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ON THE CHARGE STRUCTURE OF HIGH ENERGY MULTIPLE PRODUCTION

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ABSTRACT

Correlations between neutral and charged particles produced in high energy collisions are studied. The associated average multiplicity $\langle n_o \rangle_{n_-}$ and the associated moment $(f_2^{oo})_{n_-}$ are expressed in terms of the Mueller moments. An isospin analysis of the pion-pion correlations is performed with the help of Mueller-Regge techniques. From this analysis, the slope of $\langle n_o \rangle_{n_-}$ is predicted to be asymptotically about 0.75. This asymptotic limit should be approached already at ISR energies. Finally, predictions are given for the associated moment $(f_2^{oo})_{n_-}$, and the importance of the measurement of this quantity is emphasized.

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1. INTRODUCTION

In the past year we have witnessed considerable progress in the experimental study of the charge structure of multiple production. Experiments performed at the Serpukhov, NAL and CERN ISR laboratories have provided a wealth of new information. An important discovery is the existence of a strong positive correlation between the average number of neutral particles and the number of associated charged particles produced^{1,2)}. Another important feature revealed by the data deals with the charge transfer between hemispheres. The data show that the net transfers of zero and one unit of charge dominate, the higher transfers being strongly suppressed^{3,4)}. In a multiperipheral scheme, this means that $I \geq 2$ exchanges are small, just as one had expected from analyses of low multiplicity exclusive processes. Probably the most promising new source of information on charge correlations are the data on the differential two-body rapidity correlations⁵⁻⁷⁾. Measuring the correlations between definite charge combinations such as $+-$, $--$, -0 , etc., one can study the influence of charge and isospin. The point is that although each of these distributions taken alone is highly complicated, receiving contributions from many different sources, the difference between the individual distributions should provide us with direct information on the isospin structure underlying multiple production.

In this paper we tackle the question of the interrelation of the different experimental quantities dealing with charge correlations. We derive formulae for the associated average neutral multiplicity $\langle n_0 \rangle_{n_-}$ and the associated Mueller moment $(f_2^{00})_{n_-}$ in terms of the fully integrated quantities such as $\langle n_0 \rangle$, f_2^{0-} , etc. Our results show that an approximately linear dependence of $\langle n_0 \rangle_{n_-}$ on n_- ,

$$\langle n_0 \rangle_{n_-} \approx a + b(n_- - \langle n_- \rangle)$$

should always be expected, unless there are very strong multibody correlations. The opposite statement, that a linear rise implies absence of strong higher correlations, is in general not true. We perform a Mueller-Regge analysis of the two-body correlations. Our essential assumptions are very weak: we assume that the leading singularity has $I = 0$ and that singularities with $I \geq 2$ can be neglected. From this analysis we estimate the asymptotic value of the coefficient b to be $b_\infty = 0.75 \pm 0.1$ for non-diffractive events, i.e. events with $n_- \gtrsim \frac{1}{2} \langle n_- \rangle$. Including diffractive events, we expect the mean slope to approach 1. We also predict how this scaling limit should be approached, in good agreement with data. Finally, we give predictions for the associated moment $(f_2^{00})_{n_-}$, and emphasize the usefulness of this quantity for learning about the isospin structure of multiple production.

In this paper, only correlations between pions are studied. Moreover, only such properties are discussed that do not depend on correlations between three and more pions, since asymptotic predictions for them are presumably not yet valid at NAL energies.

The paper is organized as follows. In Section 2, we derive formulae for $\langle n_0 \rangle_{n_-}$ and $(f_2^{00})_{n_-}$ in terms of fully integrated moments. Section 3 deals with the Mueller-Regge analysis. The theoretical results are then compared with available data in Section 4. Some concluding remarks are made in Section 5.

2. CORRELATIONS BETWEEN CHARGED AND NEUTRAL PARTICLES

Let us begin our study by discussing the n_- dependence of the associated neutral multiplicity $\langle n_0 \rangle_{n_-}^{8-17}$.

