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ELECTRON POSITRON ANNIHILATION INTO HADRONS
AND QUANTUM ELECTRODYNAMICS AT HIGH ENERGY

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A B S T R A C T

A linearly raising $R(s)$ forces QED into a Lee-Wick type of field theory. We discuss experimental implications of this possibility, and hadron models where such a linear behaviour can be realized.

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Recently ¹⁾ a great deal of attention has been focused on the large and roughly constant cross-section of the process :

$$e^+ e^- \rightarrow \text{hadrons} \quad (1)$$

compared to the smaller and falling cross-section of the purely leptonic process

$$e^+ e^- \rightarrow \mu^+ \mu^- \quad (2)$$

The ratio $R(s)$ of the two cross-sections appears to increase roughly linearly with s :

$$R(s) = \frac{\sigma_R(s)}{\sigma_{\mu\mu}(s)} \approx k s \quad (3)$$

with $k \sim 0.25 \text{ GeV}^{-2}$ in the region $5 \lesssim s \lesssim 25 \text{ GeV}^2$, where the experimental data are available ²⁾. Two possibilities are open : either scaling for process (1) will be reached at some higher energies [where $R(s) \sim \text{const.}$] or the increase of R with s will hold out indefinitely, linearly in s as in Eq. (3) or perhaps with some other power of s . Leaving aside for the moment the question of the implications of an indefinitely rising R for hadron structure, we will consider the consequences for QED of the assumption that Eq. (3) is exact as $s \rightarrow \infty$. These consequences are rather startling but not necessarily disastrous.

The main consequence of an indefinitely rising R will be in the structure of the photon propagator. Let us write

$$D(s) = \frac{1}{s} \cdot \frac{1}{1 - \Pi(s)} \quad (4)$$

where $\Pi(s)$ is the so-called photon proper self-energy. To lowest order in α , the imaginary part of $\Pi(s)$ is a sum of separate contributions coming from the leptonic and hadronic channels. Since μ pair intermediate states contribute to the imaginary part of $\Pi(s)$ at high energies ($s \gg 4m_\mu^2$) an amount ³⁾ $(\alpha/3)$, the hadronic contribution $\text{Im} \Pi^h(s)$ is given by

$$\text{Im} \Pi^h(s) = \frac{\alpha}{3} R(s) \quad (5)$$

The corresponding $\Pi^h(s)$ can then be computed by a subtracted dispersion relation. One subtraction constant is determined by the charge normalization condition $\Pi(0) = 0$, as required by conventional QED. If we wish Eq. (3) to be

valid indefinitely in s , we need a second subtraction to compute $\Pi^h(s)$, and it is here that we depart from usual QED. We find

$$\Pi^h(s) = \frac{\alpha}{3\pi} ks [\ln(-ks) + a] \quad (\text{large } |s|) \quad (6)$$

where a is a number which determines the second subtraction constant. The imaginary part of $\Pi^h(s)$ as given by Eq. (6) is as required by Eqs. (5) and (3).

Therefore the photon propagator which corresponds to Eq. (3) is

$$D(s) = \frac{1}{s} \frac{1}{[1 - \frac{\alpha}{3\pi} ks [\ln(-ks) + a]]} \quad (7)$$

where we have neglected the leptonic contributions to the photon self-energy $\Pi^\mu(s) = \alpha/3\pi \ln(-s/4m_\mu^2)$ which are asymptotically negligible with respect to the hadronic ones.

We see that $D(s)$ decreases at large $|s|$ like $s^{-2}(\ln s)^{-1}$. As a result the cross-section of one-photon exchange processes becomes strongly damped at large $|s|$ both in the space-like and time-like regions. The modified propagator leads to finite electromagnetic corrections to strong and weak processes, in particular to finite electromagnetic mass differences⁴⁾. The computation of these corrections will involve the value of the subtraction constant a , which can, however, be determined experimentally as we will shortly see.

Before proceeding in our analysis of the consequences of Eq. (7), we note that the structure of $D(s)$ in Eq. (7) is closely related to that which arises in the Lee-Wick⁵⁾ version of QED, which, for the simple case of only one charged lepton, has to order α the asymptotic form

$$D_{LW}(s) = \frac{1}{s} \frac{1}{(1 - \frac{s}{M^2} - \frac{\alpha}{3\pi} \ln(-\frac{s}{m^2}))} \quad (8)$$

where M is the mass of the heavy photon, and m the lepton mass.

If we compare this with the propagator in normal QED to the same order in α :

$$D_{QED}(s) = \frac{1}{s} \frac{1}{(1 - \frac{\alpha}{3\pi} \ln(-\frac{s}{m^2}))} \quad (9)$$

we note that the Lee-Wick propagator $D_{\text{LW}}(s)$ can be simply obtained from the usual one $D_{\text{QED}}(s)$ by making a second subtraction in $\Pi(s)$, which is not demanded by the behaviour of $\text{Im}\Pi(s)$. In our case, where we assume $\text{Im}\Pi(s) = \alpha/3(ks)$ the second subtraction is unavoidable so that we are forced into the asymptotically damped form, Eq. (7).

