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SEMI-INCLUSIVE TWO-PARTICLE RAPIDITY CORRELATIONS AT ISR ENERGIES
AND INDEPENDENT CLUSTER EMISSION MODEL

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A B S T R A C T

Semi-inclusive rapidity correlations at ISR energies are studied within a two-component cluster model. For the non-diffractive component central mesonic clusters are produced independently and they decay isotropically ; their exclusive decay law is assumed to follow a Boltzmann spectrum with a temperature parameter depending on the ratio of cluster mass to decay multiplicity ; this temperature parameter is not bounded by a maximal temperature. This distribution is approximately equivalent to a Gaussian in rapidity. For the diffractive component, single excitation with subsequent anisotropic cluster decay is used.

Exclusive, inclusive and semi-inclusive k particle distributions are derived within the model. Diffractive and non-diffractive semi-inclusive two-particle correlations are displayed in the (y_1, y_2) plane. The divided semi-inclusive correlation function for rapidity $(y_1, y_2) = (0, 0)$ and $(0, 3)$ is compared with ISR data as a function of n_{ch} .

The same type of central clusters which were found to explain inclusive correlations describe consistently also semi-inclusive reactions. At $\sqrt{s} \approx 50$ GeV for the non-diffractive component in addition to the two leading clusters, three to four central clusters are produced, their average number increases like $\langle m \rangle \sim \ln \sqrt{s}/M_0$. The central clusters have an average mass $\langle M \rangle \sim 2$ GeV independent of collision energy and they decay isotropically into 2.5 to 3.5 charged particles.

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1. - INTRODUCTION

Various pieces of experimental information on correlations, such as on the integrated inclusive two-particle correlation function $f_2(s)$ ¹⁾, on the inclusive two-particle rapidity correlation $R(y_1, y_2)$ ²⁾⁻⁴⁾, on the approximate KNO form of the multiplicity distribution^{5), 6), 1e)} have strengthened the conclusion that inclusive multiparticle production at least to a large fraction proceeds via multicluster intermediate states⁷⁾⁻²²⁾.

Recently, data on semi-inclusive correlations as functions of the prong number became available. These measurements at ISR at $\sqrt{s} = 23.6$ and 62.8 GeV²³⁾ and at NAL at $p_{lab} = 205$ GeV/c²⁴⁾ give the semi-inclusive two-particle correlation function

$$\begin{aligned} \varphi_2^{(n)}(y_1, y_2, s) &= \frac{1}{\sigma_n} \frac{d^2 \sigma_n(s)}{dy_1 dy_2} - \frac{1}{\sigma_n^2} \frac{d\sigma_n}{dy_1} \frac{d\sigma_n}{dy_2} \\ &= \zeta_2^{(n)}(y_1, y_2, s) - \zeta_1^{(n)}(y_1, s) \zeta_1^{(n)}(y_2, s) \end{aligned} \quad (1.1)$$

and the divided two-particle correlation function

$$\varrho_2^{(n)}(y_1, y_2, s) = \frac{\varphi_2^{(n)}(y_1, y_2, s)}{\zeta_1^{(n)}(y_1, s) \zeta_1^{(n)}(y_2, s)} \quad (1.2)$$

as a function of n for various values of (y_1, y_2) . $\zeta_1^{(n)}(y, s)$ and $\zeta_2^{(n)}(y_1, y_2, s)$ are the one and two-particle semi-inclusive distributions, respectively. For $y_1 = y_2 = 0$ the data of both experimental groups show a decrease of $\rho_2^{(n)}$ with n , which for the ISR data might be

$$\varrho_2^{(n)}(0, 0, s) \sim \frac{1}{n} \quad (1.3)$$

The bubble chamber data from NAL give in the range $4 < n_{ch} < 14$ for charged-charged correlations very small values of $\rho_2^{(n)}(0, 0)$ which for larger n are negative. At the highest energy measurements at ISR the values for $\rho_2^{(n)}(0, 0)$ are much larger and remain positive possibly until $n_{ch} \lesssim 24$.

On the undivided correlation functions $\varphi_2^{(n)}(0,0)$, the data are rather conflicting : the NAL results for charged-charged correlations give negative $\varphi_2^{(n)}(0,0)$ increasing with n in absolute value, whereas the preliminary ISR data give increasing $\varphi_2^{(n)}(0,0)$ with n

$$\varphi_2^{(n)}(0,0) \sim n \quad (1.4)$$

at least for n up to $n \sim 20$. Problems with normalization lead to large uncertainty especially for the large n values ^{*}). For these reasons it is best to compare with the divided correlation function $\rho_2^{(n)}(y_1, y_2, s)$. Theoretical ideas based on the multiperipheral model with predominantly positive correlations and the Mueller-Regge approach ^{25), 26)} show that the undivided correlation function $\varphi_2^{(n)}(y_1, y_2, s)$ of $y_1 = y_2 = 0$ as function of n within these models has to be positive and to increase with n [in Ref. 25) $\varphi_2^{(n)}(0,0) \sim n$, in Ref. 26) $\varphi_2^{(n)}(0,0) \sim n^2$]. Therefore it is speculated ^{25), 26)} that conclusive data could help to eliminate eventually one of the current models on particle production.

The cluster model is able to reproduce detailed features of inclusive single particle distributions ^{8), 27)} and inclusive two-particle rapidity correlations ^{10), 11), 12)} in the central as well as the outer regions in the (y_1, y_2) rapidity plane. Study of inclusive correlation data has given limits on the production mechanism of clusters ^{10), 11)}. Here we use this model to study two-particle rapidity correlations as function of the multiplicity n . Our main results have already been summarized in Ref. 28) ; here we give a full and more detailed account of our work. Since our note ²⁸⁾, the cluster model has also been applied to semi-inclusive data by Berger ²⁹⁾. In order to do a numerical analysis of the ISR data, we use the general connection between inclusive, semi-inclusive and exclusive quantities without relying on the techniques of generating functions.

Our main results and our comparison with data are derived from Eqs. (4.35) and (4.36) which express the semi-inclusive two-particle correlation functions in terms of average quantities $Q_1^{av,m}(n, y)$, $Q_2^{av,m}(n, y_1, y_2)$ which describe the n particle decay of one average cluster in an event in which n clusters are produced. We also obtain and discuss results without relying on the introduction of these average quantities.

^{*}) We thank L. Foa for a discussion on this point.

The results depend in particular on such properties of the clusters as average mass of the clusters $\langle M \rangle$, average number of charged particles per cluster $n \langle 1/m \rangle$ and average number of central clusters $\langle m \rangle$ (m is the number of central clusters produced, n is the multiplicity of secondaries). The results are not very sensitive towards the decay law of the central clusters. Without attempting a detailed fit, we reproduce the trend of the ISR data on $\rho_2^{(n)}(y_1, y_2)$ at $y_1 = y_2 = 0$ and $y_1 = 0, y_2 = 3$ with central mesonic clusters with average mass $\langle M \rangle \sim 2$ GeV and an average decay multiplicity of $n \langle 1/m \rangle \sim 2.5$ to 3.5 charged particles per cluster. At these energies we need about three to four central plus two leading clusters for the non-diffractive component. At small multiplicities, $n < 10$, the diffractive component is relatively important ; we take into account only single diffraction. Our conclusion is that the same type of clusters allows to understand inclusive and semi-inclusive (or exclusive) correlations. In Section 2 we give details on the production of clusters, in Section 3 we discuss their decay. Production of secondaries via clusters for the non-diffractive component and the diffractive component is treated in Sections 4 and 5, respectively. The semi-inclusive correlations due to superposition of both components are obtained and discussed in Section 6. Our notation is introduced systematically and relevant formulae are collected in the Appendix.

2. - PRODUCTION OF CENTRAL CLUSTERS

Traditionally the concept of cluster production and decay is considered as a domain of inclusive multiparticle production. The reason for this is probably that the thermodynamic fireball model was originally formulated for inclusive particle spectra ^{30),7)}.

Cluster production and decay can, however, also be formulated for exclusive quantities. Exclusive cluster decay functions have recently been given within the statistical bootstrap model ³¹⁾⁻³⁴⁾. Exclusive cluster production was studied in particular within the multiperipheral model ³⁵⁾⁻³⁷⁾ going back to the original work of Amati, Fubini and Stanghellini ³⁸⁾ and later multiperipheral treatments of resonance production ³⁹⁾⁻⁴¹⁾. Exclusive cross-sections σ_n and the multiplicity distribution have been derived from thermodynamic particle distributions ⁴²⁾. For instance, for clusters with

mass $M \sim M_0 \ln(s/s_0)$ asymptotic KNO scaling was obtained, for clusters with mass independent of energy s and rapidity η this result cannot be derived ^{35),42)}. The short range inclusive two-particle rapidity correlations showing up in particular in the rapidity plane (y_1, y_2) around the diagonal $y_1 = y_2$ were first explained by multiperipheral production of several clusters with $\langle M \rangle \sim M_0 \ln \sqrt{s/s_0}$ or, preferably, with $\langle M \rangle \sim \text{const}$ ¹⁰⁾. The long range component which leads to detailed structure in the (y_1, y_2) plane ^{2),4)} can well be understood through correlations of the fast particle with one of the particles emerging from a diffractive cluster together with some admixture of a double diffractive component ¹²⁾. Other information, as, e.g., the inclusive two-particle rapidity correlation for charged-charged and charged-neutral particles ^{23),3)}, favours clusters with average mass independent of s and rapidity η in a wide energy range. From the central value of the inclusive correlation function $R(y_1 \approx 0, y_2 \approx 0)$ the upper limit of the cluster mass or cluster multiplicity has been deduced as $\langle M \rangle \simeq 2.2 \pm 0.3 \text{ GeV}$ or $\langle n_{\text{ch}} \rangle = 2.4 \pm 0.4 \text{ GeV}$ ^{*) 12)-15),17),18),22),43)}.

Inclusive two-particle correlations in different channels such as charged-charged, ++, +- and --, can be understood ⁴⁴⁾ in the cluster model by using the cluster mass distribution

$$F(M) \sim (M - M_0) e^{-b(M - M_0)} \quad (2.1)$$

Central clusters were found to be predominantly neutral ^{45),18)}. For emission of two equally charged particles the decay multiplicity has to be larger and correspondingly the minimal cluster mass M_0 has to be bigger than for emission of unequally charged particles. Table II.1 gives the central value of the inclusive two-particle rapidity correlation function

$$R(0,0) = \frac{\frac{1}{\sigma} \frac{d^2\sigma}{dy_1 dy_2} - \frac{1}{\sigma^2} \frac{d\sigma}{dy_1} \frac{d\sigma}{dy_2}}{\frac{1}{\sigma^2} \frac{d\sigma}{dy_1} \frac{d\sigma}{dy_2}} \quad (2.2)$$

*) The value $\langle \hat{M} \rangle \sim 3.2 \pm 0.3 \text{ GeV}$ estimated in Ref. 12) with an average total multiplicity per cluster of about five was derived for clusters containing a baryon. For mesonic clusters a good estimate might be $\langle M \rangle \sim 2.2 \pm 0.3 \text{ GeV}$.

obtained from the multiperipheral cluster model with Eq. (2.1) ^{43),44)} for various secondaries. These values are in good agreement with the data obtained at NAL ⁴⁾, ISR ^{2),3),23)} and Serpukhov ⁴⁶⁾.

In the present work we assume uncorrelated production of the central clusters. This could be imagined as in an uncorrelated jet model for cluster production or approximately in a multiperipheral model of cluster production. Using the generating functional techniques, it has been shown that factorization of inclusive quantities entails factorization of exclusive quantities ¹¹⁾.

