Archives

AMPLITUDES FOR ρ AND A_2 REGGE EXCHANGES
IN Δ PRODUCTION PROCESSES

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A B S T R A C T

In Δ production processes like $\pi^+p \rightarrow \pi^0\Delta^{++}$, $\pi^+p \rightarrow \eta\Delta^{++}$, $K^+p \rightarrow K^0\Delta^{++}$, $K^-n \rightarrow \bar{K}^0\Delta^-$ exchange degeneracy breaking between ρ and A_2 Regge poles is shown to be present in all s channel helicity amplitudes including the double-flip one. Once the dominance of two among four amplitudes and SU_3 invariance are assumed, the amplitudes are obtained in a rather model-independent way. The peculiar feature of these amplitudes are analyzed and in particular some evidence for universality in the impact parameter representation is pointed out. Possible explanations are proposed.

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INTRODUCTION

In the framework of two-body Regge phenomenology, two main features have emerged in the recent past years : the peripherality of the amplitudes and the exchange degeneracy of the Regge poles.

Peripherality of the imaginary part of the amplitude was shown by Harari ¹⁾ to be a quite general empirical rule for s channel helicity amplitudes. As regards the real part, one knows that peripherality is ruled out ²⁾, implying difficulties for two-body models ³⁾. Exchange degeneracy is a very useful tool for Regge models, since it allows to constrain their parametrization and is the basis for the important physical idea of duality between resonances and Regge poles ⁴⁾. Though correct as a first approximation, exchange degeneracy can be shown to be broken in many cases, especially when looking at line-reversed reactions ⁵⁾. Different modifications to the usual models were proposed in order to overcome these difficulties. Only a few of them seem to be systematic enough to be considered as a possible solution ⁶⁾. They were mainly applied to πN , KN scattering and hypercharge exchange reactions, sometimes for baryon-baryon scattering and vector-meson photoproduction but, to our knowledge, never for Δ production processes.

In this paper, we shall focus our attention on : $\pi^+ p \rightarrow \pi^0 \Delta^{++}$, $\pi^+ p \rightarrow \eta \Delta^{++}$, $K^+ p \rightarrow K^0 \Delta^{++}$, $K^- n \rightarrow \bar{K}^0 \Delta^-$. The first interest of these reactions is that many data are available, including the density matrix elements of the Δ . Concerning the amplitudes, the important feature of such reactions ($0^{-\frac{1}{2}+} \rightarrow 0^{-\frac{3}{2}+}$) is to allow a double s channel helicity flip, for which only little is known. In particular, the question of peripherality and exchange degeneracy is to be asked for the double flip amplitude of ρ and A_2 exchange. The second interest then is to allow a comparison with the way how ρ and A_2 are exchanged in reactions with a nucleon in the final state ($\pi^- p \rightarrow \pi^0 n$, $\pi^- p \rightarrow \eta n$, $K^+ n \rightarrow K^0 p$, $K^- p \rightarrow \bar{K}^0 n$). The fact that the curves for $d\sigma/dt$ have the same shape in both cases may be of some help since it suggests that the non-flip and the two-single flip amplitudes in Δ production might be similar to those in the above-mentioned reactions. In fact, the two sets of reactions have indeed different features as it is shown in Section 1 of our work. Line reversal symmetry and exchange degeneracy appear to be strongly broken in Δ production reactions. This fact is shown to be hard to reconcile in a consistent way with existing models.

The origin of these difficulties is investigated through an amplitude analysis based on the whole set of data available at present, with the help of some physical assumptions. The way of obtaining the amplitudes is described in Section 2. As a result, we find that exchange degeneracy is actually broken, but in all s channel amplitudes, in contrast with currently admitted ideas, as shown in Section 3. This section contains a particular study of the double-flip amplitude. In Section 4, we conclude by proposing possible interpretations of our results in the framework of models for two-body scattering or three-body effects. Finally, we select a few typical experiments which would be able to test the different theoretical proposals.