$D(s)$ as given in Eq. (7) shares the problems as well as the advantages of $D_{\text{LW}}(s)$. In particular, it has poles on the physical sheet of the s plane. When the subtraction constant $a > -(1 + \ln(3\pi/\alpha)) = -8.16$ the poles are at complex conjugate values M^2 and M^{*2} while for $a < -8.16$ the poles are at real space-like values of s . The latter possibility is clearly to be excluded.

The viability of a theory where the photon propagator has complex poles on the physical sheet has been analyzed in the context of the Lee-Wick proposal ⁶⁾. Such a theory is known to suffer from a mild case of acausality. The amplitude for processes with two photons being exchanged in the s channel is not analytical at $s = (M+M^*)^2$. This point of non-analyticity should presumably be chosen at a time-like and possibly large value of s which further restricts the acceptable range of a to $a > \log(\alpha/6) = -6.7$ for which $\text{Re} M^2 > 0$. Figure 1 shows the motion of the pole in the s complex plane as a function of a for the interesting values $a > -6.7$.

Although the propagator (7) has complex poles, it does not have the pole on the real axis which occurs in conventional QED to order α [see Eq. (9)]. The curing of the pole in conventional QED is still an open problem which has led to exciting theoretical developments based on the renormalization group ⁷⁾. Pending a solution of the fundamental problems touched above, we shall proceed by assuming that Eq. (7) gives an acceptable description of the photon propagator.

The cross-section of processes (1) and (2) in the one-photon approximation are obtained by substituting the modified propagator for the bare one :

$$\sigma_{\mu\mu} = \sigma_{\text{pointlike}} |sD(s)|^2 = \frac{4\pi\alpha^2}{3s} |sD(s)|^2 \quad (10)$$

$$\sigma_h = R(s) \sigma_{\mu\mu} = \frac{4\pi\alpha^2}{3s} |sD(s)|^2 R(s) \quad (11)$$

These expressions verify explicitly the relation ⁸⁾ between the cross-section for annihilation into a given final state and the contribution of this final state n to the absorptive part of the photon propagator :

$$\left[\text{Im } D(s) \right]_n = |D(s)|^2 (\text{Im } \Pi(s))_n = \frac{s}{4\pi\alpha} \sigma(e^+e^- \rightarrow n) \quad (12)$$

In Fig. 2 $|sD(s)|^2$ [also proportional to σ_h , see Eqs. (11) and (3)] is plotted for a few values of the subtraction constant a . Values of $a \gtrsim -2$ lead to pronounced peaks in the correction factor $|sD(s)|^2$ which could be explored with e^+e^- machines of the highest centre-of-mass energy now being proposed. Smaller values of a give less spectacular variations until extremely high s , where the cross-sections start to decrease rapidly (see, e.g., the curve for $a=-4$). Independently of the value of a , the cross-section σ_h stays safely below the unitarity limit established in Ref. 8).

Figure 3 shows the correction factor $|sD(s)|^2$ for time-like as well as space-like values of s (which can be explored with $e^+e^- \rightarrow \mu^+\mu^-$ and $e^+e^- \rightarrow e^+e^-$ respectively) in regions which are accessible at DORIS and SPEAR. The corrections to QED are relatively small, but perhaps visible in accurate experiments.

A very useful tool for investigating the consequences for QED of an increasing $R(s)$ is the following model. Assume (apart from electrons and muons) an infinite sequence of charge e fermions, f_h ($h=1,2,\dots$) with increasing masses

$$(m_h^2)^\nu = h (m_0^2)^\nu \quad (13)$$

If we neglect non-electromagnetic interactions, the cross-section for $e^+e^- \rightarrow f\bar{f}$ at order α will be proportional to the number of open channels, and we would obtain :

$$R(s) = \frac{\sigma(e^+e^- \rightarrow f\bar{f})}{\sigma(e^+e^- \rightarrow \mu^+\mu^-)} \approx \left(\frac{s}{4m_0^2} \right)^\nu \quad (14)$$

In this model we can thus obtain any power behaviour for $R(s)$. If $\nu < 1$ the model (with only electromagnetic interactions) appears to be renormalizable in the usual way. For $\nu=1$ the only new divergence⁹⁾ is in $\Pi(s)$, requiring a new subtraction as discussed above, and the theory becomes of the Lee-Wick kind.