Furthermore we here assume that clusters are produced with limited transverse momenta, which we shall neglect, such that the secondaries have approximately the same transverse momenta in the centre-of-mass as they have in the cluster rest frame.

The function

$$W^{(m)}(p_1, \dots, p_m, M_1, \dots, M_m, s) \cdot W_L(p_{L1}, p_{L2}, M_{L1}, M_{L2}) = \\ = \frac{1}{\sigma_{ce}} \frac{d\sigma_{ce}}{dp_1 \dots dp_m dM_1 \dots dM_m dp_{L1} dp_{L2} dM_{L1} dM_{L2}} \quad (2.3)$$

describes the exclusive mass and rapidity distribution of m central clusters with masses $M_1 \dots M_m$ and momenta $p_1 \dots p_m$ and two leading clusters with masses M_{L1}, M_{L2} and momenta p_{L1} and p_{L2} .

Certainly, kinematics introduces correlations among the clusters ; we attribute these completely to the leading clusters such that $W_L(p_{L1}, p_{L2}, M_{L1}, M_{L2})$ contains the δ functions for energy momentum conservation. If $W^{(m)}(p_1, \dots, p_m, M_1, \dots, M_m, s)$ is fixed, $W_L(p_{L1}, p_{L2}, M_{L1}, M_{L2})$ is determined through energy momentum conservation. As we are here not interested in the leading clusters, we integrate over their variables as well as over the transverse momenta of the central clusters. We are concerned only with the cross-section for central cluster production $W^{(m)}(\eta_1, \dots, \eta_m, M_1, \dots, M_m, s)$ as function of the cluster rapidities η_i and masses M_i . In the approximation of uncorrelated production of the central clusters the cross-section $W^{(m)}$ factorizes into one-fireball functions $F^{(m)}(\eta_i, M_i, s)$

$$\frac{1}{m!} W^{(m)}(\eta_1, \dots, \eta_m, M_1, \dots, M_m, s) = w_m(s) F^{(m)}(\eta_1, M_1, s) \dots F^{(m)}(\eta_m, M_m, s) \quad (2.4)$$

$w_m(s)$ is the multiplicity distribution of central clusters. The functions $F^{(m)}(\eta, M, s)$ are likely to depend on the cluster multiplicity m at least via their kinematic limits. We use

$$F^{(m)}(\eta, M, s) = \frac{b^2(M-M_0) \exp(-b(M-M_0))}{2 \ln\left(\frac{2\sqrt{s}}{mM}\right)} \quad (2.5)$$

in the rapidity region

$$-\ln\left(\frac{2\sqrt{s}}{mM}\right) \leq \eta \leq \ln\left(\frac{2\sqrt{s}}{mM}\right) \quad (2.6)$$

and

$$M_0 \leq M \leq \frac{\sqrt{s}}{2m} \quad (2.7)$$

with $M_0 \simeq 0.7 \text{ GeV}$ and $b \sim 1.5 \text{ GeV}^{-1}$. The functions $F^{(m)}(\eta, M, s)$ are normalized to 1,

$$\int F^{(m)}(\eta, M, s) d\eta dM = 1. \quad (2.8)$$

The function (2.5) is suggested by the arguments which led to the form (2.1). In addition to the M dependence in (2.1) we have to make an assumption for the rapidity dependence of the function $F^{(m)}(\eta, M, s)$, because we want to extend the analysis away from the central region in the rapidity plot. Due to the underlying multiperipheral ideas for the production of these clusters, we assume the flat rapidity distribution of the clusters contained in (2.6). We introduce further the dependence on the cluster multiplicity m because we want to study exclusive and semi-inclusive distributions as well.

In the context of multiperipheral cluster production and with an average cluster mass independent of energy and rapidity, the cluster multiplicity should grow logarithmically with energy.

For the average cluster mass we use approximately

$$\langle M \rangle = M_0 + \frac{2}{b} \quad (2.9)$$

where M_0 and b are the parameters of Eq. (2.5) ; thus we have clusters of $\langle M \rangle \sim 2$ GeV. For the average number of clusters $\langle m \rangle$ we use

$$\langle m \rangle = \frac{c}{\langle M \rangle} \ln \sqrt{\frac{s}{s_0}} \quad , \quad s_0 = 1 \text{ GeV}^2 \quad (2.10)$$

with c about 1.5 GeV.

At $\sqrt{s} = 50$ GeV, we obtain in the average three central clusters, $\langle m \rangle \sim 3$. In addition to this, there are the two leading clusters. If these ~ 5 clusters are equally spaced in rapidity, then the average rapidity gap between the clusters is about one unit in rapidity and it follows that the decay products of the cluster overlap ⁴⁷⁾ since each cluster spreads particles over about two units in rapidity.

Again, using the multiperipheral ideas on the production mechanism of the clusters as well as the independent cluster emission, we use for the multiplicity distribution $w_m(s)$ of the clusters a Poisson form from which the multiplicities $m < 1$ and $m > m_{\max} \approx 1.5 \ln \sqrt{s/s_0}$ are excluded. We call this distribution a truncated Poisson distribution ¹¹⁾

$$w_m(s) = \frac{\frac{\langle m(s) \rangle^m}{m!} \exp(-\langle m(s) \rangle)}{\sum_{m=1}^{m_{\max}} \frac{\langle m(s) \rangle^m}{m!} \exp(-\langle m(s) \rangle)} \quad (2.11)$$

With the functions $F^{(m)}(\eta, M, s)$ given in (2.5) and the multiplicity distribution (2.11), exclusive, inclusive and semi-inclusive cross-sections and spectra, respectively, for cluster production can be obtained according to the expressions (A.16) to (A.23) given in the Appendix.

3. - DECAY OF CLUSTERS

We assume that central clusters decay isotropically. In particular we use, for the invariant single particle spectrum due to the decay of a cluster of mass M into ℓ particles of mass m_0 , the ansatz^{*)}

$$E \frac{1}{\sigma_\ell} \frac{d^3 \sigma_\ell}{d^3 p} \equiv E \frac{d^3 u^{(\ell)}}{d^3 p} = C_\ell \exp\left(-\frac{E}{T(\ell, M)}\right) \quad (3.1)$$

$T(\ell, M)$ is a function of the mass of the cluster M and its decay multiplicity ℓ . C_ℓ is given by normalization to ℓ particles

$$\int E \frac{d^3 u^{(\ell)}}{d^3 p} \frac{d^3 p}{E} = \ell = \int C_\ell \exp\left(-\frac{E}{T(\ell, M)}\right) \frac{1}{E} 4\pi p^2 dp \quad (3.2)$$

With $p = \sqrt{E^2 - m_0^2}$ and $pd p = E dE$ the integration can be performed over the particle energy E which varies in the interval

$$m_0 \leq E \leq E_{\max} = \frac{M^2 + m_0^2 - ((\ell-1)m_0)^2}{2M} \approx \frac{M}{2}$$

For the integral, we have

$$\ell = \int_{\ell}^{E_{\max}} C_\ell 4\pi \sqrt{E^2 - m_0^2} \exp\left(-\frac{E}{T(\ell, M)}\right) dE \quad (3.3)$$

and find the normalization constant

$$C_\ell = \frac{\ell}{4\pi T(\ell, M)} \left[m_0 K_1\left(\frac{m_0}{T(\ell, M)}\right) - E_{\max} K_1\left(\frac{E_{\max}}{T(\ell, M)}\right) \right]^{-1} \quad (3.4)$$

^{*)} For a fixed prong number, a similar distribution was derived from the multiperipheral model in Ref. 48). We thank R. Hagedorn for drawing our attention to this reference.

where $K_1(x)$ is the modified Hankel function. We determine $T(\ell, M)$ from the energy sum rule

$$M = \int E \frac{d^2 u^{(e)}}{d^3 p} d^3 p = \int C_e \exp\left(-\frac{E}{T(\ell, M)}\right) 4\pi p^2 dp. \quad (3.5)$$

Extending the integral (3.5) to infinity we can approximately determine $T(\ell, M)$. In Table III.1, the value of $T(\ell, M)$ is given for various decay multiplicities ℓ as function of the cluster mass M . As $C_\ell \sim \ell$, one might deduce from Eq. (3.4) that T is a function of the ratio (M/ℓ) only, $T(\ell, M) = T(M/\ell)$; this can be seen also from Table III.1.

The decay distribution (3.1) only depends on the particle energy E and has thus no direction of preference; by adopting this decay law we have assumed explicitly isotropy in the cluster decay. The function $T(M/\ell)$ grows monotonously with (M/ℓ) .

For fixed multiplicity the mean transverse momentum grows with the cluster mass M and for fixed mass M the mean transverse momentum decreases with growing multiplicity ℓ . We assume that for large M the inclusive distributions of cluster decay are independent of M ; therefore the average multiplicity increases proportional to M .

For the multiplicity distribution $\mathcal{P}^{(e)}(\ell)(M)$ in the decay of central clusters, we assume a Poisson form from which the multiplicities $\ell < 2$ and $\ell > \ell_{\max} \approx M/m_0$ are excluded. We call this distribution a truncated Poisson distribution ¹¹⁾ with multiplicities $2 \leq \ell \leq M/m_0$ [rather similar to Eq. (2.11) where, however, the multiplicities $1 \leq m \leq m_{\max}$ are allowed]

$$\mathcal{P}^{(e)}(\ell)(M) = \frac{\frac{\langle \ell(M) \rangle^\ell}{\ell!} \exp(-\langle \ell(M) \rangle)}{\sum_{\ell=2}^{\ell_{\max}} \frac{\langle \ell(M) \rangle^\ell}{\ell!} \exp(-\langle \ell(M) \rangle)} \quad (3.6)$$

The assumption $\langle \ell(M) \rangle \sim M^q$, $q < 1$, corresponds to a model proposed in Ref. 18).

Our assumption of $\langle \ell(M) \rangle \sim M$ includes the predictions of the statistical bootstrap model (18), (28)-(30), (49), (50). Using $E = m_T \cosh y$ with the transverse mass $m_T = \sqrt{p_\perp^2 + m_0^2}$ we can obtain from expression (3.1) the rapidity distribution of one particle in the ℓ particle decay of a cluster. To obtain the modified semi-inclusive single particle rapidity distribution for the decay of one cluster, defined in Eq. (A.32), we integrate over the transverse momentum extending the integration limits in $p_\perp$ to ∞ . We then have

$$s_{z,1}^{(\ell)}(y, M, \eta) = \ell u^{(\ell)}(y, M, \eta)$$

with

$$\begin{aligned} u^{(\ell)}(y, M, \eta) &= \int \frac{d[u^{(\ell)}(y, p_\perp; M, \eta)]}{dy d^2 p_\perp} d^2 p_\perp = \\ &= C_\ell \pi \int_0^\infty dp_\perp^2 \exp\left(-\frac{\sqrt{p_\perp^2 + m_0^2} \cosh y}{T(\ell, M)}\right) \\ &= 2\pi C_\ell \exp\left(-\frac{m_0 \cosh y}{T(\ell, M)}\right) \left[\left(\frac{T(\ell, M)}{\cosh y}\right)^2 + \frac{m_0 T(\ell, M)}{\cosh y}\right] \end{aligned} \quad (3.7)$$

The normalized rapidity distribution $u^{(\ell)}(y, M, \eta)$ given in Eq. (3.7) is approximately of Gaussian form as can be seen from Fig. 3.1 in which $u^{(\ell)}(y, M, \eta=0)$ is plotted as function of y^2 for various values of the cluster mass M and decay multiplicity ℓ . The variance a of this Gaussian

$$u^{(\ell)}(y, M, \eta) \approx \frac{1}{\sqrt{2\pi}a^2} \exp\left(-\frac{(y-\eta)^2}{2a^2}\right) \quad (3.8)$$

is a function of $T(\ell, M)$, $a^2 = a^2(T(\ell, M))$. For $y \lesssim 1$, the curves for different ℓ and M do not differ much and it is justified to neglect their dependence on ℓ and M . As shown in Fig. 3.2 at high $T(\ell, M)$ a^2 becomes independent of $T(\ell, M)$. The parameter T in the decay law (3.1) is not limited by a maximal value.