- i) We first study the simple case of a linear dependence, i.e. we assume that a good description of the data is already provided by

$$\langle n_0 \rangle_{n_-} = a + b(n_- - \langle n_- \rangle) . \quad (1)$$

Combining

$$\langle n_0 \rangle = \sum_{n_-} \langle n_0 \rangle_{n_-} P(n_-) , \quad (2a)$$

$$\langle n_0 n_- \rangle = \sum_{n_-} \langle n_0 \rangle_{n_-} n_- P(n_-) , \quad (2b)$$

[$P(n_-)$ is the probability of producing n_- negative particles together with an arbitrary number of neutral ones] with the definition of the correlation integrals

$$f_2^{0-} = \langle n_0 n_- \rangle - \langle n_0 \rangle \langle n_- \rangle , \quad (3a)$$

$$f_2^{--} = \langle n_-(n_- - 1) \rangle - \langle n_- \rangle^2 , \quad (3b)$$

we get immediately^{*)}

$$a = \langle n_0 \rangle , \quad (4a)$$

$$b = \frac{f_2^{0-}}{f_2^{--} + \langle n_- \rangle} . \quad (4b)$$

*) In more specific multiperipheral models, Eq. (4) has been proved, for example by Campbell and Chang⁸⁾.

Indeed, Eq. (4) has a wider applicability than shown by this proof. Assume we have data that do not exactly satisfy the linear law Eq. (1), but make a fit to them with the ansatz (1). Then we get Eq. (4) for the fit parameters, provided we weight the data points with the prong cross-sections. The straightforward proof of this is left to the reader.

Equation (4) is our first interesting result: it shows that the behaviour of the coefficient b depends both on the isospin structure of the production amplitudes (i.e. on the relation between f_2^{0-} and f_2^{--}) and on isospin independent dynamics. Moreover, we see that, in general, b is expected to show energy dependence even if the isospin structure of the amplitudes does not change with energy. Only in the exceptional case of f_2^{0-} being proportional to $f_2^{--} + \langle n_- \rangle$ the coefficient b would be independent of energy already from low energies onwards.

ii) Next we consider the quadratic approximation

$$\langle n_o \rangle_{n_-} = a + b(n_- - \langle n_- \rangle) + c(n_- - \langle n_- \rangle)^2. \quad (5)$$

Repeating the earlier exercise we get

$$a = \langle n_o \rangle - c \cdot D, \quad (6a)$$

$$b = \frac{A f_2^{0-} - B f_3^{0--}}{\Delta}, \quad (6b)$$

$$c = \frac{-C f_2^{0-} + D f_3^{0--}}{\Delta}, \quad (6c)$$

where

$$\begin{aligned} A &= f_4^- + 5f_3^- + 4f_2^- + 2(f_2^- + \langle n_- \rangle)^2 \\ B &= f_3^- + 3f_2^- + \langle n_- \rangle \\ C &= f_3^- + 2f_2^- \\ D &= f_2^- + \langle n_- \rangle \\ \Delta &= AD - BC \end{aligned} \quad (6d)$$

Here $f_2^- = f_2^{--}$, etc. For the definitions of the higher moments see elsewhere¹⁸⁾.

One could proceed further in the above way. As one would at each step need new multibody correlations, this does not seem to be very practical. Instead we discuss simple consequences of Eq. (6). Actually we study the behaviour of the scaled quantity $\langle n_0 \rangle_{n_-} / \langle n_- \rangle$ in terms of the KNO variable $n_- / \langle n_- \rangle$:

$$\frac{\langle n_0 \rangle_{n_-}}{\langle n_- \rangle} = \tilde{a} + \tilde{b} \left[\frac{n_-}{\langle n_- \rangle} - 1 \right] + \tilde{c} \left[\frac{n_-}{\langle n_- \rangle} - 1 \right]^2. \quad (7)$$

There are two physically interesting cases in which the quadratic term is small. One is the case of strong multibody correlations but with

$$\begin{aligned} f_3^{0--} &\approx f_3^{---} \\ f_2^{0-} &\approx f_2^{--} \end{aligned},$$

in which case $\tilde{c}$ is small due to cancellations. Such a behaviour is expected in models with KNO scaling and a dominantly charge-independent production mechanism.

For the other case, let us assume that all correlations between three or more particles are absent. Then $\tilde{c}$ is not exactly equal to zero, i.e. in this case $\langle n_0 \rangle_{n_-}$ is not strictly linear, but one finds from Eq. (6)

$$\frac{\tilde{c}}{\tilde{b}} = - \frac{f_2^{--} \cdot \langle n_- \rangle}{(f_2^{--} + \langle n_- \rangle)^2} + O\left(\frac{1}{f_2^{--} + \langle n_- \rangle}\right), \quad (8)$$

which at all NAL energies is less than 0.2. It also implies a downward (convex) curvature, just as the data do.

