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HOW DO CLUSTERS LOOK IN SEMI-INCLUSIVE CROSS SECTIONS?

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ABSTRACT

It is shown how semi-inclusive quantities can be used in order to learn something about the decay of clusters in independent cluster emission models. The average multiplicity, and binomial moments, of the cluster decay distribution are proved to increase with the total multiplicity. The rate of increase can be looked at through semi-inclusive correlation measurements, and is shown to contain interesting information about the clusters.

I. INTRODUCTION

Inclusive correlations in the central region have been extensively studied during the two past years, both from the experimental ¹⁾ and theoretical ^{1),2)} point of view. They are found to be for a large part of short-range character, roughly energy independent, and present with different sizes, in various charge configurations, including the $\pi^+\pi^-$ case. This last fact is important since it reveals without any quantitative analysis being necessary that uncorrelated resonance production alone cannot be made responsible for the whole observed short-range correlations. Indeed, this was not unambiguously predictable from the observed behaviour of the Mueller integrated coefficient f_2^{--} since this behaviour could be accounted for in two component models without assuming any short-range correlation between π^- 's ³⁾. Nevertheless a quantitative estimate of short-range correlations from inclusive quantities requires a simultaneous evaluation of the long-range component, which is present at the same time, and generally attributed to diffractive processes. Data are now available on inclusive cross-sections at fixed multiplicities ⁴⁾⁻⁶⁾ (semi-inclusive cross-sections). To the extent that the average multiplicity in diffractive processes is smaller than it is in the pionization component, the study of semi-inclusive quantities for multiplicities n of the order of, or larger than, the average $\langle n \rangle$ is certainly of great help for our understanding of the structure of short-range correlations.

Independent cluster emission models (ICEM) have encountered a rather encouraging success in describing the short-range component of inclusive cross-sections ⁷⁾⁻¹²⁾. The net result is that data are in good agreement with a model in which isotropic clusters are emitted independently according to a more or less flat rapidity distribution. Once isotropy is assumed, and given a model for diffraction, data on inclusive correlations yield the value of $\tau = \langle p(p-1) \rangle / \langle p \rangle$ (for undistinguished particles), where p is the number of particles per cluster, and the symbol $\langle \rangle$ means average over a typical cluster. Model dependence in separating short-range and long-range components is made particularly clear if one considers the rather large set of values of τ proposed in published papers ⁷⁾⁻¹²⁾. Although numbers are often given for $\langle p \rangle$ in the literature, this quantity cannot be extracted independently of τ without making a definite assumption about the shape of $Q(p)$, the multiplicity distribution inside a cluster. Now for a given total charged multiplicity n , there is a number of ways to fill in the clusters, and the weight of different configurations depends on n and on $Q(p)$. Then the study of semi-inclusive rapidity correlations allows to get insight about the composition of individual clusters.

Of course, limitations in the analysis of the clusters are expected at low multiplicities, due to the presence of diffraction, and at very large multiplicities where phase space constraints prevent clusters to be independent. Both problems, which deserve attention, are not studied in detail here, so that our results are to be applied in the region, say, $\langle n \rangle \lesssim n \leq 2\langle n \rangle$. A lot of work has already been devoted to semi-inclusive correlations in the framework of cluster models or general short-range model ¹³⁾⁻¹⁷⁾. What we actually want to do here is to answer the following question : "how do clusters look when 'observed' in semi-inclusive quantities?".

In Section II, the general formalism for ICEM is recalled and things to be learnt from semi-inclusive correlations are exhibited with reference to the case studied by Berger ¹⁵⁾. The general properties in n of the correlation functions are derived in Section III, and two simple examples of multiplicity distributions are given. Models developed in Sections II and III are only meant to describe central production in the pionization component. So, apart from recalling our conclusions on that, Section IV is devoted to some remarks on the role of diffraction and of leading cluster effects.

II. FORMALISM FOR INDEPENDENT CLUSTER EMISSION MODELS

Most of the things described in this section are not new. They are included here for the sake of completeness, and we hope, for clarity.