Examples of clusters with the above decay properties are those obtained in the statistical bootstrap model. Inclusive distributions in this model have been given by Hagedorn ³⁰⁾ and Hagedorn and J. Ranft ⁷⁾ where

$$\frac{d^3 u}{d^3 p} \sim M \exp \left(- \frac{E}{T(M)} \right) \quad (3.9)$$

and $T(M)$ reaches a maximal temperature T_0 for $M \rightarrow \infty$. For exclusive decay spectra, it was shown ³³⁾ that the statistical bootstrap model leads to phase space behaviour besides of isospin factors and enhancement factors given by Yellin ⁵¹⁾.

Semi-inclusive decay distributions follow from exclusive functions. They have been given recently also in the context of the statistical bootstrap model ^{49),52)}.

These semi-inclusive decay distributions were found to behave asymptotically like (3.1) with the temperature parameter $T(\ell, M)$ depending on the cluster mass M and the decay multiplicity ℓ ⁵²⁾.

We assume that there is no correlation between the secondaries emerging from the decay of one cluster. Within the statistical bootstrap model it has been shown ³³⁾ that such correlations are weak. Under this assumption the expressions for the various distribution functions due to decay of one cluster are given in Appendix A.3. We here only quote the exclusive ℓ particle rapidity distribution

$$\frac{1}{\ell!} G^{(\ell)}(y_1, \dots, y_\ell, M, \eta) = \mathcal{P}^{(\ell)}(M) u^{(\ell)}(y_1, M, \eta) \dots u^{(\ell)}(y_\ell, M, \eta) \quad (3.10)$$

and the inclusive single particle rapidity distribution

$$f^{(1)}(y_1, M, \eta) = \sum_{\ell=1}^{\infty} \ell \mathcal{P}^{(\ell)}(M) u^{(\ell)}(y_1, M, \eta) . \quad (3.11)$$

4. - PARTICLE PRODUCTION FROM CENTRAL CLUSTERS

After having specified the production and decay functions as well as multiplicity distributions for cluster production and subsequent decay into final particles we can proceed to write down the distributions of the secondaries in the collision. We focus attention on the particles emerging from the central clusters and disregard those due to leading clusters.

4.1. - Uncorrelated production and subsequent decay of clusters

In the following we use the function $W^{(m)}(\eta_1, \dots, \eta_m, M_1, \dots, M_m, s)$ describing cluster production (introduced in Section 2) and a function $O^{(\ell)}(y_1, \dots, y_\ell, M, \eta)$ which describes the decay of one cluster with mass M and rapidity η into ℓ particles with rapidities $y_1, \dots, y_\ell$ [see Eq. (A.25)]. With these functions we can express :

i) - the exclusive distribution $\Omega^{(k)}(y_1, \dots, y_k, s)$ of k particles produced via production and decay of m clusters. It is

$$\begin{aligned} \frac{1}{k!} \frac{1}{\sigma} \frac{d^k \sigma}{dy_1 \dots dy_k} &= \frac{1}{k!} \Omega^{(k)}(y_1, \dots, y_k, s) \\ &= \sum_m \frac{1}{m!} \int \sum_{\substack{\{\ell_i\} \\ \sum \ell_i = k}} W^{(m)}(\eta_1, \dots, \eta_m, M_1, \dots, M_m, s) \cdot \\ &\quad \cdot \prod_{i=1}^m \frac{1}{\ell_i!} O^{(\ell_i)}(y_{k_i+1}, \dots, y_{k_i+\ell_i}; M_i, \eta_i) d\eta_1 \dots d\eta_m dM_1 \dots dM_m \end{aligned} \quad (4.1)$$

with

$$k_i = \sum_{j=1}^{i-1} \ell_j$$

$\{\ell_j\}$ indicates that the sum goes over all combinations of ℓ_j such that $\sum \ell_j = k$.

The assumption that the m central clusters are produced unconditionally allows us to use factorization for $W^{(m)}$ [see (A.17)]. The further assumption of uncorrelated decay of each cluster expressed by Eq. (3.10) [see also (A.25)] leads to

$$\frac{1}{k!} \Omega^{(k)}(y_1, \dots, y_k, s) = \sum_m \int \sum_{\{\ell_i\}} w_m(s) F^{(m)}(\eta_1, M_1, s) \dots F^{(m)}(\eta_m, M_m, s) \cdot \prod_{i=1}^m \mathcal{P}^{(\ell_i)}(M_i) u^{(\ell_i)}(y_{k_i+1}, M_i, \eta_i) \dots u^{(\ell_i)}(y_{k_i+\ell_i}, M_i, \eta_i) \cdot d\eta_1 \dots d\eta_m \cdot dM_1 \dots dM_m$$

with

$$k_i = \sum_{j=1}^{i-1} \ell_j \quad (4.2)$$

$w_m(s)$ is the multiplicity distribution of the m central clusters produced ; it is given in Eq. (2.11). $\mathcal{P}^{(\ell)}(M)$ is the decay multiplicity distribution of a cluster of mass M given in Eq. (3.6).

ii) - the multiplicity distribution $\mathcal{M}_k(s)$ of the observed k particles is

$$\mathcal{M}_k(s) = \int dy_1 \dots dy_k \frac{1}{k!} \Omega^{(k)}(y_1, \dots, y_k, s) \quad (4.3)$$

Inserting (4.2) into (4.3) and performing the integration over $dy_1 \dots dy_k$ by use of the normalization condition for $u^{(\ell)}(y, M, \eta)$ gives

$$\mathcal{M}_k(s) = \sum_m \int \sum_{\{\ell_i\}} w_m(s) F^{(m)}(\eta_1, M_1, s) \dots F^{(m)}(\eta_m, M_m, s) \cdot \prod_{i=1}^m \mathcal{P}^{(\ell_i)}(M_i) d\eta_1 \dots d\eta_m dM_1 \dots dM_m \quad (4.4)$$

The expression (4.4) can be simplified if we define the multiplicity distribution $P_1^{\text{av}, m}(\ell)$ of one average cluster in an event with m central clusters

$$P_1^{\text{av}, m}(\ell) = \int d\eta dM F^{(m)}(\eta, M, s) \mathcal{P}^{(\ell)}(M) \quad (4.5)$$

Then the multiplicity distribution of k particles in the collision is

$$\mathcal{M}_k(s) = \sum_m w_m(s) \sum_{\substack{\{\ell_i\} \\ \sum \ell_i = k}} \mathcal{P}_1^{av,m}(\ell_1) \dots \mathcal{P}_1^{av,m}(\ell_m) \quad (4.6)$$

For the multiplicity distribution of k particles emerging from j average clusters in an event with m central clusters, we have the recursion relation

$$\mathcal{P}_j^{av,m}(k) = \sum_{n_1=2}^{k-2(j-1)} \mathcal{P}_1^{av,m}(n_1) \mathcal{P}_{j-1}^{av,m}(k-n_1), \quad (4.7)$$

i.e., n_1 of the k particles ($k > n_1$) emerge from the first cluster and the remaining $(k - n_1)$ particles split off from the $(j - 1)$ clusters left over. In the summation we have used the condition that a cluster decays at least into two particles ($n_1 \geq 2, n_1 \leq 2-2(j-1)$). Equation (4.7) is a recursion relation ; it allows to express the multiplicity of k particles emerging from j clusters ($k \geq 2j$) in events with m central clusters in terms of the average decay multiplicity distribution $\mathcal{P}_1^{av,m}(k)$ of one cluster

$$\begin{aligned} \mathcal{P}_j^{av,m}(k) &= \sum_{n_1=2}^{k-2(j-1)} \sum_{n_2=2}^{k-n_1-2(j-2)} \sum_{n_3=2}^{k-n_1-n_2-2(j-3)} \dots \\ &\dots \sum_{n_j=2}^{k-n_1-n_2-\dots-n_{j-1}} \mathcal{P}_1^{av,m}(n_1) \mathcal{P}_1^{av,m}(n_2) \dots \mathcal{P}_1^{av,m}(n_m). \end{aligned} \quad (4.8)$$

Using (4.8) we express the multiplicity distribution as

$$\mathcal{M}_k(s) = \sum_m w_m(s) \mathcal{P}_m^{av,m}(k) \quad (4.9)$$

iii) - The inclusive rapidity distribution $\phi^{(k)}(y_1, \dots, y_k, s)$ of k particles is obtained from (4.2) by use of the general connection between inclusive and exclusive distributions [see Eq. (A.7)]

$$\phi^{(k)}(y_1, \dots, y_k, s) = \sum_{\ell=0}^{\infty} \frac{(k+\ell)!}{\ell!} \int \dots \int dy_{k+1} \dots dy_{k+\ell} \sum_{m=1}^{m_{\max}} w_m(s) \sum_{\substack{\{\ell_i\} \\ \sum_{i=1}^m \ell_i = k+\ell}} \prod_{i=1}^m \left[\int F^{(m)}(M_i, \eta_i, s) \mathcal{P}^{(\ell_i)}(M_i) u^{(\ell_i)}(y_{k+1}, M_i, \eta_i) \dots \right. \\ \left. \dots u^{(\ell_i)}(y_{k+\ell_i}, M_i, \eta_i) d\eta_i dM_i \right]$$

with

$$k_i = \sum_{j=1}^{i-1} \ell_j. \quad (4.10)$$

In particular, for the inclusive single particle distribution we have

$$\phi^{(1)}(y_1, s) = \sum_{m=1}^{\infty} n \int \dots \int dy_2 \dots dy_m \sum_m w_m(s) \sum_{\substack{\{\ell_i\} \\ \sum_{i=1}^m \ell_i = m}} \prod_{i=1}^m \left[\int F^{(m)}(\eta_i, M_i, s) \mathcal{P}^{(\ell_i)}(M_i) u^{(\ell_i)}(y_{k+1}, M_i, \eta_i) \dots u^{(\ell_i)}(y_{k+\ell_i}, M_i, \eta_i) d\eta_i dM_i \right] \\ k_i = \sum_{j=1}^{i-1} \ell_j \quad (4.11)$$

We have abbreviated $k+1 = n$.