Before leaving this section, let us write up the linear formula for the associated Mueller moment $(f_2^{00})_{n_-}$. The study of this quantity has been advocated by many authors as a sensitive test of the isospin structure of multiple production^{10,11,13)}. We find

$$(f_2^{00})_{n_-} = \alpha + \beta(n_- - \langle n_- \rangle), \quad (9a)$$

with

$$\alpha = f_2^{00} - \frac{(f_2^{0-})^2}{f_2^{--} + \langle n_- \rangle}, \quad (9b)$$

$$\beta = \frac{f_3^{00-} - b^2(f_3^{--} + 3f_2^{--} + \langle n_- \rangle)}{f_2^{--} + \langle n_- \rangle} , \quad (9c)$$

where

$$b = \frac{f_2^{0-}}{f_2^{--} + \langle n_- \rangle} .$$

3. MUELLER-REGGE ANALYSIS

In order to apply our previous formulae we must perform an isospin analysis of multiple production. Such an analysis will allow us to express the correlations between charged and neutral particles in terms of those of charged particles only. Using the relatively detailed information on the charged-charged correlations experimentally available, we then predict the energy and n_- dependence of $\langle n_0 \rangle_{n_-}$ and $(f_2^{00})_{n_-}$.

The isospin analysis is best performed by using the fully inclusive Mueller-Regge techniques. In applying these methods in the study of the correlations, however, one must proceed with the greatest caution. The reasons for this are as follows: In a "naive" Mueller-Regge analysis of two-body correlations one would assume that (i) the correlations are given by the leading Reggeon-Pomeron contributions alone, (ii) the same Reggeon-Pomeron terms control the energy behaviour of the single particle spectra in the central region, and (iii) the Pomeron is a factorizable pole¹⁹⁻²¹⁾. It seems to us, however, that none of these expectations is born out by data. Experimentally, the central region spectra are approaching the scaling limit from below. It has been argued that this rise has nothing to do with the standard Reggeon-Pomeron terms but is rather a kinematic threshold effect²²⁾. Several model calculations within the multiperipheral and the uncorrelated jet models strongly support this point of view²³⁾. Moreover, it is already well known that a considerable part of the experimentally-observed two-body correlations are caused by inelastic diffraction and are of long-range nature²⁴⁻²⁶⁾. The existence of such long-range correlations is in a direct conflict with the above assumptions (i) and (iii). For further discussion on these and other problems hampering the Mueller analysis of the central region spectra and the correlations we refer to a recent thorough study by Inami²¹⁾.

After these warnings, we proceed as follows. Define the standard correlation function

$$C_{cd}(y, y') = \rho_{cd}(y, y') - \rho_c(y) \rho_d(y') , \quad (10)$$

where ρ_c and ρ_{cd} are the single- and two-particle densities

$$\rho_c(y) = \frac{1}{\sigma_{inel}} \frac{d\sigma_c}{dy} , \quad (11a)$$

$$\rho_{cd}(y, y') = \frac{1}{\sigma_{inel}} \frac{d^2\sigma_{cd}}{dy dy'} . \quad (11b)$$

Here, y is the rapidity variable. Notice that we use the inelastic cross-section to normalize the densities. The integral of C gives the second Mueller moment f_2^{cd} :

$$f_2^{cd} = \int dy dy' C_{cd}(y, y') . \quad (12)$$

The single- and two-particle distributions are related by the generalized optical theorem to particular discontinuities of the three- and four-body elastic amplitudes¹⁹⁾:

$$\rho_c(y) = \frac{1}{\sigma_{inel}} \text{Disc } A(ab\bar{c} \rightarrow ab\bar{c}) , \quad (13a)$$

$$\rho_{cd}(y, y') = \frac{1}{\sigma_{inel}} \text{Disc } A(ab\bar{c}\bar{d} \rightarrow ab\bar{c}\bar{d}) . \quad (13b)$$

This is illustrated in Fig. 1. Since we normalize the densities to the inelastic cross-section and not to the total cross-section, we do not use the full discontinuity but that part corresponding to the summation over inelastic states only.

Let us now study the isospin analysis of these graphs in detail. The results obtained in this way are not really new²⁷⁾, but we hope to show better their generality and simplicity.