It is interesting to speculate that this simple model is relevant to the physical situation¹⁰⁾, that is that there exists an infinity of quark multiplets q_h characterized by a new quantum number $h=1,2,\dots$

In real terms this would mean, e.g., that bosons can be considered as bound states of the form :

$$B(h_1, h_2; \{\alpha\}) \sim [\bar{q}_{h_1} q_{h_2}]_{\{\alpha\}}.$$

where $\{\alpha_i\}$ denotes the set of usual quantum numbers. Electron-positron annihilation would proceed through $(J)^{CP} = (1)^{--}$ states with $h_1 = h_2$.

On the basis of our present intuition on rising Regge trajectories, dual models, etc., it would be natural to expect the new quantum numbers to contribute linear terms to mass formulae, of the kind

$$M^2 \approx A(h_1 + h_2) + \overline{M^2}(\{\alpha\}). \quad (15)$$

In turn this would correspond to a linear rise of $R(s)$, associated with successive openings of new channels. The interpretation of an indefinitely rising $R(s)$ in the frame of an infinite quark model would thus lead to prefer the linear form, Eq. (3), over slower or faster behaviours, i.e., $R(s) \sim s^\nu$ with $\nu \neq 1$.

This model also shows that the breaking of scaling implied by a rising $R(s)$ is logically independent of the validity of the parton model in deep inelastic scattering.

The main object of this note has been to point out that a linearly rising $R(s)$ does not necessarily lead to catastrophic consequences, and should therefore be considered as a serious possibility.

Our discussion has been confined to the most urgent implications raised by this possibility for QED and hadron structure. It will now be interesting to investigate the many further consequences of these ideas. A possibility is that such a mechanism can play a role in damping weak interactions at high energy.

The infinite quark model and the existence of an h quantum number also raises many questions, in particular on the structure of the axial vector current and the related question of the selection rules in pion emission. The solution of these questions may well have practical consequences for the resolution of the so-called energy crisis ¹¹⁾. If these selection rules are restrictive, given the high density of states, it is conceivable that the highly excited states produced in electron-positron annihilation might prefer to decay by photon rather than by pion emission ¹²⁾.

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R E F E R E N C E S

- 1) A. Litke et al., Phys.Rev.Letters 30, 1189 (1973);
G. Tarnopolsky et al., Phys.Rev.Letters 32, 432 (1974);
B. Richter, invited talk presented at Irvine Conference (Dec., 1973).
- 2) In Eq. (3) $\sigma_{\mu\mu}$ is the experimental cross-section, not the (point-like) theoretical one.

- 3) The lepton pair contribution to order α is

$$\text{Im } \Pi = \frac{\alpha}{3} \left(1 + \frac{2m^2}{q^2}\right) \left(1 - \frac{4m^2}{q^2}\right)^{1/2} \Theta(1 - 4m^2/q^2)$$

which has the limit $(\alpha/3)$; see :

J.D. Bjorken and S.D. Drell, "Relativistic Quantum Mechanics", p.159

R.P. Feynman, Phys.Rev. 76, 769 (1949).

- 4) The corrections to (g-2) for the muon which we find with the propagator (7) and $a=0$ are less than 1 part per million.
- 5) T.D. Lee and G.C. Wick, Nuclear Phys. B9, 209 (1969); B10, 1 (1969);
Phys.Rev. D2, 1033 (1970).
- 6) R. Cutkosky, P. Landshoff, D. Olive and J. Polkinghorne, Nuclear Phys. B12, 281 (1969);
For an illuminating review of acausal theories, see :
S. Coleman, "Acausality" in Subnuclear Phenomena, Ed. A. Zichichi,
Academic Press, N.Y., p.283 (1969).
- 7) M. Gell-Mann and F.E. Low, Phys.Rev. 95, 1300 (1954);
L.D. Landau, in "Niels Bohr and the Development of Physics", Pergamon Press
p.52 (1955).
- 8) N. Cabibbo and R. Gatto, Phys.Rev. 124, 1577 (1961).
- 9) No new primitive divergent graphs appear when $\nu=1$. It is easy to verify that photon-photon scattering is convergent for $\nu < 2$.
- 10) D. Amati and S. Fubini, CERN preprint TH.1823 (1974) have raised the possibility of a purely dynamical explanation for a raising $R(s)$.
- 11) C.H. Llewellyn Smith, CERN preprint TH.1849 (1974).
- 12) The possibility described here is similar to that raised by K. Winter (unpublished, 1970) for radiative decay of heavy baryonic resonances of high angular momentum.

FIGURE CAPTIONS

Figure 1 : Movement of pole in the complex s plane as a function of a for $a > -6.7$ s in GeV^2 .

Figure 2 : $|sD(s)|^2 \sim \sigma_h$ for $s > 0$ and different values of $-4 \leq a \leq 4$ s in GeV^2 .

Figure 3 : $|sD(s)|^2$ for "small" values of s : $-50 \leq s \leq 50$ and $a = -4, 0, 4$ s in GeV^2 .

FIG. 1

FIG. 2

FIG.3