as $\bar{I}$] of the conventional and SV model.

Our starting point is the string picture ¹¹⁾ of dual models or, more precisely, the formulation of the interacting dual string given in Ref. 12). In this work it has been shown that the interaction among strings can be

represented by switching on a suitable external field acting on a free string. Indeed this procedure allows to reproduce quite naturally the dual amplitudes in their most manageable form ¹³⁾, that is, schematically

$$A(1, 2, \dots, N) = \int dV \langle 1 | \mathcal{V}_2 \mathcal{V}_3 \dots \mathcal{V}_{N-1} | N \rangle \quad (1)$$

where the vertex $\mathcal{V}$ is the interacting part of the Lagrangian of the string and the physical states $|N\rangle$ and $\langle 1|$ are respectively the initial and the final state of the string.

Such a string picture gives a simple and intuitive description of both the conventional model (open string) and the SV model (closed string) and suggests, as noted in Ref. 12), a way to construct a unified model of interacting closed and open strings which include as special cases the conventional model and the SV model.

The detailed features of this model are discussed in I. In order to prove the equivalence between this unified model and the Pomeron-Reggeon amplitudes, we quote here the main result, that is the amplitude $A(N\mathbb{R}, M\mathbb{P})$ for the interaction among N open strings and M closed strings [see Fig. 1, where continuous (dotted) lines represent open (closed) string states] that can be written for instance in the form :

$$A(N\mathbb{R}, M\mathbb{P}) = \int \prod_{i=2}^{N-2} \frac{dx_i}{x_i} \mathcal{G}(x_{i+1} - x_i) \int \prod_{j=1}^M \frac{d^2 z_j}{|z_j|^2} \cdot$$

$$\cdot \langle \alpha_1, -p_1 | T [\mathcal{V}_{\alpha_2}(z_2; p_2) \dots \mathcal{W}_{\beta_1}(z_1, \bar{z}_1; q_1) \mathcal{W}_{\beta_2}(z_2, \bar{z}_2; q_2) \dots$$

$$\mathcal{W}_{\beta_M}(z_M, \bar{z}_M; q_M)] | \alpha_N, p_N \rangle \quad (2)$$

In Eq. (2) $\mathcal{V}_{\alpha}(x, p)$ is the usual vertex ¹³⁾ of the conventional model associated with a physical state $|\alpha, p\rangle$ of momentum p and labelled by the set of indices α ; $\mathcal{W}_{\beta}(z, \bar{z}; q)$ is the vertex associated with the closed string and is related to $\mathcal{V}$ by ^{*}

^{*}) $\bar{z}$ is the complex conjugate of z .

$$\mathcal{W}_\beta(z, \bar{z}; q) = \mathcal{V}_\alpha(z; q/2) \mathcal{V}_{\bar{\alpha}}(\bar{z}; q/2) \quad (3)$$

where α and $\bar{\alpha}$ are the two sets of indices of open string which characterize the closed string ^{*)} labelled by the indices β .

The open string variables x are integrated over the real axis ($x_{N-1} = 1$) while the closed string variables z are integrated over the upper half part of the complex plane. T indicates that the closed string vertices must be ordered among themselves and with respect to the open string ones according to the increasing modulus of the variables x and z ; instead the ordering of the open string vertices is fixed by the Θ functions.

Clearly the amplitude (2) is explicitly factorizable in a multiperipheral configuration where all the intermediate resonances are open string states. Moreover we show in I that it is factorizable in all the channels which should resonate according to the dual rules, and all the poles are associated with open or closed string resonances (ordinary or SV states, respectively).

The link between our formalism and the dual loop theory is given by the non-planar orientable graph of Fig. 2a which, at the critical value of dimension can be factorized by means of the states of the dual Pomeron sector (Fig. 2b). The analogue of the graph of Fig. 2b can be calculated with our method using the propagator of the SV states and the transition amplitude from an off-mass shell SV state to N on-mass shell Reggeons. As shown in I, both these quantities are obtained factorizing the mixed amplitude $A(NR, MP)$ [Eq. (2)] in the channel pictured in Fig. 3.