Assuming without loss of generality that the observed particle emerges from cluster 1, it is

$$\phi^{(1)}(y_1, s) = \sum_{n=2}^{n_{\max}} n \sum_{m=1}^{m_{\max}} w_m(s) \sum_{\substack{\{\ell_i\} \\ \sum \ell_i = m}} \int d\eta_1 dM_1 F^{(m)}(\eta_1, M_1, s) \mathcal{P}^{(\ell_1)}(M_1) \cdot u^{(\ell_1)}(y_1, \eta_1, M_1) \cdot \prod_{i=2}^m \left[\int d\eta_i dM_i F^{(m)}(\eta_i, M_i, s) \mathcal{P}^{(\ell_i)}(M_i) \right] \quad (4.12)$$

We define an average rapidity distribution $Q_k^{av,m}(l, y_1, \dots, y_k, s)$ of k particles which emerge together with $(l-k)$ other particles from one average cluster in an event with m central clusters, i.e., one average cluster decays into l particles, k of which have the rapidities $y_1, \dots, y_k$. In particular we have

$$Q_1^{av,m}(l, y, s) = \int d\eta dM F^{(m)}(\eta, M, s) \mathcal{P}^{(l)}(M) u^{(l)}(y, \eta, M). \quad (4.13)$$

This function is normalized such that

$$\int dy Q_1^{av,m}(l, y, s) = P_1^{av,m}(l). \quad (4.14)$$

With this average distribution $Q_k^{av,m}(l, y_1, \dots, y_k, s)$ the inclusive single particle spectrum is

$$\Phi^{(1)}(y_1, s) = \sum_{n=2}^{m_{max}} n \sum_{m=1}^{m_{max}} w_m(s) \sum_{\substack{\{l_i\} \\ \sum l_i = n}} Q_1^{av,m}(l_1, y_1, s) P_1^{av,m}(l_2) \dots P_1^{av,m}(l_m) \quad (4.15)$$

or with the relation (4.8)

$$\Phi^{(1)}(y_1, s) = \sum_{n=2}^{m_{max}} n \sum_{m=1}^{m_{max}} w_m(s) \sum_{l_1=2}^{n-2(m-1)} Q_1^{av,m}(l_1, y_1, s) P_{m-1}^{av,m}(n-l_1). \quad (4.16)$$

The inclusive two-particle distribution as derived from (4.10) is

$$\begin{aligned} \Phi^{(2)}(y_1, y_2, s) = & \sum_{n=2}^{m_{max}} n(n-1) \int \dots \int dy_3 \dots dy_n \sum_{m=1}^{m_{max}} w_m(s) \sum_{\substack{\{l_i\} \\ \sum l_i = n}} \\ & \left[\prod_{i=1}^m \left[d\eta_i dM_i F^{(m)}(\eta_i, M_i, s) \mathcal{P}^{(l_i)}(M_i) u^{(l_i)}(y_{k_i+1}, \eta_i, M_i) \cdot \dots \cdot \right. \right. \\ & \left. \left. \dots u^{(l_i)}(y_{k_i+1}, \eta_i, M_i) \right] \right] \quad (4.17) \end{aligned}$$

Both observed particles can emerge from the same or two different central clusters. There are m^2 possibilities, m of them for both particles emerging from the same and $m^2 - m$ possibilities for the two particles emerging from different clusters. Therefore, the weight of the one-cluster contribution is $1/m$ whereas the two-cluster contribution has the weight $(m-1)/m$,

$$\begin{aligned} \text{weight of one-cluster contribution} & \quad \frac{1}{m} \\ \text{weight of two-cluster contribution} & \quad \frac{m-1}{m} \end{aligned} \quad (4.18)$$

Similar to (4.13) an average rapidity distribution can be defined for two particles emerging from one average cluster

$$Q_2^{av,m}(\ell, y_1, y_2) = \int d\eta dM F^{(m)}(\eta, M, s) \mathcal{P}^{(e)}(M) u^{(e)}(y_1, \eta, M) u^{(e)}(y_2, \eta, M). \quad (4.19)$$

The two-particle distribution then reads

$$\begin{aligned} \phi^{(2)}(y_1, y_2, s) &= \sum_{n=2}^{n_{\max}} n(n-1) \left[\sum_{m=1}^{m_{\max}} \frac{W_m(s)}{m} \sum_{\ell_1=2}^{n-2(m-1)} Q_2^{av,m}(\ell_1, y_1, y_2, s) \right. \\ &\quad \cdot P_{m-1}^{av,m}(n-\ell_1) + \sum_{m=2}^{m_{\max}} W_m(s) \frac{m-1}{m} \sum_{\ell_1=2}^{n-2(m-1)} Q_1^{av,m}(\ell_1, y_1, s) \\ &\quad \left. \cdot \sum_{\ell_2=2}^{n-\ell_1-2(m-2)} Q_1^{av,m}(\ell_2, y_2, s) P_{m-2}^{av,m}(n-\ell_1-\ell_2) \right] \quad (4.20) \end{aligned}$$

iv) - The semi-inclusive rapidity distribution for k particles in an n particle reaction is [see Eqs. (A.10) and (4.2)]

$$\overline{\Omega}_k^{(m)}(y_1, \dots, y_k, s) = \frac{n!}{(n-k)!} \int \dots \int dy_{k+1} \dots dy_n \frac{1}{n!} \Omega^{(m)}(y_1, \dots, y_n, s) =$$

$$\begin{aligned}
 &= \frac{n!}{(n-k)!} \int \dots \int dy_{k+1} \dots dy_m \sum_m \int \sum_{\substack{\{\ell_i\} \\ \sum \ell_i = m}} w_m(s) F_{(\eta_1, M_1, s)}^{(m)} \dots \\
 &\dots F_{(\eta_m, M_m, s)}^{(m)} \prod_{i=1}^m \mathcal{P}_{(M_i)}^{(\ell_i)} u^{(\ell_i)}(y_{k_i+1}, M_i, \eta_i) \dots \\
 &\dots u^{(\ell_i)}(y_{k_i+\ell_i}, M_i, \eta_i) d\eta_1 \dots d\eta_m dM_1 \dots dM_m
 \end{aligned} \tag{4.21}$$

with

$$k_i = \sum_{j=1}^{i-1} \ell_j.$$

With the average quantities $Q_k^{av,m}(\ell, y_1, \dots, y_k, s)$ defined in (4.13) and (4.19) and the recursion relation (4.7) the semi-inclusive distribution of one particle in an n particle channel is

$$\left[\begin{array}{c} (n) \\ - \\ 1 \end{array} \right]_1(y_1, s) = n \sum_{m=1}^{m_{\max}} w_m(s) \sum_{\ell_1=2}^{n-2(m-1)} Q_1^{av,m}(\ell_1, y_1, s) P_{m-1}^{av,m}(n-\ell_1). \tag{4.22}$$

The semi-inclusive two-particle distribution in an n particle channel is ⁶²⁾

$$\begin{aligned}
 \left[\begin{array}{c} (n) \\ - \\ 2 \end{array} \right]_2(y_1, y_2, s) &= n(n-1) \left[\sum_{m=1}^{m_{\max}} w_m(s) \frac{1}{m} \sum_{n_1=2}^{n-2(m-1)} Q_2^{av,m}(n_1, y_1, y_2, s) \cdot \right. \\
 &\cdot P_{m-1}^{av,m}(n-n_1) + \sum_{m=2}^{m_{\max}} w_m(s) \frac{m-1}{m} \sum_{n_1=2}^{n-2(m-1)} Q_1^{av,m}(n_1, y_1, s) \cdot \\
 &\cdot \left. \sum_{n_2=2}^{n-n_1-2(m-2)} Q_1^{av,m}(n_2, y_2, s) P_{m-2}^{av,m}(n-n_1-n_2) \right]
 \end{aligned} \tag{4.23}$$

It is useful to define modified semi-inclusive single as well as two-particle distributions [see Appendix, Eq. (A.11)]

$$\mathcal{J}_1^{(n)}(y_1, s) = \frac{\overline{\mathcal{J}}_1^{(n)}(y_1, s)}{\mathcal{M}_n(s)} \quad (4.24)$$

and

$$\mathcal{J}_2^{(n)}(y_1, y_2, s) = \frac{\overline{\mathcal{J}}_2^{(n)}(y_1, y_2, s)}{\mathcal{M}_n(s)} \quad (4.25)$$

In Fig. 4.1, we show the semi-inclusive single particle distribution $\mathcal{G}_1^{(n)}(y, s)$ due to the non-diffractive component which is treated in this section. This function has been computed at $\sqrt{s} = 23.6$ and 50 GeV from Eqs. (4.23) and (4.24) without using the approximations introduced and discussed in Section 4.2. According to Eq. (1.1) and (1.2), from Eqs. (4.24) and (4.25) we can compute the divided semi-inclusive correlation function $\rho_2^{(n)}(y_1, y_2, s)$ due to the non-diffractive component. It is plotted in Fig. 4.2 for $\sqrt{s} = 62.8$ GeV. On the whole these results represent the order of magnitude and the trend of the data which, however - as discussed in the Introduction - are still rather inconclusive. Some deviations from our results obtained with the average functions $\tilde{Q}_2^{\text{av}, m}(\ell, y_1, y_2)$ introduced and discussed in Section 4.2 (see also Figs. 6.3), in particular the change of sign of $\rho_2^{(n)}(0, 0)$ for $n_{\text{ch}} \approx 12$ to 16 can be an indication of a true dynamical effect - which might be indicated by the NAL data. Due to the large numerical calculations in evaluating the very involved sums especially in Eq. (4.23), we can, however, at present not exclude that some effects in Fig. 4.2 are caused by numerical uncertainties. To settle this problem further studies are needed.

4.2. - Approximate analytic treatment

In order to obtain analytic expressions for the semi-inclusive distributions and correlation functions it is useful to neglect the dependence on the number of clusters m and the number of secondaries ℓ in the average single and two-particle distributions defined in Eqs. (4.13) and (4.19) at fixed s

$$\tilde{Q}_1^{\text{av},m}(\ell, y, s) = \frac{Q_1^{\text{av},m}(\ell, y, s)}{P_1^{\text{av},m}(\ell)} \quad (4.26)$$

and

$$\tilde{Q}_2^{\text{av},m}(\ell, y_1, y_2, s) = \frac{Q_2^{\text{av},m}(\ell, y_1, y_2, s)}{P_1^{\text{av},m}(\ell)} \quad (4.27)$$

$\tilde{Q}_k^{\text{av},m}(\ell, y_1, \dots, y_k, s)$ is the normalized rapidity distribution of k particles emerging from one average cluster which is produced in an m cluster event at fixed s and which decays into ℓ particles, e.g.,

$$\int \tilde{Q}_1^{\text{av},m}(\ell, y, s) dy = 1$$

and

$$\int \tilde{Q}_2^{\text{av},m}(\ell, y_1, y_2, s) dy_1 dy_2 = 1 \quad (4.28)$$

In most cases the neglect of the ℓ and m dependence is quite a good approximation since part of the ℓ and m dependence of the factors in (4.26) and (4.27) cancels.

Using the expressions for cluster production and cluster decay as given in Sections 2 and 3, we calculate numerically $\tilde{Q}_1^{\text{av},m}(\ell, y, s)$ and $\tilde{Q}_2^{\text{av},m}(\ell, y_1, y_2, s)$. Some examples for these functions are plotted in Figs. 4.3 and 4.4, respectively. From Eq. (2.10) giving $\langle m \rangle \sim 3$ at $\sqrt{s} \sim 50$ GeV and from the average decay multiplicity of about $n_{\text{ch}} \sim 2.5$ to 3.5 per cluster we expect that the curve for $\tilde{Q}_1^{\text{av},4}(3, y, s)$ in Fig. 4.3 is a good representative for a single particle spectrum due to decay of a central average cluster. At fixed s , the curves for different cluster numbers m and different multiplicities ℓ at the plateau around $y \approx 0$ do not differ by more than a factor of 2. The curves in Fig. 4.4 are cuts in the (y_1, y_2) plane parallel to the y_1 axis at various values of y_2 . From here the central bumps along the axis $y_1 = y_2$ as well as its extensions can be seen. Again, although with increasing ℓ and m the spread in rapidity shrinks, the height of the plateau in $\tilde{Q}_2^{\text{av},m}(\ell, y_1, y_2, s)$ does not change more than by a factor of 2. In Fig. 4.4, we show a schematic plot of contours in the (y_1, y_2) plane.