Consider first Fig. 1a. Since $p\bar{p}$ can have only isospin 0 and 1, the two blobs can carry only these isospins when analysed in the $p\bar{p}$ and $p'\bar{p}'$ channels. Denoting by $\rho_c^{(I, I')}(y)$ the contributions from fixed isospin in these channels we have

$$\rho_{\pm}(y) = \rho_+^{(0,0)}(y) \pm \rho_+^{(1,0)}(y) \pm \rho_+^{(0,1)}(y) + \rho_+^{(1,1)}(y) \quad (14a)$$

and

$$\rho_0(y) = \rho_+^{(0,0)}(y) + \rho_0^{(1,1)} . \quad (14b)$$

Thus we find

$$Q_0(y) = \frac{1}{2} (Q_+(y) + Q_-(y)) + Q_0^{(1,1)}(y) - Q_+^{(1,1)}(y). \quad (15)$$

Up to here, everything is completely general and exact. Our dynamical assumption now is that the leading $I = 1$ singularity has intercept $\alpha_\rho(0) \approx \frac{1}{2}$. From this it follows that

$$Q_c^{(1,1)}(y) \sim s^{-1/2}, \quad (16)$$

and thus

$$Q_0(y) = \frac{1}{2} Q_{ch}(y) + O(s^{-1/2}), \quad (17a)$$

and after integration

$$\langle n_0 \rangle = \frac{1}{2} \langle n_{ch} \rangle + O(s^{-1/2}). \quad (17b)$$

Notice that this result does not assume any factorization and is also valid in the fragmentation region and in the presence of diffraction dissociation.

A similar analysis for $\rho_{cd}(y, y')$ is more complicated due to possible $I = 2$ and $I = 3$ exchange in the central blob. Our additional assumption is that such exchanges are negligible, in analogy with our experience from exclusive and charge transfer results. With this assumption, arguments completely analogous to those leading to Eq. (17) lead to

$$Q_{ch, ch}(y, y') = 2 Q_{o, ch}(y, y') + O(s^{-1/2}) \quad (18a)$$

$$= 4 Q_{o, o}(y, y') + O(s^{-1/2}). \quad (18b)$$

Combining Eqs. (10), (17) and (18), we find

$$C_{ch, ch}(y, y') = 2 C_{o, ch}(y, y') + O(s^{-1/2}) \quad (19a)$$

$$= 4 C_{o, o}(y, y') + O(s^{-1/2}), \quad (19b)$$

and after integration over rapidities

$$f_2^{ch, ch} = 2 f_2^{o, ch} + O(s^{-1/2}) \quad (20a)$$

$$= 4 f_2^{o, o} + O(s^{-1/2}). \quad (20b)$$

Using charge conservation, this can be expressed in terms of negative pion correlation and multiplicity

$$f_2^{00} \approx f_2^{0-} \approx f_2^{--} + \frac{1}{2} \langle n_- \rangle . \quad (21)$$

Notice that at this stage, we have introduced an error at low energies, since charge is not strictly conserved for the pion component of the final state: some of the incoming protons will turn into neutrons, giving thus an additional charge to the pions. This effect, which in the Mueller-Regge description stems from $I = 1$ exchange in the fragmentation regions, could only be taken into account using experimental multiplicity distribution of charged pions, or using a detailed Mueller-Regge model for non-leading trajectories. Since the former are not yet measured and the latter do not seem to work particularly well²¹⁾, we have taken Eq. (21) without corrections.

Using our previous formulae (1) and (4) together with this result, we get

$$\langle n_0 \rangle_{n_-} \approx \langle n_0 \rangle + \frac{f_2^{--} + \frac{1}{2} \langle n_- \rangle}{f_2^{--} + \langle n_- \rangle} (n_- - \langle n_- \rangle) . \quad (22)$$

This is an extremely interesting result. From it we see immediately that the asymptotic value of the linear coefficient depends on the nature and strength of the $\pi^- \pi^-$ correlations. If the correlations are of long-range nature, $|f_2^{--}|$ will eventually grow much faster than the average multiplicity $\langle n_- \rangle$. Consequently, the scaling value of the linear coefficient -- which will be reached very slowly -- will be $b_\infty = 1$. In the case of the correlations being of short range only, $|f_2^{--}|$ will grow proportionally to $\langle n_- \rangle$. In such a case the asymptotic limit of the coefficient b will be below one, and might already be reached at present energies.