One gets :

$$A(NR, MP) = \sum_{\lambda, \mu} \langle 0 | \langle 0 | \mathcal{Q}(1, 2 \dots N) | \lambda, q/2 \rangle_b | \mu, q/2 \rangle_c \cdot \int \frac{d^2 z}{|z|^2} \langle \lambda, \frac{q}{2} | z^{-L_0-1} | \lambda, \frac{q}{2} \rangle \langle \lambda, \frac{q}{2} | \mathcal{B}(1, 2, \dots N) | \mu, -\frac{q}{2} \rangle \langle \mu, -\frac{q}{2} | \bar{z}^{-L_0-1} | \mu, -\frac{q}{2} \rangle \quad (4)$$

^{*)} Actually the space of the physical SV states of momentum q is the direct product of two spaces of physical states of the open string belonging to the same level and with momentum $q/2$. The vertex (3) is obtained from the usual SV vertex [given, e.g., in (6)] by identifying the two sets of oscillators appearing in it.

where :

i) the amplitude

$$\begin{aligned} & \langle 0 | \langle 0 | Q(1, 2 \dots M) | \lambda, \frac{q}{2} \rangle_b | \mu, \frac{q}{2} \rangle_c = \\ & = \int \prod_{i=1}^{M-1} \frac{d^2 z_i}{|z_i|^2} \langle 0 | \langle 0 | T[\mathcal{W}_{b_c}^{\beta_1}(z_1, \bar{z}_1; q_1) \dots \mathcal{W}_{b_c}^{\beta_M}(z_M, \bar{z}_M; q_M)] | \lambda, \frac{q}{2} \rangle_b | \mu, \frac{q}{2} \rangle_c \end{aligned} \quad (5)$$

is the $(M+1)$ SV state amplitude (with the state of momentum q off-mass shell) ; in (5) the integration is extended over the whole complex plane and

$$\mathcal{W}_{b_c}^{\beta}(z, \bar{z}; q) = \mathcal{V}_b^{\alpha}(z, \frac{q}{2}) \mathcal{V}_c^{\bar{\alpha}}(\bar{z}, \frac{q}{2}) \quad (6)$$

is the usual SV vertex ;

ii) the quantity

$$\begin{aligned} & \int_{\Gamma} \frac{d^2 z}{|z|^2} \langle \lambda, \frac{q}{2} | z^{-L_0-1} | \lambda, \frac{q}{2} \rangle \langle \mu, -\frac{q}{2} | \bar{z}^{-L_0-1} | \mu, -\frac{q}{2} \rangle \equiv \\ & \equiv \int_{\Gamma} \frac{d^2 z}{|z|^2} \langle \lambda, \frac{q}{2} | \langle \mu, \frac{q}{2} | z^{-L_0-1} \bar{z}^{-L_0-1} | \lambda, \frac{q}{2} \rangle_b | \mu, \frac{q}{2} \rangle_c \end{aligned} \quad (7)$$

where the integration is extended inside the unitary circle in the complex plane of z , is the propagator of the SV states ^{*)} ;

*) Let us notice that the propagator (7) forces the Reggeon states, $|\lambda, q/2\rangle$ and $|\mu, q/2\rangle$ to belong to the same eigenvalue of L_0 ; hence their direct product is a SV state. Moreover, due to the projective invariance properties of $Q(1, 2, \dots, M)$ the only states $|\lambda, q/2\rangle$ and $|\mu, q/2\rangle$ that contribute to the sum in (4) are the physical ones.

iii) the amplitude

$$\langle \lambda, q/2 | B(1, 2, \dots, N) | \mu, -q/2 \rangle =$$

$$= e^{i\pi q^2/2} \int_0^{2\pi} \prod_{i=2}^N d\sigma_i \mathcal{G}(\sigma_i - \sigma_{i-1}).$$

$$\cdot \langle \lambda, q/2 | \mathcal{V}_{\alpha_1}(1; p_1) \mathcal{V}_{\alpha_2}(e^{i\sigma_2}; p_2) \dots \mathcal{V}_{\alpha_N}(e^{i\sigma_N}; p_N) | \mu, -q/2 \rangle \quad (8)$$

is the amplitude that connects the off-mass shell SV state of momentum q with N external Reggeons ^{*}).