For an approximate treatment we drop the labels l and m in the functions (4.26) and (4.27), and have

$$\tilde{Q}_k^{av,m}(l, y_1, \dots, y_k, s) \approx \tilde{Q}_k^{av}(y_1, \dots, y_k, s) . \quad (4.29)$$

Using (4.26) in (4.22) gives the modified semi-inclusive single particle distribution

$$\mathcal{J}_1^{(n)}(y, s) = n \tilde{Q}_1^{av}(y, s) . \quad (4.30)$$

The function $\tilde{Q}_1^{av}(y, s)$ corresponds therefore up to the factor n to the semi-inclusive single particle distribution. For the semi-inclusive two-particle distribution we have from (4.23), (4.27) and (4.7)

$$\begin{aligned} \mathcal{J}_2^{(n)}(y_1, y_2, s) &= n(n-1) \tilde{Q}_2^{av}(y_1, y_2, s) \sum_{m=1}^{m_{\max}} \frac{w_m(s)}{m} P_m^{av,m}(n) \\ &+ n(n-1) \tilde{Q}_1^{av}(y_1, s) \tilde{Q}_1^{av}(y_2, s) \sum_{m=2}^{m_{\max}} w_m(s) \frac{m-1}{m} P_m^{av,m}(n) . \end{aligned} \quad (4.31)$$

We observe that

$$\frac{\sum_{m=1}^{m_{\max}} \frac{w_m(s)}{m} P_m^{av,m}(n)}{\mathcal{M}_n(s)} = \left\langle \frac{1}{m} \right\rangle \quad (4.32)$$

and

$$\frac{\sum_{m=2}^{m_{\max}} w_m(s) \frac{m-1}{m} P_m^{av,m}(n)}{\mathcal{M}_n(s)} = 1 - \left\langle \frac{1}{m} \right\rangle . \quad (4.33)$$

$\langle 1/m \rangle$ is the average inverse number of central clusters. The quantity $n \langle 1/m \rangle$ gives approximately the number of charged particles resulting from the decay of one average cluster.

The modified semi-inclusive two-particle distribution is then

$$\begin{aligned} \zeta_2^{(n)}(y_1, y_2, s) = & n(n-1) \tilde{Q}_2^{av}(y_1, y_2, s) \langle \frac{1}{n} \rangle \\ & + n(n-1) \tilde{Q}_1^{av}(y_1, s) \tilde{Q}_1^{av}(y_2, s) \left(1 - \langle \frac{1}{n} \rangle\right). \end{aligned} \quad (4.34)$$

For the semi-inclusive two-particle rapidity correlation function defined in Eq. (1.1), we obtain

$$\begin{aligned} \varphi_2^{(n)}(y_1, y_2, s) = & n(n-1) \tilde{Q}_2^{av}(y_1, y_2, s) \langle \frac{1}{n} \rangle \\ & - n \left[1 + (n-1) \langle \frac{1}{n} \rangle\right] \tilde{Q}_1^{av}(y_1, s) \tilde{Q}_1^{av}(y_2, s) \end{aligned} \quad (4.35)$$

and for the divided correlation function defined in (1.2) we have

$$\rho_2^{(n)}(y_1, y_2, s) = \frac{n(n-1)}{n^2} \langle \frac{1}{n} \rangle \frac{\tilde{Q}_2^{av}(y_1, y_2, s)}{\tilde{Q}_1^{av}(y_1, s) \tilde{Q}_1^{av}(y_2, s)} - \frac{1 + (n-1) \langle \frac{1}{n} \rangle}{n} \quad (4.36)$$

The first term in (4.35) and (4.36) is determined by $\tilde{Q}_2^{av}(y_1, y_2, s)$ and describes the short range behaviour in the rapidity plane ; the second term gives a flat long range contribution.

If $n \langle 1/n \rangle$ is approximately independent of s and n as expected in the model, we find from our approximate treatment that $\varphi_2^{(n)}(0,0)$ and $\rho_2^{(n)}(0,0)$ behave as $\varphi_2^{(n)}(0,0) \sim n$ and $\rho_2^{(n)}(0,0) \sim 1/n$. Similarly as found in the ISR experiment ²³⁾.

5. - THE DIFFRACTIVE COMPONENT

In good approximation the diffractive particle production proceeds via formation of one cluster (Fig. 5.1) of large mass M and rapidity η . It holds approximately

$$M = M(\eta, s) \approx \sqrt{s} e^{-|\eta|}. \quad (5.1)$$

The spectrum of the diffractive particle [denoted by $(k+1)$ in Fig. 5.1] with a value $x = p_{\parallel}/(\sqrt{s}/2)$ near 1 can well be described by the triple Regge expression. In particular, the proton spectrum in the reaction $pp \rightarrow p+M$ has been fitted with the function ⁵³⁾⁻⁵⁶⁾

$$\frac{d\sigma}{dM^2} = \frac{1.2}{M^3} + \frac{0.5}{M^2} \quad (5.2)$$

at NAL and ISR energies.

At the same time this function gives the mass distribution $F_D^{(1)}(M, \eta, s)$ of the diffractive cluster. We therefore have

$$\begin{aligned} F_D^{(1)}(M, \eta, s) &= \frac{d\sigma}{dM} \\ &= C \left(\frac{2.4}{M^2} + \frac{1}{M} \right) \delta \left(|\eta| - \ln \frac{\sqrt{s}}{M} \right). \end{aligned} \quad (5.3)$$

Normalization to one diffractive cluster,

$$\int_{M_0}^{M_{\max}} F_D^{(1)}(M, \eta, s) dM = 1$$

results in

$$C = \left[2.4 \left(\frac{1}{M_0} - \frac{1}{M_{\max}} \right) + \ln \frac{M_{\max}}{M_0} \right]^{-1}. \quad (5.4)$$

We use $M_{\max} \propto \sqrt{s}$. For single diffractive production, the exclusive, the inclusive and the semi-inclusive cluster distribution functions are equal ; they are identical to (5.3)

$$\begin{aligned} W_D^{(n)}(\eta, M, s) &= V_D^{(n)}(\eta, M, s) = S_D^{(n)}(\eta, M, s) = \mathcal{T}_D^{(n)}(\eta, M, s) \\ &\equiv F_D^{(n)}(\eta, M, s) . \end{aligned} \quad (5.5)$$

Diffractive particle production via a one-cluster intermediate state can be described by anisotropic cluster decay⁵⁷⁾. The ℓ particle decay distribution can then be written as

$$\frac{d^3 u_D^{(\ell)}}{dy d^2 p_{\perp}} = \ell \frac{1}{\sqrt{2\pi L^2(\ell, M)}} \exp\left(-\frac{(y-\eta)^2}{2 L^2(\ell, M)}\right) f(p_{\perp}) \quad (5.6)$$

where

$$\int f(p_{\perp}) d^2 p_{\perp} = 1 .$$

Contrary to the isotropic decay used in Eq. (3.1), the transverse momentum distribution here does not change with the cluster mass M and with the decay multiplicity ℓ . Therefore, we can integrate easily over the $p_{\perp}$ distribution for all M and ℓ and obtain the normalized rapidity distribution for one particle of the ℓ body decay of the cluster

$$u_D^{(\ell)}(y, M, \eta) = \frac{1}{\sqrt{2\pi L^2(\ell, M)}} \exp\left(-\frac{(y-\eta)^2}{2 L^2(\ell, M)}\right) . \quad (5.7)$$

This function is normalized to 1.

Different from the isotropic case, the parameter $L(\ell, M)$ and the average decay multiplicity $\langle \ell(M) \rangle$ are determined completely through normalization and the energy sum rule

$$\int E u_D^{(\ell)}(y, M, \eta) dy \sim \frac{M}{\ell} \quad (5.8)$$

For $M/\ell \gg 1$ GeV, it behaves like $L(\ell, M) \sim \ln(M/\ell m_0)$. The multiplicity distribution $\mathcal{P}_D^{(\ell)}(M) = \mathcal{P}^{(\ell)}(M)$ of the cluster decay is again assumed to be a truncated Poisson distribution given in Eq. (3.6). For small cluster masses the fireball treated here becomes isotropic, i.e., also the average transverse momentum $\langle p_\perp \rangle$ decreases with the cluster mass.

As we did for central clusters before, we assume the diffractive cluster to emit particles in an uncorrelated manner. Then we obtain factorized expressions for the diffractive parts of the exclusive ℓ particle rapidity distribution $O_D^{(\ell)}(y_1, \dots, y_\ell, M, \eta)$, of the inclusive k particle rapidity distribution $f_D^{(k)}(y_1, \dots, y_k, M, \eta)$, of the semi-inclusive k particle rapidity distribution $S_{Dzk}^{(n)}(y_1, \dots, y_k, M, \eta)$ as well as the modified semi-inclusive distributions $s_{Dzk}^{(n)}(y_1, \dots, y_k, M, \eta)$. These factorized expressions are given in Appendix A.3, Eqs. (A.25) to (A.33). Into these expressions, we have to insert for $u^{(\ell)}(y, M, \eta)$ the diffractive decay distribution given in Eq. (5.6).

In the particle distributions given below for the diffractive collision we do not make explicit the dependence on particle $(k+1)$ which does not emerge from the cluster. The correlation effects between this particle and the particle from the cluster have been studied in Ref. 12). The various particle distributions for the diffractive component are obtained in analogy to those of the non-diffractive component, Eqs. (4.2), (4.3), (4.10), (4.22) and (4.23) with the diffractive cluster mass distribution $F_D^{(1)}(M, \eta, s)$ [Eq. (5.3)] and the cluster decay distribution $u_D^{(\ell)}(y, M, \eta)$ [Eq. (5.7)].

