CM-P00060468

Archives

Ref.TH.1881-CERN

A NEW UNITARITY BOUND ON e^+e^- ANNIHILATION

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A B S T R A C T

Unitarity relates σ_h the electron-positron annihilation cross-section into hadrons and σ_μ the cross-section into muon pairs. This relation provides much more restrictive bounds on σ_h than the simple unitarity bound, for the case in which the hadronic channels dominate the total annihilation cross-section.

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Recent observations ¹⁾ indicate that the annihilation cross-section σ_h of electron-positron pairs into hadrons is roughly constant as a function of s , the centre-of-mass energy squared, for $s \leq 25 \text{ GeV}^2$. The annihilation cross-section into muon pairs σ_μ appears to be in agreement with the lowest order QED prediction

$$\sigma_\mu = \sigma_{\text{point}} = \frac{4 \pi \alpha^2}{3 s} \quad (1)$$

The present data are consistent with a constant σ_h

$$\sigma_h \approx 2 \times 10^{-32} \text{ cm}^2 \quad (2)$$

and a linearly rising ratio

$$R \equiv \sigma_h / \sigma_\mu = K s \quad (3)$$

with $K \approx 0.25 \text{ GeV}^{-2}$. Up to what energy can this behaviour continue ? One unitarity limit has previously been given ²⁾

$$\sigma_h + \sigma_\mu \leq 3 \pi / s \quad (4)$$

The constant cross-section of Eq. (2) would become incompatible with this bound at $s \approx 2 \cdot 10^5 (\text{GeV})^2$ corresponding to colliding e^+e^- beams with beam energy $E \approx 200 \text{ GeV}$.

In this note we point out that unitarity also leads to the bound

$$q_h \leq \frac{12 \pi}{s [R + 4 + 4/R]} \quad (5)$$

which is much more restrictive than Eq. (4) for the case in which R is large ; that is, hadronic channels dominate the annihilation cross-section. The values of σ_h and R given by Eqs. (2) and (3) would become incompatible with Eq. (5) for $s \approx 1700 (\text{GeV})^2$ corresponding to a colliding beam energy $E \approx 20 \text{ GeV}$. Thus, Eq. (5) could become of interest for the next generation of e^+e^- experiments.

The new bound like the old one is based on the idea that the annihilation is dominated by the one-photon annihilation graph. In addition we assume a form of electron-muon universality. Formally, our assumptions and approximations are the following.

(A) - All annihilation occurs from the $J^{PC} = 1^{--}$ state with opposite helicities for e^+ and e^- :

$$|e\rangle = (|e_L^+ e_R^- J=1\rangle + |e_R^+ e_L^- J=1\rangle) / \sqrt{2} \quad (6a)$$

Similarly the only $\mu^+ \mu^-$ state produced is

$$|\mu\rangle = (|\mu_L^+ \mu_R^- J=1\rangle + |\mu_R^+ \mu_L^- J=1\rangle) / \sqrt{2} \quad (6b)$$

(B) - Neglect of final states with photons.

In discussing the import of this approximation, let us separate the case of hard photons from that of soft photons. Hard photon emission is expected to contribute in the unitarity relation terms which are $C(\alpha)$ smaller than those which are kept.

The neglect in the unitarity relation of states containing soft photons requires more care. In order to neglect real soft photons without spoiling the unitarity relations, we should at the same time neglect the contribution of virtual soft photons to the amplitudes which enter this relation. The unitarity bounds we will obtain under hypothesis (B), do not therefore apply to the physical cross-sections, but to those cross-sections obtained from the raw experimental data after subtracting radiative corrections, due to both real and virtual soft photons. This can be done by

standard methods. We note that the cross-section σ_h, σ_μ so defined should be independent of the lepton mass, as we have eliminated the obvious dependences of the form $\ln s/m_e$ which are due to radiative correction.

(C) - Neglect of $J^{PC} = 1^{--}$ final states with lepton pairs plus hadrons.

We omit in our unitarity relation the amplitudes for processes such as

$$e^+ + e^- \rightarrow e^+ + e^- + \text{hadrons} \quad (7)$$

$$e^+ + e^- \rightarrow \mu^+ + \mu^- + \text{hadrons} \quad (8)$$

Process (7) is expected to have a large cross-section from two-photon exchange, but this is essentially a peripheral process so that the cross-section for the $J^{PC} = 1^{--}$ state is expected to be small (of order α^2). Our derivation actually requires only the weaker assumption of neglecting the amplitude for process (8).

(D) - Muon-electron universality.

All S matrix elements are assumed ³⁾ to be invariant under the interchange $e^+e^- \rightleftharpoons \mu^+\mu^-$. This depends on the physical assumption that μ and e have the same interaction and on our conclusion that σ_h and σ_μ as defined in (B) are independent of the lepton mass for large s .