In an n body event, c clusters are produced with a probability $P(c)$ (P will be a Poisson law in subsequent applications), and contain respectively $p_1, p_2, \dots, p_c$ particles with probabilities $Q(p_i)$. The probability of an n event is then

$$\Pi(n) = \sum_{c, p_i}^{(n)} P(c) Q(p_1) \dots Q(p_c), \quad (1)$$

the summation $\sum^{(n)}$ being restricted to

$$\sum_{i=1}^c p_i = n \quad (2)$$

Now let $\Gamma^{(1)}(y)$ be the probability that one particle present in a cluster has rapidity y , $\Gamma^{(2)}(y_1, y_2)$ that two particles present in the same cluster have rapidities y_1 and y_2 . In principle $\Gamma^{(1)}$ and $\Gamma^{(2)}$ may depend on the number c of clusters in the event, and on p , the number of particles

present in the cluster from which the observed ones originate. We believe that dependence on p can be safely neglected because in the model the clusters decay essentially according to phase space, and higher p will just correspond to clusters heavier than average. Independence on c is a stronger assumption because energy momentum conservation will tend to increase the cluster density in the central region in events involving many clusters. If, for example, in a c cluster event, the cluster density is represented by a Gaussian with width σ_c , the energy sum rule implies¹²⁾

$$\sigma_c = \left(2 \log \frac{\sqrt{s}}{c M_\perp} \right)^{1/2}, \quad (3)$$

where $M_\perp$ is an average transverse mass for a cluster. This effect, which can be thought of as a consequence of the leading particle effect, is experimentally observed as a narrowing of the one-particle inclusive distribution with increasing multiplicities, but it is small if we confine ourselves to the central region, as we shall do in the following. Note that since the one-particle density $\rho_n^{(1)}(y)$ is then given by

$$\rho_n^{(1)}(y) \equiv \frac{1}{\sigma_n} \frac{d\sigma_n}{dy} = n \Gamma^{(1)}(y), \quad (4)$$

the statement that $\Gamma^{(1)}$ is independent of p and c , amounts to the statement that $\rho_n^{(1)}(0)$ is linear in n . From now on, we shall consider $\Gamma^{(1)}$ and $\Gamma^{(2)}$ as independent of n in the central region. We can now construct the two-particle density $\rho_n^{(2)}(y_1, y_2)$

$$\begin{aligned} \rho_n^{(2)}(y_1, y_2) &\equiv \frac{1}{\sigma_n} \frac{d\sigma_n}{dy_1 dy_2} = \frac{1}{\Pi(n)} \sum_{c, p_i}^{(n)} P(c) Q(p_1) \dots Q(p_c) \times \\ &\times \left\{ \sum_{i \neq j=1}^c p_i p_j \Gamma^{(1)}(y_1) \Gamma^{(1)}(y_2) + \sum_{i=1}^c p_i (p_i - 1) \Gamma^{(2)}(y_1, y_2) \right\} \end{aligned} \quad (5)$$

The meaning of this formula is obvious : the first sum inside the brackets corresponds to all possible ways of finding two particles in two different clusters, the second one to the configuration with the two particles in the same cluster. Using $\sum_i p_i = n$ and the independence of $\Gamma^{(i)}$ on p 's and c , it is convenient to rearrange the different terms under the form

$$\rho_n^{(2)}(y_1, y_2) = n \left[n \Gamma^{(1)}(y_1) \Gamma^{(1)}(y_2) - \Gamma^{(2)}(y_1, y_2) \right] +$$

$$+ \frac{\langle c \rangle}{\Pi(n)} \sum_p p^2 Q(p) \Pi(n-p) \times \left[\Gamma^{(2)}(y_1, y_2) - \Gamma^{(1)}(y_1) \Gamma^{(1)}(y_2) \right] \quad (6)$$

Here we have made use of $cP(c) = \langle c \rangle P(c-1)$ when P is a Poisson law.

The correlation function

$$C_n(y_1, y_2) = \rho_n^{(2)}(y_1, y_2) - \rho_n^{(1)}(y_1) \rho_n^{(1)}(y_2) \quad (7)$$

the integral of which is $-n$, reads

$$C_n(y_1, y_2) = \gamma_n^2 \left[\Gamma^{(2)}(y_1, y_2) - \Gamma^{(1)}(y_1) \Gamma^{(1)}(y_2) \right] - n \Gamma^{(1)}(y_1) \Gamma^{(1)}(y_2) \quad (8)$$

with

$$\gamma_n^2 = \langle c \rangle \sum_p p(p-1) \frac{Q(p) \Pi(n-p)}{\Pi(n)}. \quad (9)$$

All the information which can be extracted from semi-inclusive two-body correlations is contained in the coefficient $\gamma^2(n)$. Note that $\Gamma^{(2)}$ and $\Gamma^{(1)}$ can be related to fully inclusive quantities, just because of the assumption that, at least in the central region, these functions are independent of n . The general relationship between semi-inclusive and inclusive quantities has been especially discussed in Refs. 14) and 17). However, one should keep in mind that the process of central production we are analyzing here has to be supplemented by some other mechanisms like diffraction, responsible for the long-range part of the correlations. So, $\Gamma^{(2)}$ and $\Gamma^{(1)}$ are actually related to that part only of the inclusive quantities which is due to central production.