By sewing together two of these amplitudes B by means of the SV propagator (7) one can write the following expression for the many Reggeon $\rightarrow$ SV state $\rightarrow$ many Reggeon amplitude depicted in Fig. 4

$$B(NR \rightarrow P \rightarrow MR) = \sum_{\lambda, \mu} \int \frac{d^2 z}{|z|^2} \int_0^{2\pi} \prod_{i=2}^N d\sigma_i \mathcal{G}(\sigma_i - \sigma_{i-1}) \int_0^{2\pi} \prod_{j=2}^M d\sigma_{N+j} \cdot$$

$$\cdot \mathcal{G}(\sigma_{N+j} - \sigma_{N+j-1}) e^{i\pi q^2} \langle \lambda, q/2 | \bar{z}^{-L_0-1} \mathcal{V}_{\alpha_1}(1; p_1) \mathcal{V}_{\alpha_2}(e^{i\sigma_2}; p_2) \dots \mathcal{V}_{\alpha_N}(e^{i\sigma_N}; p_N) P | \mu, -q/2 \rangle \cdot$$

$$\cdot \langle \mu, -q/2 | \bar{z}^{-L_0-1} \mathcal{V}_{\alpha_{N+1}}(1; p_{N+1}) \mathcal{V}_{\alpha_{N+2}}(e^{i\sigma_{N+2}}; p_{N+2}) \dots \mathcal{V}_{\alpha_{N+M}}(e^{i\sigma_{N+M}}; p_{N+M}) | \lambda, q/2 \rangle \quad (9)$$

In this expression we have introduced the projection operator P on the physical Reggeon states because, as the factorization (4) shows, the off-mass shell states $|\lambda, q/2\rangle$ and $|\mu, -q/2\rangle$ must be physical.

After use of the transformation property of the vertices

$$z^{-L_0} \mathcal{V}_{\alpha}(z; p) z^{L_0} = \mathcal{V}_{\alpha}\left(\frac{z}{z}; p\right) \text{ with } z = r e^{-i\sigma_1} \quad (10)$$

^{*}) In I it is shown that when the states $|\lambda, q/2\rangle$ and $|\mu, -q/2\rangle$ are on their mass shell, the amplitude (8) becomes the mixed amplitude (2) for N Reggeon 1SV state. The proof is performed by writing $\langle \lambda, q/2 |$ as

$$\lim_{z \rightarrow 0} \langle 0 | \mathcal{V}\left(\frac{z}{z}; -q/2\right) \quad \text{and} \quad |\mu, -q/2\rangle = \lim_{z \rightarrow 0} \mathcal{V}\left(1/\bar{z}; -q/2\right) | 0 \rangle$$

and then sending the unitary circle into the real axis by means of the projective transformation $z \rightarrow z' = -i(z+1)/(z-1)$.

and the change of variables $\sigma_i + \sigma_1 \rightarrow \sigma_i$ ($i = 2, 3, \dots, N$) the expression (9) becomes

$$\begin{aligned} \mathcal{B}(NR \rightarrow P \rightarrow MR) &= e^{i\pi q^2} \sum_{\lambda, \mu} \int dV \langle \lambda, q/2 | \mathcal{V}_{\alpha_1}(\frac{e^{i\sigma_1}}{r}; P_1) \mathcal{V}_{\alpha_2}(\frac{e^{i\sigma_2}}{r}; P_2) \cdots \\ &\mathcal{V}_{\alpha_N}(\frac{e^{i\sigma_N}}{r}; P_N) r^{-2L_0-2} P | \mu, -\frac{q}{2} \rangle \langle \mu, -\frac{q}{2} | \mathcal{V}_{\alpha_{N+1}}(1; P_{N+1}) \mathcal{V}_{\alpha_{N+2}}(e^{i\sigma_{N+2}}; P_{N+2}) \cdots \\ &\cdots \mathcal{V}_{\alpha_{N+M}}(e^{i\sigma_{N+M}}; P_{N+M}) | \lambda, \frac{q}{2} \rangle \end{aligned} \quad (11)$$