To prove Eq. (5) we consider the S matrix in the subspace of states $|e\rangle, |\mu\rangle$ and $|h_\alpha\rangle$ where $|h_\alpha\rangle$ is the set of $J^{PC} = 1^{--}$ hadronic states distinguished by the subscript α . From muon-electron universality (D)

$$\langle e | S | e \rangle = \langle \mu | S | \mu \rangle = \eta e^{2i\delta} \quad (9a)$$

$$\langle e | S | \mu \rangle = \langle \mu | S | e \rangle = i f \quad (9b)$$

$$\langle e | S | h_\alpha \rangle = \langle \mu | S | h_\alpha \rangle = i f_\alpha \quad (9c)$$

The cross-sections for unpolarized e^+e^- beams are

$$\sigma_u = \frac{3\pi}{s} |f|^2 \quad (10a)$$

$$\sigma_h = \frac{3\pi}{s} \sum_{\alpha} |f_{\alpha}|^2 \quad (10b)$$

The S matrix can be written

$$S = \begin{pmatrix} \eta e^{2i\delta} & if & if_{\alpha} \\ if & \eta e^{2i\delta} & if_{\alpha} \\ .. & .. & .. \end{pmatrix} \quad (11)$$

From the first two rows we find two unitarity relations

$$\sum_i |\langle e | S | i \rangle|^2 = \eta^2 + |f|^2 + \sum_{\alpha} |f_{\alpha}|^2 = 1 \quad (12a)$$

$$\begin{aligned} \sum_i \langle e | S | i \rangle \langle u | S | i \rangle^* = \\ -2\eta \operatorname{Im} (f e^{-2i\delta}) + \sum_{\alpha} |f_{\alpha}|^2 = 0 \end{aligned} \quad (12b)$$

The first of these leads directly to the simple unitarity limit (4) since $\eta^2 \geq 0$. The second gives rise to the following inequality

$$\begin{aligned} \left(\sum_{\alpha} |f_{\alpha}|^2 \right)^2 &= 4\eta^2 [\operatorname{Im} (f e^{-2i\delta})]^2 \\ &\leq 4\eta^2 |f|^2 = 4|f|^2 (1 - |f|^2 - \sum_{\alpha} |f_{\alpha}|^2) \end{aligned}$$

The new unitarity bound Eq. (5) is easily obtained from Eq. (13) by substituting Eqs. (10)

It is interesting to note that the new unitarity bound is satisfied in the field theoretical evaluation of σ_h and σ_μ using only one-photon annihilation graphs with the corrected photon propagator :

$$\frac{1}{s} \rightarrow \frac{1}{s} \frac{1}{1 + \Pi(s)} \quad (14)$$

Here $\Pi(s)$ represents the proper self-energy part ²⁾. The absorptive part of Π is given to first order in α by

$$\text{Im } \Pi(s) = (\alpha/3) (2+R) \quad (15)$$

The contribution $2\alpha/3$ comes from the lepton channels and the rest from the hadron channels. In fact, Eq. (15) follows from the unitarity of the S matrix and muon-electron universality. From Eqs. (1), (14) and (15)

$$\begin{aligned} \sigma_\mu &= \sigma_{\text{point}} |1 + \Pi|^{-2} \\ &= \frac{4\pi\alpha^2}{3s} \left[(1 + \text{Re } \Pi)^2 + \frac{\alpha^2}{9} (R+2)^2 \right]^{-1} \end{aligned} \quad (16a)$$

The bound (5) is now easily obtained

$$\begin{aligned} \sigma_h &= R \sigma_\mu \leq R \frac{4\pi\alpha^2}{3s} \left[\frac{\alpha^2}{9} (R+2)^2 \right]^{-1} \\ &= \frac{12\pi R}{s (R+2)^2} \end{aligned} \quad (16b)$$

It is interesting to note that the bound is saturated when $(1 + \text{Re } \Pi) = 0$; that is, the propagator becomes purely imaginary.

A basic difference between Eq. (5) and the simple unitarity limit Eq. (4) is that Eq. (5) is a relation between two cross-sections σ_h and σ_μ ^{*)}. For large R this can be written

*) In fact Eq. (5) does provide a limit on σ_h alone, $\sigma_h \leq (3\pi/2s)$ which is a factor 2 below that of Eq. (5).

$$\sigma_h^2 \leq 3\pi \chi^2 \sigma_\mu \quad (17)$$

which closely resembles well-known relations between inelastic and elastic cross-sections. By virtue of universality σ_μ here plays the rôle of the elastic cross-section for leptonic channels. ^μ As a result of the optical theorem the amplitude f must have an imaginary part proportional to σ_h ³⁾. Thus a constant σ_h becomes incompatible with a falling σ_μ when the unitarity limit (5) is reached.

The failure of this unitarity limit would indicate that one of the assumptions (A) through (D) has failed. It would seem to us that a significant failure of any of these assumptions at a moderate value of s (such as $2 \cdot 10^3 \text{ GeV}^2$) would indicate a breakdown of normal QED for leptons ^{*}).

ACKNOWLEDGEMENTS

One of us (G.K.) wishes to thank H. Satz for a conversation and N.R.C. and I.P.P. Canada for financial research support.

^{*}) If weak neutral currents contribute to annihilation, the inequality (5) must be modified. However, this contribution is not expected to be important for values of s as low as $2 \cdot 10^3 \text{ GeV}^2$.

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