The main features of $C_n(y_1, y_2)$ are well exhibited by considering the simple model ¹⁵⁾ where $Q(p)$ is well concentrated around $p_0 = \langle p \rangle$. In this case, one finds

$$C_n(y_1, y_2) = n \left[(p_0 - 1) \Gamma^{(2)}(y_1, y_2) - p_0 \Gamma^{(1)}(y_1) \Gamma^{(1)}(y_2) \right] \quad (10)$$

Let us recall the simple properties which emerge¹⁵⁾ from this narrow width approximation (NWA) model.

- i) As a function of y_1 and y_2 , C_n is the sum of a positive term proportional to $\Gamma^{(2)}$, which falls off rapidly in $|y_1 - y_2|$, and of a negative, rather flat, term proportional to the product of one particle densities. The relative size of the two terms is governed by the value of p_0 , the number (here fixed) of particles per cluster.
- ii) $\Gamma^{(2)}$ and $\Gamma^{(1)}$ are roughly proportional to $1/\log s$, as a consequence of an approximate scaling of the cluster density. So, at fixed n and y 's, $C_n(y_1, y_2)$ behaves like $1/\log s (1 - O(1/\log s))$.
- iii) $C_n(y_1, y_2)$ is exactly linear in n . Note that this result holds in the central region only, $|y_1| \lesssim 1$, say, because of the expected narrowing of the Γ functions at large n .

We shall now focus on the last point, namely the n dependence of C_n in the central region in general cluster models. From the preceding example, it is clear that the interesting things lie in the deviation of $\sqrt{v^2(n)/n}$ from a constant according to the choice of $Q(p)$.

III. THE n DEPENDENCE OF THE SEMI-INCLUSIVE CORRELATION FUNCTIONS

We shall use here the generating function formalism. Let $\tilde{f}(z)$ be the generating function of a sequence $f(n)$

$$\tilde{f}(z) = \sum_n z^n f(n). \quad (11)$$

The inversion formula is

$$f(n) = \frac{1}{2\pi i} \oint \frac{\tilde{f}(z)}{z^{n+1}} dz, \quad (12)$$

where the integration contour surrounds $z=0$. In what follows, the integral (12) will be evaluated using the saddle point method (SPM)^{*)}. As a first example, let us consider the multiplicity distribution $\Pi(n)$ itself. From Eq. (1) we have

*) We thank M. Le Bellac for pointing out to us the interest of using this method in the cluster context.

$$\tilde{\Pi}(z) = \exp[\langle c \rangle (\tilde{Q}(z) - 1)] . \quad (13)$$

The SPM yields

$$\Pi(n) = \frac{1}{z_0^n} \frac{\exp[\langle c \rangle (\tilde{Q}(z_0) - 1)]}{[2\pi \langle c \rangle (z_0^2 \tilde{Q}''(z_0) + z_0 \tilde{Q}'(z_0))]^{1/2}} , \quad (14)$$

with z_0 , a function of n , being the positive solution of

$$z_0 \tilde{Q}'(z_0) = \langle p \rangle \frac{n}{\langle n \rangle} . \quad (15)$$

This solution always exists and is unique, as a consequence of $\tilde{Q}(z)$ being positive as well as all its derivatives $\tilde{Q}^{(k)}(z)$ [positivity of all $Q(p)$]. Note that $z_0(n)$ only depends on $n/\langle n \rangle$.

It will be useful to consider the conditional probability $Q(p;n)$ for a cluster in an n body event to contain p particles

$$Q(p;n) = \frac{K(p,n)}{\Pi(n)} . \quad (16)$$

$$K(p,n) = Q(p) \sum_{c, p_1, \dots, p_{c-1}}^{(n-p)} Q(p_1) \dots Q(p_{c-1}) P(c) \quad (17)$$

is the probability for getting n particles with exactly p particles in one of the clusters. The two-variable generating function of K is

$$\tilde{K}(u,z) = \frac{\tilde{Q}(uz)}{\tilde{Q}(z)} \exp[\langle c \rangle (\tilde{Q}(z) - 1)] \quad (18)$$

and that of $Q(p;n)$ is (using the SPM)