Finally, let us write up the corresponding formula for the associated Mueller moment $(f_2^{00})_{n_-}$. Without knowledge of the three-body correlations we can only determine it at $n_- = \langle n_- \rangle$ from Eqs. (9) and (18):

$$(f_2^{00})_{n_- = \langle n_- \rangle} \approx \frac{1}{2} \langle n_- \rangle \frac{f_2^{--} + \frac{1}{2} \langle n_- \rangle}{f_2^{--} + \langle n_- \rangle} . \quad (23)$$

4. COMPARISON WITH EXPERIMENT

The available data on the charged-neutral correlations are still rather scanty. Hence we do not attempt a detailed analysis of the data. The following phenomenology should be regarded only as a crude check that our previous reasoning has not led us far from the right track.

As previously mentioned, the experimental data exhibit clear long-range correlation effects. Such effects are mainly due to the simultaneous existence of diffractive and non-diffractive production²⁴⁻²⁶). The diffractive component is known to be important at low multiplicities ($n_- \lesssim \frac{1}{2} \langle n_- \rangle$) only. Thus we should correctly describe the experimental behaviour of the associated quantities around $n_- \approx \langle n_- \rangle$, when the diffractive component is subtracted. When we use the quadratic ansatz for $\langle n_0 \rangle_{n_-}$, we can even have some hope that subtracting the diffractive component will not change the result much. This will be the case if $\langle n_0 \rangle_{n_-}$ has a convex shape, in which case adding diffractive events will increase the mean over-all slope only by giving extra weight to the low-multiplicity points.

Let us first discuss the results with the diffractive events included. The first check to be done is whether Eq. (18) is satisfied. All existing data are consistent with it, though there is some slight indication that $n_0 > \frac{1}{2} \langle n_{ch} \rangle$, over the whole known energy range^{*)}. The next test is that of our assumption that only $I \leq 1$ exchange is important in Fig. 1b. One way to test this is to calculate f_2^{0-} from the associated multiplicities measured at Serpukhov and NAL, using

$$f_2^{0-} = \frac{1}{6_{inel}} \sum_{n_-} \sigma_{n_-} (\langle n_0 \rangle_{n_-} - \langle n_0 \rangle) \cdot n_- \quad (24)$$

The results are shown in Fig. 2. The agreement of our relation (21) with the data is seen to be reasonably good^{**)}. The other curves shown are those of the so-called ϵ^- and ρ^- -models¹¹⁾. For these models:

$$f_2^{0-} = f_2^{--} \quad (\epsilon^- \text{-model}), \quad (25)$$

$$f_2^{0-} = f_2^{--} + \frac{3}{4} \langle n_- \rangle \quad (\rho^- \text{-model}), \quad (26)$$

*) This small excess of neutral pions can result, for example, from η production. Each η decays on the average to $0.29 \pi^+ + 1.17 \pi^0 + 0.87 \gamma$, equivalent to $0.29 \pi^+ + 1.60 \pi^0$.

**) The agreement would be even better if the theoretical curve were shifted slightly down. A proper treatment of the fragmentation region contributions could easily provide such a small constant correction to formula (21). As we have already mentioned, such a treatment would need experimental charged pion multiplicity distributions.

It is interesting to notice that both these models give $I = 2$ exchange when analysed in Mueller language. The same will in general hold in multiperipheral models with $I = 0, 1$ input trajectories: the "I-model" of Caneschi and Schwimmer²⁸⁾ gives, for example, $f_2^{0-} = f_2^{--} + 1/3 \langle n_- \rangle$ ⁸⁾. On the other hand, our formula $f_2^{0-} = f_2^{--} + \frac{1}{2} \langle n_- \rangle$ is obtained in models of charge-independent emission of pions, as in the π -model of Ref. (11), in the critical fluid model of Thomas¹³⁾, and in the fireball models of Ref. 12. [There are also other fireball models where it does not hold²⁹⁾]. It is also obtained in multiperipheral models with $I = 1/2$ exchange [the H-model of Refs. 8 and 28; see also Craigie et al.³⁰⁾].

The $I < 2$ hypothesis can be tested also using data on differential two-body rapidity correlations. The CERN-Hamburg-Vienna Collaboration finds over all ISR energies⁷⁾

$$R_{o, ch}(0,0) = \frac{1}{\rho_o(0) \rho_{ch}(0)} \quad C_{o, ch}(0,0) = 0.65 \pm 0.08 \quad (27)$$

while the Pisa-Stony Brook experiment⁶⁾ gives in the same energy range

$$R_{ch, ch}(0,0) = \frac{1}{[\rho_{ch}(0)]^2} \quad C_{ch, ch}(0,0) = 0.65 \pm 0.06. \quad (28)$$

Using Eq. (17a), this means that Eq. (19a) is exactly fulfilled at $y = y' = 0$. Since also the shape of the correlation is the same in both cases, we see that $I = 2$ exchange is indeed absent. The same conclusion follows also using $\pi^+ \pi^-$ correlation data from NAL⁵⁾, which are more exact but which cannot as directly be compared with the CHV results.