Then we have for the diffractive component :

- the exclusive k particle rapidity distribution

$$\begin{aligned} \frac{1}{k!} \Omega_D^{(k)}(y_1, \dots, y_k, s) &= \frac{1}{k!} \frac{1}{\sigma_D} \frac{d\sigma_D^{(k)}}{dy_1 \dots dy_k} \\ &= \int dM d\eta F_D^{(1)}(\eta, M, s) \mathcal{P}_D^{(k)}(M) u_D^{(k)}(y_1, M, \eta) \dots u_D^{(k)}(y_k, M, \eta) \quad (5.9) \end{aligned}$$

- the multiplicity distribution

$$M_{Dk}(s) = \int dM d\eta \mathcal{P}_D^{(k)}(M) F_D^{(1)}(\eta, M, s) \quad (5.10)$$

- the inclusive k particle rapidity distribution

$$\begin{aligned} \Phi_D^{(k)}(y_1, \dots, y_k, s) &= \sum_{\ell=0}^{\infty} \frac{(k+\ell)!}{\ell!} \int dM d\eta F_D^{(1)}(\eta, M, s) \cdot \\ &\quad \mathcal{P}_D^{(k+\ell)}(M) u_D^{(k+\ell)}(y_1, M, \eta) \cdot \dots \cdot u_D^{(k+\ell)}(y_k, M, \eta) \end{aligned} \quad (5.11)$$

- the semi-inclusive k particle rapidity distribution

$$\begin{aligned} \overline{\Sigma}_{Dk}^{(n)}(y_1, \dots, y_k, s) &= \frac{n!}{(n-k)!} \int dM d\eta F_D^{(1)}(\eta, M, s) \mathcal{P}_D^{(n)}(M) \cdot \\ &\quad u_D^{(n)}(y_1, M, \eta) \cdot \dots \cdot u_D^{(n)}(y_k, M, \eta) \end{aligned} \quad (5.12)$$

- and the modified semi-inclusive k particle rapidity distribution

$$\mathcal{T}_{Dk}^{(n)}(y_1, \dots, y_k, s) = \frac{\overline{\Sigma}_{Dk}^{(n)}(y_1, \dots, y_k, s)}{M_n(s)} \quad (5.13)$$

Similarly as for the non-diffractive component, we might introduce quantities $\tilde{Q}_{1D}^{av}(n, y_1, s) \approx \tilde{Q}_{1D}^{av}(y_1, s)$ and $\tilde{Q}_2^{av}(n, y_1, y_2, s) \approx \tilde{Q}_2^{av}(y_1, y_2, s)$ in analogy to Eqs. (4.26) and (4.27). Then the modified diffractive semi-inclusive single particle distribution is

$$\mathcal{T}_{1D}^{(n)}(y, s) = n \tilde{Q}_{1D}^{av}(y, s) \quad (5.14)$$

and similarly the two-particle distribution

$$\zeta_{2D}^{(n)}(y_1, y_2, s) = n(n-1) \tilde{Q}_{2D}^{av}(y_1, y_2, s) . \quad (5.15)$$

The two-particle correlation function of the diffractive component is

$$\begin{aligned} \varphi_{2D}^{(n)}(y_1, y_2, s) &= n(n-1) \tilde{Q}_{2D}^{av}(n, y_1, y_2, s) - \\ &- n^2 \tilde{Q}_{1D}^{av}(n, y_1, s) \tilde{Q}_{1D}^{av}(n, y_2, s) . \end{aligned} \quad (5.16)$$

From here, the divided semi-inclusive correlation function $\rho_{2D}^{(n)}(y_1, y_2, s)$ of the diffractive component can be obtained.

In Fig. 5.2, we show the semi-inclusive single particle distribution $\zeta_{1D}^{(n)}(y, s)$ for the diffractive component as function of the rapidity for various values of the multiplicity n . For small n , a dip is pronounced ; here particle production is due to rather light clusters. At larger n the clusters produced need to be heavy, having smaller rapidity η ; they tend to fill in the dip. Then a plateau in the rapidity distribution of $\zeta_{1D}^{(n)}(y, s)$ is seen as is the case for the semi-inclusive single particle distributions of the non-diffractive component $\zeta_{1ND}^{(n)}(y, s) = n \tilde{Q}_{1ND}^{av, m}(n, y, s)$; the functions $\tilde{Q}_{1ND}^{av, m}(l, y, s)$ were given in Fig. 4.1 for various m and l . The functions $\zeta_{1ND}^{(n)}(y, s)$ and $\zeta_{1D}^{(n)}(y, s)$ are related in a simple way [see also Eq. (5.14)] to the normalized average quantities $\tilde{Q}_{1ND}^{av}(y, s)$ and $\tilde{Q}_{1D}^{av}(y, s)$. Therefore, the approximation $\tilde{Q}_{1ND}^{av}(y, s) \approx \tilde{Q}_{1D}^{av}(y, s)$ adopted in Ref. 26) is quite valid, although it is no longer necessary in our present treatment.

In Fig. 5.3, we plot the diffractive semi-inclusive two-particle distribution $\zeta_{2D}^{(n)}(y_1, y_2, s)$ at $\sqrt{s} = 50$ GeV as function of y_1 for various values of y_2 and for a reaction with $n=4$ (Fig. 5.3a) and $n=14$ particles (Fig. 5.3b). Of course, the diffractive cross-section for $n=14$ is very small ; this value of n is chosen to emphasize the change of $\zeta_{2D}^{(n)}(y_1, y_2, s)$ as function of n . These graphs should be compared with those in Fig. 4.4,

where the functions $\tilde{Q}_{2ND}^{av,m}(\ell, y_1, y_2, s)$ - which are related to the non-diffractive semi-inclusive two-particle distributions $\zeta_{2ND}^{(n)}(y_1, y_2, s)$ by Eq. (4.34) - are displayed in a similar way. These plots correspond to cuts parallel to the y_1 axis at $y_2 = 0, 1, 2, 3$ and 4 through the functions $\zeta_{2D}^{(n)}(y_1, y_2, s)$ in the (y_1, y_2) rapidity plane. At $y_1 = y_2 = 0$ the diffractive component is rather small. Due to symmetry, curves for $y_2 < 0$ are not given. From these graphs we deduce the sketch in Fig. 5.4 for the contours in the (y_1, y_2) plane for the diffractive component, which shows the two-bump structure of the functions $\zeta_{2D}^{(n)}(y_1, y_2, s)$. For larger n the two bumps are less pronounced leading almost to a plateau along the diagonal $y_1 = y_2$. The non-diffractive component at all n leads to a plateau along the diagonal $y_1 = y_2$. The average quantities $\tilde{Q}_{2ND}^{av}(y_1, y_2, s)$ and $\tilde{Q}_{2D}^{av}(y_1, y_2, s)$ defined in (4.27) and (5.15) therefore differ in form at small y_1 and y_2 (around $y_1 \approx y_2 \approx 0$) especially if only few particles emerge from one cluster (small n or ℓ , respectively in diffractive or non-diffractive case).

In Fig. 5.5, we plot the semi-inclusive two-particle correlation function $\varphi_{2D}^{(n)}(y_1, y_2, s)$ for the diffractive component at $\sqrt{s} = 50$ GeV as function of the rapidity y_1 and various values of y_2 . Figure 5.5a is for a reaction with multiplicity $n = 8$ and Fig. 5.5b for $n = 14$. Again, such large values of n are used to emphasize the changes as function of n ; at small n the correlation function $\varphi_{2D}^{(n)}(y_1, y_2, s)$ is rather uniform in the (y_1, y_2) plane, although the basic maximum-minimum structure shown in Figs. 5.5 and 5.6 is present, however, much less pronounced.

It is seen that the correlation is small for $y_1 \approx y_2 \approx 0$, and large for $y_1 \approx y_2$ around 2. For $y_2 > 0, y_1 < 0$ the correlation is negative and in absolute values as big as for $y_1 \approx y_2$. The situation is visualized in Fig. 5.6 in a schematic plot. In absolute values the diffractive correlation $\varphi_{2D}^{(n)}$ is larger for larger n .

6. - TWO-COMPONENT SEMI-INCLUSIVE DISTRIBUTIONS

After having derived the non-diffractive as well as the diffractive semi-inclusive one- and two-particle distributions (4.24), (4.25) and (5.14), (5.15) we can proceed to combine these two contributions with weights $\alpha^{(n)} = (\sigma_{n,ND})/(\sigma_{n,ND} + \sigma_{n,D})$ and $(1-\alpha^{(n)})$ (56). We have chosen the parameter c in the average number of central clusters $< m >$, Eq. (2.10), such that we reproduce approximately the tail of the multiplicity distribution at the higher NAL energies ; no fit was attempted. We further use that in pp collisions of $\sigma_{inel} \approx 33$ mb the diffractive component contributes about 6 to 7 mb (c), (1d), (59), (60). This gives the fraction $\alpha^{(n)}$ of the non-diffractive part as a function of n , displayed in Fig. 6.1. For $n_{ch} \gtrsim 14$, $\alpha^{(n)}$ is practically 1 and the cross-section is essentially non-diffractive. Although the model asymptotically has no KNO scaling, practical KNO scaling is found at accelerator energies. The multiplicity distribution due to the non-diffractive component is shown in Fig. 6.2 in KNO form.

The two-particle semi-inclusive rapidity correlation function by superposition of the diffractive and non-diffractive component (61) is

$$\begin{aligned} \varphi_2^{(n)}(y_1, y_2, s) = & \alpha^{(n)} \varphi_{2ND}^{(n)}(y_1, y_2, s) + (1 - \alpha^{(n)}) \varphi_{2D}^{(n)}(y_1, y_2, s) \\ & + \alpha^{(n)}(1 - \alpha^{(n)}) \left[\int_{1D}^{(n)}(y_1, s) - \int_{1ND}^{(n)}(y_1, s) \right] \left[\int_{1D}^{(n)}(y_2, s) - \int_{1ND}^{(n)}(y_2, s) \right]. \end{aligned} \quad (6.1)$$

In general, the third term does not vanish. We might, however, introduce the functions $\tilde{Q}_{1ND}^{av}$ and $\tilde{Q}_{1D}^{av}$ into the distributions $\zeta_{1ND}^{(n)}$ and $\zeta_{1D}^{(n)}$. In good approximation it is $\tilde{Q}_{1D}^{av}(y, s) \approx \tilde{Q}_{1ND}^{av}(y, s)$, as discussed in Section 5 (see Figs. 4.1 and 5.2), i.e., the diffractive and the non-diffractive part of the semi-inclusive single particle distributions are approximately equal. Then the third term in (6.1) cancels. This approximation has been used in Ref. 28).

In Section 4.1 we have discussed the problems encountered by evaluating the non-diffractive contribution (see Fig. 4.2) according to Eq. (4.23). In the following we use the average quantities $\tilde{Q}_{1ND}^{av}(y_1, s)$ and $\tilde{Q}_{2ND}^{av}(y_1, y_2, s)$ introduced in Section 4.2 in order to compute the non-

diffractive two-particle distribution $\psi_{2ND}^{(n)}(y_1, y_2, s)$ according to Eq. (4.35) as a function of the average number of charged particles per cluster $n < 1/m >$. From Eqs. (6.1) and (1.2) we deduce the divided semi-inclusive correlation function $\rho_2^{(n)}(y_1, y_2, s)$ which is shown in Figs. 6.3a and 6.3b for $y_1 = y_2 = 0$ and $y_1 = 0, y_2 = 3$, at $\sqrt{s} = 62.8$ and 23.6 GeV. The data on $\rho_2^{(n)}(0, 3)$ are especially rather uncertain. From the graphs on $\rho_2^{(n)}(0, 0)$ we judge that $n < 1/m > \sim 3$ to 3.5 might be a good value, although $n < 1/m > \sim 2.5$ does represent the data as well. Since leading clusters are not treated explicitly in the present version of the model, no firm predictions on low multiplicities can be obtained.

We conclude that correlations at ISR energy in semi-inclusive channels can be described by a superposition of a diffractive and a non-diffractive component. In the non-diffractive component central cluster of average mass $< M > \sim 2$ GeV are produced independently ; they decay isotropically into 2.5 to 3.5 charged particles in the average. The average cluster mass remains independent of energy and at higher energy more central clusters are produced, the average multiplicity of the central clusters increases with $< m > \sim \ln \sqrt{s/s_0}$.

It is very appealing that multiparticle production on the inclusive as well as exclusive (semi-inclusive) level can be understood via production and decay of the very same type of intermediate objects.

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APPENDIX - PARTICLE PRODUCTION WITHIN THE INDEPENDENT CLUSTER
EMISSION MODEL

In this Appendix we set up our notation for exclusive, semi-inclusive and inclusive quantities and give their expressions obtained with the assumption of uncorrelated production of non-diffractive clusters.