where the integration domain is given by

$$\int dV = \int_0^1 \frac{dr}{r} \int_0^{2\pi} d\sigma_1 \int_{\sigma_1}^{\sigma_1+2\pi} \prod_{i=2}^N d\sigma_i \mathcal{J}(\sigma_i - \sigma_{i-1}) \int_0^{2\pi} \prod_{j=2}^M d\sigma_{N+j} \mathcal{J}(\sigma_{N+j} - \sigma_{N+j-1}) \quad (12)$$

and corresponds to the unitary circle for the variables of the vertices $N+1, N+2, \dots, N+M$ and to a larger circle (of radius $1/r$ with r integrated between 0 and 1) for the variables of the vertices $1, 2, \dots, N$.

The completeness of the states $|\lambda\rangle$ and $|\mu\rangle$ allows to write the sum over $|\lambda\rangle$ and $|\mu\rangle$ as a trace in the harmonic oscillator space without zero mode :

$$\begin{aligned} \mathcal{B}(NR \rightarrow P \rightarrow MR) &= e^{i\pi q^2} \int dV r^{-\frac{q^2}{2}-2} \cdot \\ &\text{Tr} \left[\langle \frac{q}{2} | P \mathcal{V}_{\alpha_1}(\frac{e^{i\sigma_1}}{r}; P_1) \cdots \mathcal{V}_{\alpha_N}(\frac{e^{i\sigma_N}}{r}; P_N) r^{2R} | -\frac{q}{2} \rangle_0 \cdot \right. \\ &\left. \cdot \langle -\frac{q}{2} | \mathcal{V}_{\alpha_{N+1}}(e^{i\sigma_{N+1}}; P_{N+1}) \cdots \mathcal{V}_{\alpha_{N+M}}(e^{i\sigma_{N+M}}; P_{N+M}) | \frac{q}{2} \rangle_0 \right] \end{aligned} \quad (13)$$

where the vectors $|\pm q/2\rangle_0$ ($\langle \mp q/2|$) represents an incoming (outgoing) state of momentum $\pm q/2$ in the space of the zero mode only and $L_0 = -(p_0^2/2) - R$.

The trace appearing in (13) is formally that of the planar loop ; then the projection operator P is equivalent (at the critical value of dimension) to the factor

$$f(r^2)^2 = \prod_{n=1}^{\infty} (1 - r^{2n})^2 \quad (14)$$

Now one can easily recognize that (13) gives exactly the same amplitude ^{*)} of the graph of Fig. 2b at the critical value of the space-time dimension [see, e.g., Refs. 5) and 6)]. It is quite amusing that expression (13) is obtained in the dual loop theory after the Jacobi transformation in the graph of Fig. 2a ; however, this fact is not surprising because our integration variables have been chosen from the very beginning in such a way as to make evident the Pomeron poles. In our formalism the amplitude (13) can be written in a self-consistent way also for a number of dimensions less than the critical one ^{**)} ; however, if $d \neq 26$ it is different from the expression obtained in the loop theory ; it seems interesting to investigate the meaning of that amplitude in such a case.

Our procedure for constructing the twisted loop diagram shows that the Pomeron is a factorizable pole and that the corresponding decay amplitude of a Pomeron into N Reggeons is given by the expression (8), which, on the mass-shell of the Pomeron, coincides with the 1SV state - N Reggeon amplitude as given by (2). It would be interesting to study more closely the relationship between our procedure of factorization and the one used in Refs. 5) and 6).

More generally, it is immediate to prove that (2) is actually the M Pomeron - N Reggeon amplitude (when all the external particles are on their mass shell). In fact, factorizing the one-Pomeron decay amplitude one gets the vertex ^{+) of an on-mass shell Pomeron between the two off-mass shell physical Reggeons (Fig. 5) :}

^{*)} Due to different conventions about the zero mode, the quantity

$$e^{i q \cdot (\sum_{i=1}^N p_i)} \langle 0 | \mathcal{V}(p_1) \mathcal{V}(p_2) \dots \mathcal{V}(p_N) | 0 \rangle$$

given in Ref. 6) is equivalent to our $\langle 0 | q/2 | \mathcal{V}(p_1) \mathcal{V}(p_2) \dots \mathcal{V}(p_N) | -q/2 \rangle_0$ with $q = \sum_{i=1}^N p_i$.