The last test consists in comparing the associated neutral multiplicity with our quadratic ansatz Eq. (5). The three-body correlation f_3^{0--} was estimated from the π -model which gave the same two-body correlations as our model. Its effect was rather small. The predictions are shown in Fig. 3 together with the data, and good agreement is found. In particular, the curves have average slope less than one and negative curvature, as expected before.

As said earlier, the experimentally observed long-range correlations are mainly due to inelastic diffraction and not a property of non-diffractive production. Since we are particularly interested in the charge structure of the latter, we now repeat the above analysis, using the non-diffractive correlations only. They were estimated as follows. We took the experimentally measured multiplicity distributions at 50-400 GeV/c³¹⁾. From these distributions we subtracted the diffractive multiplicity distributions using the estimates due to Roberts and Roy³²⁾. The resulting "pionization" prong cross-sections were smoothed by fitting separately over the relevant energy range. These smoothed cross-sections were then

used to calculate the non-diffractive correlation integral $f_{2\pi}^{--}$. At energies above 400 GeV/c we estimated the correlation integral from the data of the Pisa-Stony Brook Collaboration⁶⁾. The relevant decomposition of the correlation integral reads:

$$f_2^{--} = \alpha_D f_{2D}^{--} + \alpha_\pi f_{2\pi}^{--} + \alpha_D \alpha_\pi (\langle n_- \rangle_\pi - \langle n_- \rangle_D)^2. \quad (29)$$

Here, f_{2D}^{--} and $f_{2\pi}^{--}$ are the correlations from diffractive and non-diffractive components, respectively, and $\alpha_D = \sigma_D/\sigma_{inel}$ and $\alpha_\pi = \sigma_\pi/\sigma_{inel}$ are the relative strengths of the two components. The decomposition of f_2^{--} into its various components is shown in Fig. 4.

The behaviour of the non-diffractive correlation integral $f_{2\pi}^{--}$ is very interesting. At low energies $f_{2\pi}^{--}$ is negative and varies relatively slowly with energy. Around 200 GeV/c $f_{2\pi}^{--}$ crosses to positive values and begins to grow rapidly. The physics behind this behaviour is probably the following. That the correlations are negative at low energies is caused by kinematic "phase-space" effects. Increasing the energy, the fragmentation regions move apart and the central plateau begins to develop. A pair of negative pions belonging to this plateau feel a positive short-range correlation. Around 200 GeV/c the plateau is sufficiently developed that this positive correlation is strong enough to overcome the negative kinematic correlations and $f_{2\pi}^{--}$ starts to grow. In the present day fashionable language of cluster production, the strength of the experimentally-observed positive correlation corresponds to an average cluster size around 1.2 negative particles per cluster and Poisson distribution of negative particles within each cluster²⁶⁾. Correspondingly, f_2^{--} will grow asymptotically as

$$f_{2\pi}^{--} \sim 1.2 \langle n_- \rangle_\pi \quad (30)$$

At two energies (69 and 303 GeV/c), we show in Fig. 3 also the prediction for non-diffractive events only. They are nearly identical with the predictions for all events.

With the above result (30) we can estimate the asymptotic value of the slope of $\langle n_0 \rangle_{n_-}$ near $n_- \approx \langle n_- \rangle$ to be

$$b_\infty = \lim_{s \rightarrow \infty} \frac{f_{2\pi}^{--} + \frac{1}{2} \langle n_- \rangle_\pi}{f_{2\pi}^{--} + \langle n_- \rangle_\pi} \approx 0.75. \quad (31)$$

The error of this should be not worse than 10%. Notice that this would correspond to an error of 50% in our estimate of $f_{2\pi}^{--}$. Thus, we expect the asymptotic value of the coefficient b to be clearly below one.

The energy dependence of the slope parameter b is summarized in Fig. 5, where the experimental values are plotted together with our theoretical prediction. The figure is an updated version of the compilation of Dao and Whitmore²⁾. From this figure we see the very interesting prediction of our model that the scaling limit should be approached within 10% already at the ISR energies. The only available ISR data on $\langle n_0 \rangle_{n_-}$ are those of the CERN-Hamburg-Vienna Collaboration⁷⁾. The CHV result for b is 0.74 ± 0.04 at $\sqrt{s} = 53$ GeV/c. The experiment, however, covers only a limited solid angle. Hence the CHV numbers should probably not be directly compared to the values shown in Fig. 5.