A.1. - Distribution functions for particles

In a collision the exclusive distribution of k particles with the momenta $p_1, \dots, p_k$

$$\frac{1}{\sigma(s)} \frac{d^k \sigma(s)}{dp_1 \dots dp_k} \equiv \Omega^{(k)}(p_1, \dots, p_k, s) \quad (\text{A.1})$$

is related to the multiplicity distribution

$$\omega_k(s) = \frac{\sigma_k(s)}{\sigma(s)} = \frac{1}{k!} \int \dots \int dp_1 \dots dp_k \Omega^{(k)}(p_1, \dots, p_k, s) \quad (\text{A.2})$$

The inelastic cross-section $\sigma_{\text{inel}}(s) \equiv \sigma(s)$ is

$$\sigma(s) = \sum \sigma_k(s) \quad (\text{A.3})$$

and further it is

$$\sum \omega_k(s) = 1 \quad (\text{A.4})$$

We abbreviate $dp = d^3p/E$.

We assume that the exclusive distribution function of k particles factorizes

$$\frac{1}{k!} \Omega^{(k)}(p_1, \dots, p_k, s) = \omega_k(s) f^{(k)}(p_1) \dots f^{(k)}(p_k) \quad (\text{A.5})$$

The single particle functions $f^{(k)}(p)$ are normalized to 1

$$\int f^{(k)}(p) dp = 1. \quad (A.6)$$

We do not indicate explicitly their s dependence. The label k indicates that in the considered reaction k particles are produced. Of course, kinematics will introduce correlations which destroy over-all factorization. We therefore limit the ansatz (A.5) to central cluster only, the momenta of the leading particles or leading clusters can be chosen such that over-all energy and momentum is conserved.

The inclusive distribution of k particles is

$$\begin{aligned} \frac{1}{\sigma(s)} \frac{d^k \sigma(s)}{dp_1 \dots dp_k} &\equiv \Phi^{(k)}(p_1, \dots, p_k, s) \\ &= \sum_{\ell=0}^{\infty} \frac{1}{\ell!} \int \dots \int dp_{k+1} \dots dp_{k+\ell} \Omega^{(k+\ell)}(p_1, \dots, p_k, p_{k+1}, \dots, p_{k+\ell}). \end{aligned} \quad (A.7)$$

Using the factorization hypothesis (A.5) for $\Omega^{(k+\ell)}$, the inclusive distribution becomes

$$\begin{aligned} \Phi^{(k)}(p_1, \dots, p_k, s) &= \sum_{\ell=0}^{\infty} \frac{1}{\ell!} \int \dots \int dp_{k+1} \dots dp_{k+\ell} (k+\ell)! \omega_{k+\ell}(s) \cdot \\ &\quad f^{(k+\ell)}(p_1) \dots f^{(k+\ell)}(p_k) f^{(k+\ell)}(p_{k+1}) \dots f^{(k+\ell)}(p_{k+\ell}). \end{aligned} \quad (A.8)$$

After performing the integration, Eq. (A.8) simplifies to

$$\Phi^{(k)}(p_1, \dots, p_k, s) = \sum_{\ell=0}^{\infty} \frac{(k+\ell)!}{\ell!} \omega_{k+\ell}(s) f^{(k+\ell)}(p_1) \dots f^{(k+\ell)}(p_k). \quad (A.9)$$

The semi-inclusive distribution of k observed particles in a reaction with n particles is

$$\begin{aligned}
 \frac{1}{\sigma_n(s)} \frac{d^k \sigma_n(s)}{dp_1 \dots dp_k} &\equiv \overline{\sum}_k^{(n)} (p_1, \dots, p_k, s) \\
 &= \frac{1}{(n-k)!} \int \dots \int dp_{k+1} \dots dp_n \Omega^{(n)}(p_1, \dots, p_n, s) \\
 &= \frac{n!}{(n-k)!} \omega_n(s) f^{(n)}(p_1) \dots f^{(n)}(p_k) .
 \end{aligned} \tag{A.10}$$

It is also useful to define a modified semi-inclusive distribution

$$\overline{\sum}_k^{(n)}(p_1, \dots, p_k, s) = \frac{n!}{(n-k)!} f^{(n)}(p_1) \dots f^{(n)}(p_k) . \tag{A.11}$$

We remark that the semi-inclusive distribution $\zeta_k^{(n)}(p_1, \dots, p_k, s)$ is normalized to $n!/(n-k)!$. The semi-inclusive two-particle correlation function is

$$\varphi_2^{(n)}(p_1, p_2, s) = \overline{\sum}_2^{(n)}(p_1, p_2, s) - \overline{\sum}_1^{(n)}(p_1, s) \overline{\sum}_1^{(n)}(p_2, s) . \tag{A.12}$$

By means of factorization expressed in Eq. (A.11), it is seen that the two-particle correlation function $\varphi_2^{(n)}(p_1, p_2, s)$ is normalized to $-n$

$$\begin{aligned}
 \varphi_2^{(n)}(p_1, p_2, s) &= n(n-1) f^{(n)}(p_1) f^{(n)}(p_2) - n^2 f^{(n)}(p_1) f^{(n)}(p_2) \\
 &= -n f^{(n)}(p_1) f^{(n)}(p_2) .
 \end{aligned} \tag{A.13}$$

The divided correlation function also is in use

$$\wp_2^{(n)}(p_1, p_2, s) = \frac{\varphi_2^{(n)}(p_1, p_2, s)}{\overline{\sum}_1^{(n)}(p_1, s) \overline{\sum}_1^{(n)}(p_2, s)} = \frac{\overline{\sum}_2^{(n)}(p_1, p_2, s)}{\overline{\sum}_1^{(n)}(p_1, s) \overline{\sum}_1^{(n)}(p_2, s)} - 1 . \tag{A.14}$$

A.2. - Distribution functions for the production of central clusters

The rapidity distribution of one cluster with mass M and rapidity η in a reaction with m central clusters is $F^{(m)}(\eta, M, s)$. This function is normalized to 1

$$\int_{M_0}^{M_{\max}} \int_{\eta_{\min}}^{\eta_{\max}} F^{(m)}(\eta, M, s) dM d\eta = 1. \quad (A.15)$$

The integration is limited to such intervals of η and M for which central clusters are found. The label (m) in $F^{(m)}(\eta, M, s)$ indicates that kinematic restrictions can distinguish $F^{(m)}(\eta, M, s)$ slightly from $F^{(m')}(\eta, M, s)$.

The exclusive distribution of m clusters is

$$\frac{1}{\sigma_{cl}(s)} \frac{d\sigma_{cl}(s)}{d\eta_1 \dots d\eta_m dM_1 \dots dM_m} \equiv W^{(m)}(\eta_1, \dots, \eta_m, M_1, \dots, M_m, s). \quad (A.16)$$

$\sigma_{cl}(s)$ is the cross-section for production of central clusters.

It has been shown ¹¹⁾ that from a factorized distribution function for inclusive cluster production follows a factorized distribution for exclusive cluster production. With the factorization assumption for inclusive cluster production, the exclusive cluster distribution is

$$\begin{aligned} & \frac{1}{m!} W^{(m)}(\eta_1, \dots, \eta_m, M_1, \dots, M_m, s) \\ &= w_m(s) F^{(m)}(\eta_1, M_1, s) \dots F^{(m)}(\eta_m, M_m, s). \end{aligned} \quad (A.17)$$

Due to the normalization (A.15) the multiplicity distribution of the cluster is

$$w_m(s) = \int \dots \int \frac{1}{m!} W^{(m)}(\eta_1, \dots, \eta_m, M_1, \dots, M_m) d\eta_1 \dots d\eta_m dM_1 \dots dM_m. \quad (A.18)$$

The inclusive distribution of k clusters with the factorization assumption becomes

$$\begin{aligned} \frac{1}{\sigma_{cl}(s)} \frac{d\sigma_{cl}(s)}{d\eta_1 \dots d\eta_k dM_1 \dots dM_k} &= V^{(k)}(\eta_1, \dots, \eta_k, M_1, \dots, M_k, s) \\ &= \sum_{\ell=0}^{\infty} \frac{1}{\ell!} \int \dots \int d\eta_{k+1} \dots d\eta_{k+\ell} dM_{k+1} \dots dM_{k+\ell} \cdot \\ &\cdot (k+\ell)! w_{k+\ell}(s) F^{(k+\ell)}(\eta_1, M_1, s) \dots F^{(k+\ell)}(\eta_{k+\ell}, M_{k+\ell}, s). \end{aligned} \quad (A.19)$$

Using (A.15) the inclusive cluster distribution is

$$V^{(k)}(\eta_1, \dots, \eta_k, M_1, \dots, M_k, s) = \sum_{\ell=0}^{\infty} \frac{(k+\ell)!}{\ell!} w_{k+\ell}(s) F^{(k+\ell)}(\eta_1, M_1, s) \dots F^{(k+\ell)}(\eta_{k+\ell}, M_{k+\ell}, s). \quad (A.20)$$

The semi-inclusive distribution of k clusters in an event with m clusters is

$$\begin{aligned} \frac{1}{\sigma_{cl}(s)} \frac{d^k \sigma_{cl}(s)}{d\eta_1 \dots d\eta_k dM_1 \dots dM_k} &= S_k^{(m)}(\eta_1, \dots, \eta_k, M_1, \dots, M_k, s) \\ &= \frac{1}{(m-k)!} \int \dots \int d\eta_{k+1} \dots d\eta_m dM_{k+1} \dots dM_m \cdot \\ &\cdot W^{(m)}(\eta_1, \dots, \eta_m, M_1, \dots, M_m, s). \end{aligned} \quad (A.21)$$

With factorization of $W^{(m)}(\eta_1, \dots, \eta_m, M_1, \dots, M_m, s)$ expressed in (A.17) and after performing the integration it is

$$S_k^{(m)}(\eta_1, \dots, \eta_k, M_1, \dots, M_k, s) = \frac{m!}{(m-k)!} F^{(m)}(\eta_1, M_1, s) \dots F^{(m)}(\eta_k, M_k, s) \quad (A.22)$$

The modified semi-inclusive cluster distribution is

$$\mathcal{I}_k^{(m)}(\eta_1, \dots, \eta_k, M_1, \dots, M_k, s) = \frac{m!}{(m-k)!} F^{(m)}(\eta_1, M_1, s) \dots F^{(m)}(\eta_k, M_k, s). \quad (\text{A.23})$$

A.3. - The decay distribution of one cluster

The function $u^{(\ell)}(y, M, \eta)$ describes the decay of a cluster with mass M and rapidity η into a particle with rapidity y where ℓ particles are produced altogether in the cluster decay. We do not make the dependence on $p_\perp$ explicit, see Eq. (3.7). The function $u^{(\ell)}(y, M, \eta)$ is normalized to 1

$$\int_{-\infty}^{\infty} dy u^{(\ell)}(y, M, \eta) = 1. \quad (\text{A.24})$$

The exclusive rapidity distribution $\mathcal{O}^{(\ell)}(y_1, \dots, y_\ell, M, \eta)$ of ℓ particles due to the decay of one cluster in an uncorrelated fashion is

$$\frac{1}{\ell!} \mathcal{O}^{(\ell)}(y_1, \dots, y_\ell, M, \eta) = \mathcal{P}^{(\ell)}(M) u^{(\ell)}(y_1, M, \eta) \dots u^{(\ell)}(y_\ell, M, \eta). \quad (\text{A.25})$$

$\mathcal{P}^{(\ell)}(M)$ is the multiplicity distribution of the particles emerging from this cluster,

$$\int \dots \int \frac{1}{\ell!} \mathcal{O}^{(\ell)}(y_1, \dots, y_\ell, M, \eta) dy_1 \dots dy_\ell = \mathcal{P}^{(\ell)}(M). \quad (\text{A.26})$$