1. - THE EXPERIMENTAL EXCHANGE DEGENERACY BREAKING : A PUZZLE FOR USUAL MODELS

The experimental data on the considered Δ production processes consist in differential cross-sections and density matrix elements for the decay of the resonance. Among them, we first consider those obtained at P_{lab} values around 5 GeV/c, for which we shall give the results of our amplitude analysis ⁷⁾. The data on differential cross-sections are given in Fig. 1 for $\pi^+p \rightarrow \pi^0\Delta^{++}$, in Fig. 2 for $\pi^+p \rightarrow \eta\Delta^{++}$, and in Fig. 3 for the s - u crossed reactions $K^+p \rightarrow K^0\Delta^{++}$ and $K^-n \rightarrow \bar{K}^0\Delta^-$. This last figure shows a striking line-reversal breaking. The log log plot of Fig. 4 is particularly convincing.

Note that a pure Regge behaviour would give straight lines on this plot. The value of $|t|$ is chosen in such a way that it corresponds to the region with the highest statistics. In fact, normalization errors are often present, particularly in reactions involving neutron scattering off deuterium. But the systematic error quoted ⁸⁾ of 20% is not sufficient to give essential modifications. Note that with the normalizations given, the SU_3 relations between all four reactions are satisfied ⁷⁾. We shall assume in the next sections that the data have correct normalizations.

At first sight, this exchange degeneracy breaking is not surprising. The situation is quite similar to that of hypercharge exchange reactions - $KN \rightarrow \pi Y$ and $\pi N \rightarrow KY$ (reactions a) - where the first reactions

("exotic" in the framework of duality diagrams) have bigger cross-sections than the second ones ⁵⁾. In the analogous nucleon reactions (b), $K^-p \rightarrow \bar{K}^0 n$ and $K^+n \rightarrow K^0 p$ exchange degeneracy is broken, but at smaller energy ⁹⁾. Above 5 GeV/c, the two s-u crossed reactions seem to coincide, and this fact has been considered as an argument in favour of exchange degeneracy. What seems new in the case of Δ production (c) is the relatively large effect obtained at 3-5 GeV/c, difficult to explain consistently with other reactions. For instance, the RPR model gives the correct features for reactions (b) ^{6c)} and (a) ¹⁰⁾ but not for reactions (c) (see Figs. 1,2,3,4) where no important line reversal breaking is obtained. As in similar cases ^{6c,d)}, exchange degeneracy breaking is due to Regge cuts expected to appear mainly in non-flip amplitudes. These are not strong enough in Δ production to get the correct size. Thus, the situation would favour models starting with non-degenerate poles ^{6a,e)}, but the different behaviour of nucleon and Δ production is then explained only by a technical and ad hoc choice of the parameters. This is not true for models related to an s channel approach ^{6f)} which would be interesting to consider.

In view of these difficulties for most of the usual two-body models, we shall now try to get out more precise features of the amplitudes through a more or less model-independent way completing the already considered set of data with the observed density matrix elements of the Δ resonance.

2. - DETERMINATION OF THE AMPLITUDES

The experimental data on the density matrix elements - for instance for the reaction $\pi^+p \rightarrow \pi^0 \Delta^{++}$ (Fig. 5) - show the features predicted by the M_1 dominance model of Stodolsky and Sakurai ¹¹⁾. In this model, the ρ coupling to N- Δ essentially contributes to the magnetic dipole moment M_1 . Using the same arguments as Renninger and Sarma ¹²⁾, we shall assume that one invariant amplitude B [see Eq. (1) : the transition operator] is dominant in Δ production reactions.

In order to improve the fit near the forward direction, we add the amplitude D, as a small correction. Apart from a rather successful fit, the argument of Renninger and Sarma to use invariant amplitudes was to

avoid kinematical singularities. One of the aims of our work is to discuss their hypothesis, since M_1 dominance does not provide us with a unique ansatz, and we know that s channel helicity amplitudes, for instance, are also free of kinematical singularities. In fact, we shall give our results for these helicity amplitudes.