$$\tilde{Q}(u;n) = \frac{\tilde{Q}(z_0 u)}{\tilde{Q}(z_0)} , \quad (19)$$

with z_0 always given by Eq. (15). All what we want to know about the structure of an average cluster in n body events is contained in the very simply formula (19). In particular, we can compute any moment $B_k(n) = \langle p(p-1)\dots(p-k+1) \rangle_n$ of the conditional distribution $Q(p;n)$. The result is obvious :

$$B_k(n) = z_0^k \frac{\tilde{Q}^{(k)}(z_0)}{\tilde{Q}(z_0)}. \quad (20)$$

Now, from Eqs. (9) and (20), one finds easily

$$\gamma^2(n) = n \frac{\langle p(p-1) \rangle_n}{\langle p \rangle_n} \quad (21)$$

The interest of this formula is to make explicit the departure from linearity in n of $\gamma^2(n)$. The useful quantity is $\tau = \langle p(p-1) \rangle_n / \langle p \rangle_n$, in the same way as it is $\tau = \langle p(p-1) \rangle / \langle p \rangle$ in inclusive two-body correlations.

III.1 General properties of ICEM's

- i) If we set $n = \langle n \rangle$, $z_0 = 1$ [Eq. (15) and definition of $\tilde{Q}(z)$]. Then from Eq. (19), it follows that

$$Q(p; \langle n \rangle) = Q(p) \quad (22)$$

The clusters in $n = \langle n \rangle$ events are representative of the average clusters as "seen" in inclusive experiments. In particular

$$\gamma^2(\langle n \rangle) = \langle n \rangle \tau$$

The relationship between $C_{\langle n \rangle}(y_1, y_2)$ and non-diffractive inclusive quantities is particularly simple ¹⁴⁾.

- ii) It is straightforward to prove that, due to the positivity of $\tilde{Q}'(z)$ for z positive, z_0 is an increasing function of n . Moreover, we can prove the very interesting result that $\gamma^2(n)/n$ is a non-decreasing function of n ^{*}, i.e., the narrow width case [Eq. (10), $\gamma^2(n) = n(p_0 - 1)$] corresponds to the smallest possible rate of increase of $\gamma^2(n)$. Due to the expression (8) of C_n , the semi-inclusive correlation is at least linear in n in the central region.

^{*}) We thank A. Martin for providing us with the following simple proof. Due to Eqs. (20), (21) and (15), proving $d/dn(\gamma^2(n)/n) \geq 0$ is equivalent to prove that $\tilde{Q}'(z)\tilde{Q}''(z) - z(\tilde{Q}''^2(z) - \tilde{Q}'(z)\tilde{Q}'''(z))$ is positive. This property follows from the Schwartz inequality applied to the Taylor expansion of $\tilde{Q}''$: setting $\tilde{Q}' = \sum q \lambda_q z^{q-1}$, then $\tilde{Q}''^2 = (\sum q \lambda_q z^{q-1})^2$ is less than

$$\left(\sum_q \lambda_q z^q \right) \times \left(\sum_q q^2 \lambda_q z^{q-2} \right) = \tilde{Q}'(z) \times (\tilde{Q}'''(z) + \tilde{Q}''(z)/z)$$

because all λ_q are positive.

- iii) One can also linearize $\gamma^2(n)$ around $n = \langle n \rangle$, and so obtain information about the clusters by a local study of the semi-inclusive correlation near $n = \langle n \rangle$. The result can be written as

$$\left. \frac{d}{dn} C_n(y_1, y_2) \right|_{n=\langle n \rangle} = \left(1 + \frac{2\tau + \langle p(p-1)(p-2) \rangle / \langle p \rangle}{1 + \tau} \right) \left(\Gamma_{(y_1, y_2)}^{(2)} - \Gamma_{(y_1)}^{(1)} \Gamma_{(y_2)}^{(1)} \right). \quad (23)$$

Equation (23) is a nice example of how semi-inclusive measurements yield more information [here the size of the third binomial moment, relatively to $\langle p \rangle$, of the distribution $Q(p)$]. Recall that only $\tau = \langle p(p-1) \rangle / \langle p \rangle$ is given by inclusive measurements. In this respect, the measurement of $(d/dn)C_n|_{n=\langle n \rangle}$ is equivalent to that of the three-particle inclusive correlation function. The gain here is that, in the latter case, we need a model for the diffractive processes, a difficulty which can be escaped here by evaluating $\Gamma^{(2)}$ and $\Gamma^{(1)}$ directly from a model for the cluster rapidity distribution. This is quite a safe procedure as long as we keep close to the central region.