^{**)} The only difference is that the projection operator is no longer $f(r^2)^2$, due to the presence of the Brower states ¹⁴⁾.

^{+) As shown in I, the vertex (15) can be further factorized in a Pomeron-Reggeon transition vertex and in a three-Reggeon vertex ; hence it may happen that the expression (15) diverges because q^2 is just on the pole of a Reggeon ; in such a case, it is understood that the residue must be taken.}

$$\langle \lambda, -p_1 | \mathcal{V}_\beta(q) | \mu, p_2 \rangle = \langle \lambda, -p_1 | \int_0^\pi d\sigma \mathcal{V}_\alpha(e^{i\sigma} \frac{q}{2}) \mathcal{V}_\alpha(e^{-i\sigma} \frac{q}{2}) | \mu, p_2 \rangle \quad (15)$$

These vertices can be sewed together and with Reggeon vertices (by means of Reggeon propagators attached to the off-mass shell Reggeon legs) to form a tree ^{*)} representing an M Pomeron - N Reggeon transition amplitude ; the amplitude so built coincides with the M SV state - N Reggeon amplitude. Moreover, the unified model amplitude (2) factorizes in the SV amplitude [see I and also Eq. (4)] ; hence the N on-mass shell Pomeron amplitude coincides with the SV amplitude.

We can then conclude that if from the loop theory one can extract in a consistent way Pomeron-Reggeon amplitudes, the spectrum and the interaction of the Pomerons coincide with the ones of the SV states.

Let us stress that our results are not the complete many-loop dual theory ^{**)}. We have only formulated the theory (complete at the lowest order ^{+) in g - i.e., at the tree level) of the interaction of the particles of the Pomeron sector among themselves and with Reggeons ; these results could also be obtained, in principle, by considering the dual}

^{*)} If the tree is ended by physical states there is no trouble with spurious states, due to the good projective transformation properties of the $\mathcal{V}$'s and the $\mathcal{W}$'s [see Appendix A of I].

^{**)} The amplitudes we have written in this work (and in I) can be topologically characterized according to the number of borders of the world surface of the interacting strings. The interaction amplitude among Pomerons only (the SV amplitude) do not involve open strings and hence corresponds to a surface without borders ; the Reggeon amplitude (the conventional model) and the mixed Pomeron-Reggeon amplitude [given in I and written in Eq. (2)] correspond to surfaces with one border ; the one-loop amplitude of Fig. 4 (and the analogous with some external Pomerons, that we can easily write) corresponds to a surface with two borders. It is not immediate to write in a closed form amplitudes with a larger number of borders.

⁺) The three-Reggeon coupling constant g determines uniquely also the coupling constants g_P (three-Pomeron vertex), g_{PR} (Pomeron-Reggeon transition) and g_{PRR} (Pomeron-Reggeon-Reggeon vertex). In fact from the dual identities

and from the identity of Fig. 2a and Fig. 2b, one gets (with $\alpha'_R = 1$) :
 $g_{PR} \simeq g$; $g_{PRR} = g_P \simeq g^2$.

Reggeon graphs containing only non-planar orientable loops and taking their residua when all the Pomerons are on-mass shell.

Because in all the treatment of this work (and of I), only the projective properties of the vertices $\mathcal{V}$ and $\mathcal{W}$ have been used, the extension of our results to the Neveu-Schwarz model ought to be straightforward.

ACKNOWLEDGEMENTS

We acknowledge useful discussions with M. Ademollo, D. Amati, E. Del Giudice, S. Fubini, E. Galzenati, R. Musto, F. Nicodemi and D. Olive. We thank D. Olive for a critical reading of the manuscript.

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FIG. 1

FIG. 2a

FIG. 2b

FIG. 3

FIG. 4

FIG. 5