2.1. - The transition operator

The expression of the transition operator is

$$T = \gamma^5 [A Q + 4 B \gamma^5 L + 2 C K + 4 D \not{K} K] \quad (1)$$

where (Fig. 6)

$$K = \frac{k+k'}{2}, \quad Q = k-k', \quad P = \frac{p+p'}{2}, \quad L = \epsilon_{\mu\nu\rho\sigma} P^\nu Q^\rho K^\sigma$$

2.2. - Parametrization of Regge poles and cuts

The amplitudes B and D , free of kinematic singularities are parametrized as follows.

i) - The Regge poles (ρ, A_2) are taken to be exchange degenerate

$$R = a e^{bt} P(\alpha) \xi(\alpha) v^{\alpha-h} \quad (2)$$

with

$$\xi(\alpha) = i + \operatorname{tg} \frac{\pi\alpha}{2} \quad \text{for positive signature}$$

$$\xi(\alpha) = i - \cotg \frac{\pi\alpha}{2} \quad \text{for negative signature}$$

$$P(\alpha) = \alpha(\alpha+1)(\alpha+2)(\alpha+3)$$

$\alpha(t)$ is the usual ρ trajectory (the factor h is fixed for each invariant amplitude, in order to obtain the same over-all power law for the s channel helicity amplitudes : $h=0,1,2$, respectively for A , B or C , and D).

ii) - The Regge cuts are taken as phenomenological cuts (with no reference to a definite model) in order to perform the amplitude analysis. (This kind of approach was already used in other reactions ¹³.) Only the cut trajectory is fixed to be the well-known "convolution" cut ρ pole * Pommeranchuk pole ($\rho * P$)

$$\alpha_c = \alpha_\rho + \frac{\alpha'_\rho \alpha'_P}{\alpha'_\rho + \alpha'_P} t$$

With the same notations as in Eq. (2), we write

$$C_\pm = e^{b_\pm t} P(\alpha_c) \frac{\xi(\alpha_c) \nu^{\alpha_c - \frac{1}{2}}}{a_\pm (\log \nu - i\frac{\pi}{2}) + c_\pm} \quad (3)$$

Eight parameters are to be determined for each amplitude (B and D) :
a, b (for the Regge poles) ; $a_\pm$, $b_\pm$, $c_\pm$ (for the cuts).

2.3. - SU₃ constraints

In order to reduce the number of unknown parameters, we shall relate the different reactions ($\pi^+ p \rightarrow \pi^0 \Delta^{++}$, $\pi^+ p \rightarrow \eta \pi^{++}$, $K^+ p \rightarrow K^0 \pi^{++}$, $K^- n \rightarrow \bar{K}^0 \Delta^-$) by exact SU₃ invariance of all Regge couplings, for poles and cuts. This assumption can be based on the fact that the following SU₃ relation on the differential cross-sections is verified ⁷⁾

$$\frac{d\sigma}{dt} (\pi^+ p \rightarrow \pi^0 \Delta^{++}) + 3 \frac{d\sigma}{dt} (\pi^+ p \rightarrow \eta \pi^{++}) = \frac{d\sigma}{dt} (K^+) + \frac{d\sigma}{dt} (K^-) \quad (4)$$

The list of parameters is given in the Table.

3. - RESULTING AMPLITUDES AND DISCUSSION

The results of the fit are given for the observables in Figs. 1,2,3,4 ($d\sigma/dt$) and 5 (ρ_{ij}). Having then checked that they agree with the data, we can discuss our amplitudes after performing a transformation to the more physically meaningful s channel helicity amplitudes : $M_{\frac{1}{2},\frac{1}{2}}$ (non-flip) $M_{\frac{3}{2},\frac{1}{2}}$ and $M_{-\frac{1}{2},\frac{1}{2}}$ (single flip) $M_{\frac{3}{2},-\frac{1}{2}}$ (double flip). The last one is of particular interest, as being reached for the first time.

The ρ and A_2 resulting amplitudes are presented for $p_{lab} = 5$ GeV/c as functions of the impact parameter $M(b)$ (Figs. 7a,b,c,d) and in the complex plane, as functions of $t : M(t)$ (Figs. 8a,b,c,d). Let us now comment and discuss these curves.