- iv) Equation (19) is of course the general answer to the question : "how do clusters look in n body events ?". Using the same kind of trick as above for proving that C_n/n increases, we can prove that in general, all binomial moments $B_k(n)$ are non-decreasing functions of n ^{*}). So is $\langle p \rangle_n$ for example. It means that in high multiplicity events ($n > \langle n \rangle$), clusters are more populated than they are on the average. In this respect, the study of high multiplicity events ($n > \langle n \rangle$) is a way to learn something about clusters heavier than the average ^{**}).

III.2 Two simple examples

All the above described properties were independent of the particular choice of $Q(p)$, and thus not quantitative. In order to get estimates of the effects of the finite width of the clusters, let us consider two simple examples of neutral clusters ^{***}).

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- ^{*}) It is also easy to show from Eq. (19) that if $Q(p)$ is a Poisson law, so is $Q(p;n)$.
- ^{**}) Another method is to look at events containing one or more heavy particles ¹⁸⁾.
- ^{***}) It is in the spirit of independent cluster models to take them neutral. If they are not, correlations between them have to be taken into account, at least of the short-range type, thus spoiling the simplicity of the whole analysis. But of course this problem deserves attention and more data about semi-inclusive correlations between different kinds of particles are needed.

E1 : "One or two pair model" : a cluster may decay into two charged or four charged particles with probability α and $1-\alpha$, respectively.

E2 : "Poisson model" : $Q(p)$ is a Poisson law in $p/2$.

In order to make the comparison meaningful, we compare E1 and E2 for the same value of $\tau = \langle p(p-1) \rangle / \langle p \rangle$, in such a way that they both lead to the same inclusive correlations. Then

$$\begin{cases} \alpha = (6 - 2\tau) / (5 - \tau) \\ \langle p \rangle = \tau - 1 \end{cases} \quad (24)$$

We may first compare the behaviour of C_n near $n = \langle n \rangle$. From Eqs. (23) and (24), one can compute the ratio $r = \lambda(E2) / \lambda(E1)$ of the slopes $\lambda = (dC_n / dn) |_{n=\langle n \rangle}$ in both cases. One finds that r varies from 1.1 to 1.2 as τ increases from 2 to 2.8. The effect is probably too small to be very useful.

More interesting is the behaviour of C_n at large n , namely the behaviour of $\gamma^2(n)/n$ as a function of n . One has :

$$\begin{aligned} (E_1) \quad X_1 &= \frac{\gamma^2(n)}{n} = \frac{(6-2\tau) + 6(\tau-1)z_{o(1)}^2}{(6-2\tau) + 2(\tau-1)z_{o(1)}^2} \\ (E_2) \quad X_2 &= \frac{\gamma^2(n)}{n} = 1 + z_{o(2)}^2 (\tau-1) \end{aligned} \quad (25)$$

$z_{o(1),(2)}$ are the corresponding solutions of Eq. (15). $z_o^2 \rightarrow \infty$ as $n \rightarrow \infty$ in both cases. Then x_1 should increase from τ at $n = \langle n \rangle$ to 3 (independent of τ) at large n . So in E1 the change in slope is predicted to be important above $\langle n \rangle$ if τ is not close to 3. This last value is fairly large, and would be compatible with inclusive data only if the long-range correlations are small in the central region. In the case E2, $x_2 \rightarrow \infty$ like $2/(\tau-1) \log n / \langle n \rangle$ but this behaviour is reached for non-realistic values of $n / \langle n \rangle$. For $\tau = 2$, for example, $z^2 = 1.5$ corresponds to $n / \langle n \rangle \sim 2$. In this case x_2 goes from 2 to 2.5 as n is increased from $\langle n \rangle$ to $2\langle n \rangle$.

IV. GENERAL COMMENTS AND OUTLOOK

We have shown how, in principle, the study of semi-inclusive correlations yields important information about the cluster composition. This information is essentially concentrated in the way how the behaviour of C_n at fixed rapidities differs from a simple $n/\langle n \rangle$ variation as a function of n and $\langle n \rangle$. As a consequence, we suggest that a better function which should be plotted against n and $\langle n \rangle$ is

$$X_n(y_1, y_2) = \frac{\langle n \rangle}{n} \left[C_n(y_1, y_2) + \frac{1}{n} \rho_n^{(1)}(y_1) \rho_n^{(1)}(y_2) \right]. \quad (26)$$

This function is normalized to zero upon integration over y_1 and y_2 , and is a true measurement of the correlation in the rapidity space once n particles are produced. Moreover, it is a function of y_1 and y_2 only (in the central region), in the limiting case of the infinitely narrow cluster model ¹⁵⁾ which appears to be a useful reference model.