In Fig. 6, we show our prediction for $(f_2^{00})_{n_- = \langle n_- \rangle}$ at $n_- = \langle n_- \rangle$, in the energy range 50-400 GeV/c. There we use f_2 and $\langle n_- \rangle$ of the pionization component only. As we see, $(f_2^{00})_{n_- = \langle n_- \rangle}$ is expected to grow rapidly with increasing energy. The behaviour of the slope, however, cannot be predicted reliably on the basis of the available correlation data. It depends critically on the rate of growth of the three-body correlations [see Eq. (9c)], and experimentally these correlations do not yet exhibit their asymptotic trends.

For comparison we show in Fig. 6 also the predictions for $(f_2^{00})_{n_- = \langle n_- \rangle}$ of the ϵ - and ρ - and I-models. They are:

$$(f_2^{00})_{n_- = \langle n_- \rangle} = \begin{cases} 2\langle n_- \rangle \frac{f_2^{--} + \frac{1}{2}\langle n_- \rangle}{f_2^{--} + \langle n_- \rangle} & (\epsilon\text{-model}) & (32a) \\ -\frac{1}{4}\langle n_- \rangle \frac{f_2^{--} + 2\langle n_- \rangle}{f_2^{--} + \langle n_- \rangle} & (\rho\text{-model}) & (32b) \\ \langle n_- \rangle \frac{f_2^{--} + \frac{5}{9}\langle n_- \rangle}{f_2^{--} + \langle n_- \rangle} & (\text{I-model}) & (32c) \end{cases}$$

The other models discussed before give the same predictions as ours. It has already been pointed out by other authors^{10,11,13)} that $(f_2^{00})_{n_-}$ is a very useful quantity to distinguish between different isospin models. The reason is clear: at $n_- = \langle n_- \rangle$, it depends (roughly) only on two-particle correlations. Thus the charge-independent part of the dynamics is well understood. Compared to the two-particle correlations f_2^{00} and f_2^{0-} themselves it has however an important advantage. Due to the presence of diffraction, these grow rapidly, the leading part being the same in all models. Thus all models give very similar predictions for f_2^{ab} . For $(f_2^{00})_{n_- = \langle n_- \rangle}$, however, the leading part drops out. This can be seen from Eq. (9b),

and is also clear since at $n_- = \langle n_- \rangle$ there is hardly any diffraction. Thus, the predictions for $(f_2^{00})_{n_-}$ differ strongly in different models.

No high-energy data on $(f_2^{00})_{n_-}$ are yet available. An accurate measurement of this quantity at NAL-ISR energies is of great interest and will probably yield important new information on the isospin structure of multiple production.

5. DISCUSSION AND CONCLUSION

We have investigated the interrelation between the associated neutral multiplicity $\langle n_0 \rangle_{n_-}$, the second neutral moment $(f_2^{00})_{n_-}$ and the fully integrated Mueller moments. The associated quantities are expanded as Taylor's series around the mean multiplicity, and the coefficients of these expansions are expressed in terms of the Mueller moments. From the explicit formulae of these coefficients we have written up, it is readily seen that if strong correlations between three or more particles are absent the expansions converge very rapidly.

The correlations between charged and neutral particles depend both on the isospin structure of the multi-body amplitudes and on isospin independent dynamics. In order to study the influence of the latter effects we have performed an isospin analysis of the pion-pion correlations, with the help of the fully inclusive Mueller techniques. The application of such techniques to the analysis of the central region spectra and the correlations encounters some well-known difficulties, when only known trajectories are used.

Hence, we make as weak assumptions as possible. Essentially the only questionable assumption of our analysis is that of the absence of the $I = 2$ exchanges in the Mueller amplitudes. The available multibody data are well consistent with this assumption, but they are not yet accurate enough to provide us with a stringent test of its validity. Additional support for the validity of such an assumption may be found from the analyses of two-body scattering and multibody charge transfer data which show that amplitudes involving $I = 2$ exchanges are strongly suppressed. When high statistics correlation data become available, the validity of this assumption crucial to our analysis should be thoroughly tested.