3.1. - Comparison with πN amplitudes

The b representation of the non-flip amplitude (Fig. 7a) and the two non-flip amplitudes (Figs. 7c,d) of the ρ pole are very similar to the results of the amplitude analysis in π nucleon reactions ²⁾. In the Argand plane $F(t)$, the situation is more puzzling. In particular, the non-flip amplitude (Fig. 8a) shows small loops near the forward direction and no peripheral zeros in the imaginary part (near $|t| \approx 0.2 \text{ GeV}^2$).

This unexpected feature can be attributed to the amplitude B being too dominant in our solution near $t=0$. Though fitting the data, the choice of B and D may not give the best description of M_1 dominance at small t : the compatibility of the geometrical properties of the amplitudes with the Sakurai-Stodolsky model for Δ production remains an open problem.

Nevertheless, we think that the main features of our resulting amplitudes as illustrated by the b representation are correct (Fig. 7). Small "accidents" in the forward direction are hidden. Moreover, we improve our results in b by fitting the data at relatively large $|t|$ ⁷⁾ (see, for instance, Fig. 1).

3.2. - Peripherism and b universality

At first look on Figs. 7a,b,c,d, the empirical rule of Harari seems to be well verified : the imaginary parts of the ρ amplitudes are peripheral (near 1 Fermi). Note that the tensor exchange does also show a peripheral structure. In this respect, our amplitudes are more in favour of dual absorptive models and/or s channel approach (Figs. 6b,f) than of other models.

But peripherality deserves more study. In particular, one must not forget the kinematical b^ν factors which are present in an amplitude with net s channel helicity flip equal to ν . Removing these factors (Fig. 9), one obtains the very interesting result that all amplitudes for a given exchanged pole are, up to a normalization, approximately equal, namely

$$M_\nu(b) \approx g_\nu b^\nu M(b) \quad (5)$$

where $M(b)$ is a "universal" function. "Universality" here means that apart from the factor $g_\nu b^\nu$, the amplitudes do not depend on the s channel helicity. But we may extend this property to amplitudes corresponding to the same trajectory in other reactions, since the universal curve obtained (Fig. 9) is very similar to that of π nucleon amplitudes ^{2),14)}. Hence, this property - which could be due only to the dominance of one invariant amplitude (M_1 dominance) in Δ production, seems to be more general. This property of "b universality", was already proposed as a general empirical rule for two-body reactions ^{14),6f)}. All amplitudes then are of the same type. Note that a model based on b universality would probably fit the data, but the g_ν are to be taken different for ρ and A_2 amplitudes, implying exchange degeneracy breaking.

3.3. - Explanation of line-reversal breaking

The interesting point which was at the beginning of our analysis is the typical line-reversal breaking of $K^+p \rightarrow K^0 \Delta^{++}$ (exotic reaction) and $K^-n \rightarrow \bar{K}^0 \Delta^-$ (non-exotic). The amplitudes of Fig. 8 show the mechanism for line-reversal breaking. The condition for the exotic reaction (" ρ - A_2 ") to be larger than the non-exotic one (" ρ + A_2 "), is the following : the angle θ between ρ and A_2 at given t has to be greater than 90° (and

smaller than 270°). The imaginary parts of ρ and A_2 being approximately equal, the effect is mainly due to the real parts. Namely, in all amplitudes, the real part in the negative signature is bigger (in absolute value), or anti-absorbed, in the language of absorption models. In particular, the double zeros predicted by a pure ρ exchange in the real parts, are absent in all amplitudes. This fact was already known for non-flip amplitudes ²⁾ and is here shown to be common to all helicity amplitudes. This is what we learn from Δ production and is hard to explain in models for which line-reversal breaking is due to cut contributions ^{6c,d,e)}, because they are too small in flip amplitudes. Universality in b , which is obtained approximately here, is a good candidate to be the mechanism which propagates line reversal breaking in all amplitudes.

4. - THEORETICAL OUTLOOK AND PRACTICAL TESTS

We have already pointed out that most of the two-body models would face difficulties in describing both nucleon scattering and Δ production in a natural way. The line-reversal breaking seems bigger for the second reactions, leading to real part effects in all s channel helicity amplitudes and not only in the non-flip one, as commonly expected.