Our answer to the question asked in the introduction can be summarized as follows : in general, clusters are more (respectively, less) populated at multiplicities higher (respectively, smaller) than the average. The n dependence of this population inside the clusters may allow to discriminate between various possible multiplicity distributions. At that point, we would like to stress that all of the equations of this paper can be applied not only to the full multiplicities, but also to multiplicities in a well-defined solid angle which can be smaller than 4π and for example leaving aside the fragmentation region. This remark, which has been made already ¹⁵⁾, may help not only because of allowing for a lighter experimental arrangement, but also because the observed multiplicity has more chance to be produced through the central cluster emission process.

Now, in making quantitative analyses of the data in terms of cluster emission models, one has of course to take into account other known features of multiparticle production. We shall recall here two important things which have both to do with the so-called leading particle effect.

- i) The most obvious leading particle effect has to do with diffraction, especially single diffraction, generally associated with Pomeron exchange. The associated average multiplicities are of the order of $\frac{1}{2}$ the total average multiplicity, so that, as already remarked ¹⁵⁾, this effect is not expected to spoil too much the behaviour of C_n for $n \gtrsim \langle n \rangle / 2$, on which we have especially focused our study. One has only

to take care that in all our equations, $\langle n \rangle$ always mean an average over non-diffractive processes, and is then larger by 20 or 25% than the experimentally observed average multiplicity. Let us also remark that for very low multiplicities, particles associated with diffraction will generally be decay products of light recoil clusters, and thus have less chance to reach the central region. If it is so, the correlation is higher than that given by cluster production. As a final remark on this subject, let us point out that it may well happen that the n dependence of the corresponding correlation follows roughly the same law as the one associated with non-diffractive events. Such a result can be expected in a model like the one proposed by Berger and Fox¹⁰⁾ where the recoil cluster has the same properties as a non-diffractive event in a general hadron-hadron collision. Preliminary numerical calculations confirm this statement.

- ii) More generally, one may expect the existence of "leading cluster" effects. By this we mean clusters which on the average have a rapidity spectrum which scales in the projectile or target frame, or equivalently, in x . The fast decrease of the ratio π^+/π^- at $x=0$ in pp collisions, as well as the observed asymmetry in $\pi p \rightarrow \pi + X$ inclusive distributions, could be well described by such a mechanism, which is of course present in a multiperipheral scheme where the leading clusters contain essentially one particle. So if this leading cluster concept is valid, one expects that in any n body event, we have some probability $L(\beta)$ that β particles belong to such clusters and then cannot be seen in the central region. The model for central production we have developed before is then to be applied in each event to $n-\beta$ particles only, with a weight proportional to $L(\beta)$. As a consequence, any semi-inclusive cross-section is expected to be very small in the central region for n less than $\langle \beta \rangle^*$. In particular, $1/\sigma_n (d\sigma_n/dy)|_{y=0}$ is predicted to be roughly proportional to $n - \langle \beta \rangle$ rather than to n . This is how we explain that in the Pisa-Stony Brook experiment on $R_n(0,0)$ ¹⁾, where the correlation is divided by the square of the one-particle density, nR_n decreases as a function of n for $n \lesssim 10$, whereas an increase would be rather expected in any ICEM with finite cluster width. We would like also to recall that the existence of leading clusters is a mechanism which was proposed in Ref. 19) to explain the observed quasi KNO scaling over a large range of energies.

*) Note that since the total charge is around 0 in the central region at high enough energies, $\langle \beta \rangle$ should be at least equal to 2. It is interesting to know whether it is actually larger than that. $\langle \beta \rangle$ larger than 2 means that $L(\beta > 2)$ is not negligible, thus justifying the concept of leading cluster.

While completing this work, we have received a paper by Le Bellac and Hayot ²⁰⁾ who also study some aspects of cluster models, and reach conclusions similar to ours. However, we would like to point out here that most of the general properties of the ICEM's we have derived in this work are based on the positivity properties of the function $\tilde{Q}(z)$. General short-range models have not these properties ^{*)}. If, for example, $C_n(0,0)$ happened to be less than linear as a function of n , ICEM's would be ruled out whereas general short-range correlation models could survive.

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^{*)} We thank B. Petersson for this remark.

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