The inclusive rapidity distribution of k particles due to the decay of one cluster again factorizes

$$f^{(k)}(y_1, \dots, y_k, M, \eta) = \sum_{\ell=0}^{\infty} \frac{(k+\ell)!}{\ell!} \mathcal{P}^{(\ell+k)}(M) u^{(\ell+k)}(y_1, M, \eta) \dots u^{(\ell+k)}(y_k, M, \eta). \quad (\text{A.27})$$

In particular we have the inclusive single particle distribution due to one-cluster decay

$$f^{(1)}(y, M, \eta) = \sum_{\ell=1}^{\infty} \ell \mathcal{P}^{(\ell)}(M) u^{(\ell)}(y, M, \eta) \quad (\text{A.28})$$

and the inclusive two-particle distribution

$$f^{(2)}(y_1, y_2, M, \eta) = \sum_{\ell=2}^{\infty} \ell(\ell-1) \mathcal{P}^{(\ell)}(M) u^{(\ell)}(y_1, M, \eta) u^{(\ell)}(y_2, M, \eta). \quad (\text{A.29})$$

Observing only k particles in the ℓ particle decay of a cluster with mass M and rapidity η leads to the semi-inclusive cluster decay distribution

$$S_{2k}^{(\ell)}(y_1, \dots, y_k, M, \eta) = \frac{\ell!}{(\ell-k)!} \mathcal{P}^{(\ell)}(M) u^{(\ell)}(y_1, M, \eta) \dots u^{(\ell)}(y_k, M, \eta). \quad (\text{A.30})$$

The modified semi-inclusive decay distribution is

$$S_{2k}^{(\ell)}(y_1, \dots, y_k, M, \eta) = \frac{\ell!}{(\ell-k)!} u^{(\ell)}(y_1, M, \eta) \dots u^{(\ell)}(y_k, M, \eta). \quad (\text{A.31})$$

For the single particle and two-particle semi-inclusive decay distributions due to one-cluster decay we have

$$\mathcal{S}_{z_1}^{(e)}(y, M, \eta) = \ell u^{(e)}(y, M, \eta) \quad (\text{A.32})$$

and

$$\mathcal{S}_{z_2}^{(e)}(y_1, y_2, M, \eta) = \ell(\ell-1) u^{(e)}(y_1, M, \eta) u^{(e)}(y_2, M, \eta) . \quad (\text{A.33})$$

	ch-ch	ch-0	+ +	- -	+ -
pp 102 GeV/c pp 205 GeV/c NAL 4a), 4b)	0.65 ± 0.05 0.50 ± 0.05		0.5 ± 0.08 0.25 ± 0.05	0.4 ± 0.08 0.38 ± 0.06	0.7 ± 0.08 0.67 ± 0.06
pp $\sqrt{s} = 23$ to 63 GeV ISR 2), 3)	0.65 ± 0.05	0.6			
π^- -p 40 GeV/c Serpukhov 46)			0.26 ± 0.10	0.26 ± 0.10	0.65 ± 0.07
Cluster model 43), 44)	0.52	0.6	0.3	0.3	0.6

TABLE II.1 Central value of the inclusive two-particle correlation function $R(y_1 = 0, y_2 = 0)$ for various secondaries.

$M(\text{GeV})$ ℓ	0.8	1.3	1.8	2.3	2.8	3.3	3.8	4.3	4.8	5.3
2	0.130	0.235	0.340	0.443	0.547	0.649	0.752	0.854	0.957	1.059
4	0.0605	0.125	0.187	0.247	0.309	0.370	0.432	0.492	0.553	0.614
6	-	0.076	0.118	0.161	0.202	0.244	0.287	0.329	0.371	0.413
8	-	0.048	0.082	0.113	0.145	0.177	0.207	0.239	0.271	0.303
10	-	-	0.060	0.085	0.110	0.135	0.160	0.186	0.210	0.236
12	-	-	0.040	0.066	0.087	0.107	0.128	0.149	0.171	0.192
14	-	-	-	0.053	0.070	0.088	0.106	0.123	0.142	0.160
16	-	-	-	0.032	0.058	0.073	0.089	0.104	0.120	0.136
18	-	-	-	-	0.048	0.062	0.075	0.089	0.103	0.117
20	-	-	-	-	0.012	0.053	0.065	0.077	0.090	0.102
22	-	-	-	-	-	0.045	0.057	0.068	0.079	0.090
24	-	-	-	-	-	-	0.050	0.060	0.070	0.080
26	-	-	-	-	-	-	0.042	0.053	0.063	0.072
28	-	-	-	-	-	-	-	0.048	0.056	0.065
30	-	-	-	-	-	-	-	0.038	0.057	0.059

TABLE III.1 The parameter $T(\ell, M)$ (in GeV) in the exclusive cluster decay law (3.1) as function of the cluster mass M for various decay multiplicities ℓ .

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$$\begin{aligned}
 \overline{P}_2^{(n)}(y_1, y_2, s) &= \sum_{m=1}^{m_{\max}} w_m(s) m \sum_{n_1=2}^{n-2(m-1)} n_1(n_1-1) Q_2^{av, m}(n_1, y_1, y_2, s) \times \\
 &\times P_{m-1}^{av, m}(n-n_1) + \sum_{m=2}^{m_{\max}} w_m(s) m(m-1) \sum_{n_1=2}^{n-2(m-1)} Q_1^{av, m}(n_1, y_1, s) \times \\
 &\times \sum_{n_2=2}^{n-n_1-2(m-2)} Q_1^{av, m}(n_2, y_2, s) n_1 n_2 P_{m-2}^{av, m}(n-n_1-n_2)
 \end{aligned}$$

The formula (4.23) used in our calculation is a good approximation.
We thank A. Morel for discussion of this point.

FIGURE CAPTIONS

Figure 3.1 The decay distribution $u^{(\ell)}(y, M, \eta)$ Eq. (3.1) of an isotropically decaying central cluster with mass M and rapidity $\eta = 0$ plotted as function of y^2 for the parameters cluster mass M and decay multiplicity ℓ .

Figure 3.2 The parameter $a^2(T)$ in the Gaussian cluster decay law, Eq. (3.8) as function of the parameter $T = T(M/\ell)$ in the cluster decay law, Eq. (3.1).

Figure 4.1 The non-diffractive semi-inclusive single particle distributions $\zeta_1^{(n)}(y, s)$ at
a) $\sqrt{s} = 23.6$ GeV, and
b) $\sqrt{s} = 50$ GeV, and
various values of the multiplicity n . This function has been computed from Eqs. (4.22) and (4.24) without use of the approximations introduced later in Section 4.2.

Figure 4.2 The non-diffractive component of the semi-inclusive two-particle correlation function $\rho_2^{(n)}(y_1, y_2)$ computed from Eqs. (1.2) and (4.23) to (4.25).
a) $\rho_2^{(n)}(0, 0)$ at $\sqrt{s} = 62.8$ GeV ;
b) $\rho_2^{(n)}(0, 3)$ at $\sqrt{s} = 62.8$ GeV.

Figure 4.3 The normalized average function $\tilde{Q}_1^{av, m}(\ell, y, s)$ introduced in Eqs. (4.13) and (4.26). It describes a one-particle rapidity distribution of an ℓ particle decay of an average cluster in an event with m central clusters at energy $\sqrt{s} = 50$ GeV. The curves are labelled by (ℓ, m) ; no leading clusters are taken into account.

Figure 4.4 The normalized average function $\tilde{Q}_2^{av, m}(\ell, y_1, y_2, s)$ is introduced in Eqs. (4.1g) and (4.27). It describes the rapidities of two particles in an ℓ particle decay of an average cluster in an event with m central clusters at energy $\sqrt{s} = 50$ GeV. The curves are cuts in the rapidity plane at $y_2 = -3.8, -2.8, -1.8, -0.8, +0.2, 1.2, 2.2$ and 3.2 . They are given for various values of (ℓ, m)

Figure 4.4

Ctd.

- a) $(\ell, m) = (3.2)$;
- b) $(\ell, m) = (3.4)$;
- c) $(\ell, m) = (6.2)$;
- d) $(\ell, m) = (3.2)$;
- e) $(\ell, m) = (9.6)$.

No leading clusters are taken into account.

Figure 4.5

Contour plots for the functions $\tilde{Q}_2^{av,m}(\ell, y_1, y_2, s)$ as deduced from Figure 4.2 for

- a) $(\ell, m) = (3.2)$;
- b) $(\ell, m) = (3.4)$.

Figure 5.1

The diffractive graph.

Figure 5.2

The diffractive semi-inclusive single particle distribution $\zeta_1^{(n)}(y, s)$ as function of y for various values of n at $\sqrt{s} = 50$ GeV, $n = 4, 6, 8$ and 12 .

Figure 5.3

The diffractive semi-inclusive two-particle distribution $\zeta_{2D}^{(n)}(y_1, y_2, s)$ as function of y_1 for various values of y_2 , $y_2 = 0, 1, 2, 3, 4$ at $\sqrt{s} = 50$ GeV for

- a) $n = 4$, and
- b) $n = 14$.

Figure 5.4

Contour plots for the two-particle semi-inclusive distribution of the diffractive component deduced from Figures 5.3a), b) for

- a) $n = 4$, and
- b) $n = 14$.

Figure 5.5

The diffractive semi-inclusive two-particle correlation $\varphi_2^{(n)}(y_1, y_2, s)$ at $\sqrt{s} = 50$ GeV as function of y_1 for various values of y_2

- a) for $n = 8$ and $y_2 = 0, 2$ and 4 , and
- b) for $n = 14$ and $y_2 = 0, 1, 2, 3$ and 4 .

Figure 5.6

Contour plot for the diffractive semi-inclusive two-particle correlation function $\varphi_{2D}^{(n)}(y_1, y_2, s)$ deduced from Figure 5.5 for $n = 14$.

Figure 6.1 The parameter $\alpha^{(n)} = (\sigma_{ND}^{(n)})/(\sigma_{ND}^{(n)} + \sigma_D^{(n)})$ as function of the charged multiplicity $n = n_{ch}$.

Figure 6.2 The multiplicity distribution of the non-diffractive component (for central clusters only) in KNO form for $p_{lab} = 100, 300$ and 1250 GeV/c in pp collisions compared with data at Serpukhov and NAL [see Refs. 1].

Figure 6.3 The divided semi-inclusive two-particle correlation function $\rho_2^{(n)}(0,0)$ and $\rho_2^{(n)}(0,3)$ as function of the charged multiplicity n_{ch} and for the number of charged particles per central cluster as parameter $n < 1/m > = 2.5, 3$ and 3.5. The solid line is $\rho_{2tot}^{(n)}$; the broken line indicates the non-diffractive part $\rho_{2ND}^{(n)}$. The data are from Ref. 23) for
a) $\sqrt{s} = 62.8$ GeV ;
b) $\sqrt{s} = 23.5$ GeV.

FIG. 31

FIG. 3.2

FIG. 4.1 a)

FIG. 4.1 b

FIG. 4.2

FIG. 4.3

FIG. 4.4 d+e)

e)

FIG. 4.5

FIG. 5.1

FIG. 5.2

FIG. 5.3 a

FIG. 5.3 b)

FIG. 5.4 a

FIG. 5.4 b)

FIG. 5.5a

FIG. 5.5 b

FIG 6.2

FIG. 6.3 a)

FIG. 6.3 b)