In fact, we saw that the empirical law of b universality seems to agree with available data. A model, using such an empirical rule, is then favoured by our results. In particular the s channel approach of B. and F. Schrempp seems to agree with two typical features of our amplitudes : b universality and peripherality of tensor trajectories. Such a model is obtained from a "resonance" pole in the complex b plane of the non-exotic amplitude. As a check we have drawn the non-flip amplitude for $K^+p \rightarrow K^0 \Delta^{++}$ and $K^-n \rightarrow \bar{K}^0 \Delta^-$ in this plane. Figure 10 shows a real exotic amplitude and a rotating non-exotic one, in good agreement with the working hypothesis of the model. A more quantitative work with an s channel approach would be very interesting.

However, one should keep in mind that an s channel approach should not be contradictory to a t channel approach. Both have to converge to the physical amplitude ! Moreover, b universality is at present only a useful tool to generate all s channel amplitudes from one of them, but it still lacks more theoretical foundation. In fact, other types of effects have

to be considered which would be present in Δ production, because it corresponds to a three-body final state and not a two-body final state and not a two-body one.

The first possibility is the violation of line-reversal symmetry for resonance production processes, proposed by Berger and Fox¹⁵⁾. The violation is due to a coherent interference between the resonance and the background as obtained in dual phenomenological "B₅" amplitudes. A specific feature of this effect would be that it survives at large energy (asymptotic violation), and it decreases when varying the invariant mass away from the resonance position.

Another possibility is to relate meson-nucleon scattering to Δ production through a diagram of type (Fig. 11). In such a scheme Δ production at energy $\sqrt{s}$ is related to meson-nucleon scattering at lower energies $\sqrt{s_1}$. In the case of $KN \rightarrow K\Delta$, line reversal breaking would appear as a reflection of that observed in $KN \rightarrow KN$ at lower energies. In contrast with what happens in the Berger-Fox¹⁵⁾ model, the effect would disappear rapidly with increasing energy, but not with increasing the Δ invariant mass ($\sqrt{s_1}$). This possibility deserves more study, including perhaps other contributions.

As a conclusion about experiments, one would like to stress first the theoretical and experimental interest of studies of Δ or similar production processes. In particular, more precise experiments on the energy and invariant mass dependence of the reactions would be of great help for discussing the different theoretical ideas involved in the problem. For instance, new preliminary results on $K^+p \rightarrow K^0\Delta^{++}$ and $K^-n \rightarrow \bar{K}^0\Delta^-$ seem in favour of an s dependence of line-reversal breaking¹⁶⁾ (see Fig. 4), but one has to wait for definitive results, including also higher energies and different invariant masses (out of the Δ resonance). From a phenomenological point of view, the following conclusions can be drawn.

- 1) - The ρ and A_2 channel helicity amplitudes for Δ production verify approximately the "universality" in terms of impact parameter b

$$M_{\nu}^{\rho, A_2}(b) = g_{\nu}^{\rho, A_2} b^{\nu} M^{\rho, A_2}(b)$$

- 2) - The resulting amplitudes are very similar to those in meson-nucleon scattering [the "universal" amplitude $M^{\rho, A_2}(b)$ is the same]. But, due namely to a strongly coupled double-flip amplitude, the line-reversal breaking is persistent at larger energy.

- 3) - Though giving good results in the b representation, and fitting the data, the description in terms of the invariant amplitudes B and D of Renninger and Sarma¹²⁾ fails at very small transfer values t . Imposing the Stodolsky-Sakurai model (M_1 dominance) over s channel helicity amplitudes should give better results.

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Regge exchange	Amplitudes	Parameters
A_2, ρ	B	$a = 2.3$ $b = -0.27$
	D	$a = -0.92$ $b = 1.92$
ρ_{cut}	B	$a_- = -1.55$ $b_- = 4.25$ $c_- = -1.6$
	D	$a_- = -0.18$ $b_- = -0.61$ $c_- = 0.28$
$A_2 \text{ cut}$	B	$a_+ = -0.021$ $b_+ = 1.96$ $c_+ = -0.23$
	D	$a_+ = -0.033$ $b_+ = 0.81$ $c_+ = 0.81$
Pole trajectory : $0.485 + 0.88t = \alpha(t)$ Cut trajectory : $0.485 + 0.293t = \alpha_c(t)$		
χ^2 : 1.063 for 605 points		

TABLE : Parameters of the fit

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FIGURE CAPTIONS

Figures 1, 2, 3

Differential cross-sections. The p_{lab} values are taken near 5 GeV, the energy at which the analysis is performed. The continuous line is the fit with the "RPR" model, the dashed one corresponds to the analysis of Section 3 (model " Δ ").