Here a remark must be made on the implications of the semi-inclusive scaling law on the charged neutral correlations. This problem was recently studied by Dao and Whitmore²⁾. These authors showed that if semi-inclusive scaling is valid already at present machine energies, then the negative-negative correlations should asymptotically exceed the negative-neutral ones (i.e. asymptotically $f_2^{--} > f_2^{0-}$). In our opinion it is extremely unlikely that such a change in the charge ordering of the correlations should occur. We rather regard this strange prediction of the semi-inclusive scaling law as strong evidence against the law being satisfied already at present machine energies. Our analysis, of course, predicts $f_2^{0-} > f_2^{--}$ at

all energies. A violation of this inequality would mean a complete breakdown of our assumption of the suppression of the $I = 2$ amplitude (or of that of the $I = 0$ nature of the Pomeron).

An extremely interesting conclusion of our analysis is that the asymptotic slope of the associated neutral multiplicity depends on the nature of the two-body correlations within the non-diffractive production component. If the correlations are of long-range nature the asymptotic value of the slope -- which should be reached very slowly -- should be one. In the case of only short-range correlations being present the slope should much faster approach an asymptotic limit below one. The exact value of this limit depends on the strength of the correlations. From our numerical analysis we predict the limit to be 0.75 ± 0.1 . These considerations are valid for multiplicities around the mean multiplicity only. At low multiplicities the behaviour of $\langle n_0 \rangle_{n_-}$ is modified by the presence of the diffractive component, such that the average slope over low and average multiplicities should be one. At very high multiplicities, on the other hand, the long-range correlations due to energy-momentum conservation become important, and also the dynamical multibody correlations should not be ignored.

At present machine energies the slope of $\langle n_0 \rangle_{n_-}$ rises slowly and monotonically, being about 0.6 at the NAL energies. This energy dependence is well reproduced by our model calculation. It is important to notice that the isospin structure of our model is independent of the incoming energy. In the model calculation the s -dependence of the slope of $\langle n_0 \rangle_{n_-}$ is caused solely by the s -dependence of the isospin independent correlations. We firmly believe that this aspect of our calculations is a realistic one, i.e. that most of the s -dependence of $\langle n_0 \rangle_{n_-}$ experimentally observed is not caused by changes in the isospin structure of the production amplitudes but by the s -dependence of the isospin independent dynamics.

Finally, we have presented numerical predictions for the associated neutral moment $(f_2^{00})_{n_-}$ in the energy range 50-400 GeV/c. The study of this quantity has been earlier advocated by several authors as a sensitive test of the isospin structure of multiple production. Our results support this point of view, since our predictions differ to a considerable degree from those of some popular isospin models. An accurate measurement of $(f_2^{00})_{n_-}$ in the NAL-ISR energy range is clearly of great interest.

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Figure captions:

- Fig. 1 : Inclusive cross-sections and correlations in terms of the generalized optical theorem.
- Fig. 2 : Correlation integral between negative and neutral pions in proton-proton collisions. Data are from Ref. 1. The full line is our prediction; the dashed and dashed-dotted ones are the predictions of the ρ and ϵ models of Ref. 11, respectively.
- Fig. 3 : a-d) Average number of neutral pions versus negative particle number. Data are from Ref. 1. Data points with broken error bars are those for which σ_{n_-} is less than 3% of σ_{inel} . Dashed-dotted curves at 69 and 303 GeV/c are our predictions for the non-diffractive component only.
e) Predictions for $\langle n_0 \rangle_{n_-}$ at $p_{lab} = 1500$ GeV/c (dashed-dotted line) and at $p_{lab} \rightarrow \infty$ (full line).
- Fig. 4 : Correlation integral between negative particles, decomposed into its various components. Full line: total correlation function. Dashed line: diffractive contribution. Dashed-dotted line: "interplay" term. Dashed-double-dotted line: non-diffractive contribution. Data points are from Ref. 1.
- Fig. 5 : Slope parameter b defined precisely by Eq. (4b) as a function of energy. Data points are computed from Refs. 1 and 32. The dashed lines are our predictions for the pionization component; the solid lines are our predictions for all events including diffractive ones.
- Fig. 6 : Predictions for the correlation integral between neutral pions at $n_- = \langle n_- \rangle$. Full line: our prediction. Dashed line: ρ model. Dashed-dotted line: ϵ model. Dashed-double-dotted line: I-model.

a)

$$C_{cd}(y, y') = \rho_{cd}(y, y') - \rho_c(y) \rho_d(y')$$

b)

FIG. 1

FIG. 2

FIG. 3

FIG. 4

FIG 5

FIG. 6