Figure 4

Line-reversal breaking at $|t|$ fixed. The preliminary results ¹⁶⁾ are given with a special sign ($\nearrow$).

Figure 5

Density matrix elements for $\pi^+ p \rightarrow \pi^0 \Delta$ (Gottfried-Jackson frame) (all reactions give very similar data). Results for the " Δ " model of Section 3 (dashed line) and M_1 dominance model ¹¹⁾ (continuous line).

Figure 6

"Meson-nucleon $\rightarrow$ meson- Δ resonance" scattering.

Figure 7

Impact parameter representation of the amplitudes at 5 GeV/c.

- a) $M_{\frac{1}{2}, \frac{1}{2}}(b)$;
- b) $M_{\frac{1}{2}, -\frac{1}{2}}(b)$;
- c) $M_{\frac{3}{2}, \frac{1}{2}}(b)$;
- d) $M_{\frac{3}{2}, -\frac{1}{2}}(b)$.

Note that $\text{Re } \rho$, $\text{Im } A_2$, $\text{Im } \rho$, but $-\text{Re } A_2$ are drawn for convenience.

Figure 8

Argand plots of the amplitudes at 5 GeV/c.

- a) $M_{\frac{1}{2}, \frac{1}{2}}(t)$;
- b) $M_{\frac{1}{2}, -\frac{1}{2}}(t)$;
- c) $M_{\frac{3}{2}, \frac{1}{2}}(t)$;
- d) $M_{\frac{3}{2}, -\frac{1}{2}}(t)$.

The exotic amplitude is " $\rho-A_2$ " ; the non-exotic one " $\rho+A_2$ ".
The numbers along the curves are $|t|(\text{GeV}^2) \times 10$.

Figure 9 Comparison of the amplitudes and b universality.

a) ρ amplitudes

The drawn curves correspond to :

$$\frac{M_{\frac{1}{2},\frac{1}{2}}}{(M_0)} \approx \frac{-M_{\frac{3}{2},\frac{1}{2}}}{(M_1)} / 2b \approx \frac{-M_{\frac{1}{2},-\frac{1}{2}}}{(M_1')} / b \approx \frac{-M_{\frac{3}{2},-\frac{1}{2}}}{(M_2)} / 5b^2$$

b) A_2 amplitudes

$$\frac{M_{\frac{1}{2},\frac{1}{2}}}{(M_0)} / 2 \approx \frac{-M_{\frac{3}{2},\frac{1}{2}}}{(M_1)} / 3b \approx \frac{-M_{\frac{1}{2},-\frac{1}{2}}}{(M_1')} / 3b \approx \frac{-M_{\frac{3}{2},-\frac{1}{2}}}{(M_2)} / 7b^2$$

Figure 10 The non-flip amplitudes in the complex b plane for
 $K^+p \rightarrow K^0 \Delta^{++}$, $K^-p \rightarrow \bar{K}^0 \Delta^-$ the numbers are $|b| \times 10$ (Fermis).

Figure 11 Possible mechanisms for $(N\pi)$ production near the Δ .

a) Nucleon exchange ;

b) Pion exchange.

FIG. 1

FIG. 2

FIG.3

FIG. 5

FIG. 6

FIG. 7 a

FIG. 7 b

FIG. 7c

$M_{3/2, -1/2} P_{\text{LAB}} = 5$

FIG. 7d

FIG. 8 a

$$M_{1/2,-1/2}(t) \quad P_{\text{lab}} \quad 5 \text{ GeV}/c$$

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FIG. 8 c

FIG. 9b

$M_{1/2, 1/2}(b)$ $P_{lab} = 5 \text{ GeV}$

FIG. 10

FIG. 11 a

FIG. 11 b