CM-P00060462

Archives

Ref.TH.1875-CERN

ON THE TRANSFORMATION BETWEEN CURRENT AND CONSTITUENT QUARKS
AND CONSEQUENCES FOR POLARISED ELECTROPRODUCTION STRUCTURE FUNCTIONS

F.E. Close
 CERN - Geneva

A B S T R A C T

The transformation of Buccella et al. and of Melosh is a rotation in spin space, so spin dependent matrix elements are sensitive to this transformation, while spin averaged quantities are not. This is explicitly shown by applying this transformation to the nucleon constituent space $SU(6)$ wave function and the generated current space wave function is seen to be simply a spin rotation of the constituent state. Matrix elements of current operators are then calculated with special emphasis on polarised electroproduction where the results of Kuti and Weisskopf, which ignored this transformation, are vitiated. It is shown that the current quarks generated by this transformation cannot be identified with quark partons and the properties of a more general operator which is a candidate for such a transformation are discussed. The nucleon octet states are decomposed into $SU(6)$ parton configurations and elastic and inelastic matrix elements within the octet are computed. The following sum rule is derived for the asymmetry in polarised electroproduction on polarised protons

$$\int_0^1 dx G_1^{\delta P}(x) \approx \frac{1}{3} \frac{g_A}{g_V}$$

and the relation with previous sum rules is clarified. In the limit $x \rightarrow 1$ we predict that the polarisation asymmetries from proton and neutron will be equal.

Ref.TH.1875-CERN

21 May 1974

ERRATUM AND ADDENDUM

Due to a change in Section IV C) at proof stage, the following changes are necessary in the text.

- P. 13 Delete footnote.
P. 26 Line 21 : delete "and also the baryon magnetic moments".
P. 30 Delete Eq. (A.7).

Eq. (3.1) Replace Σ by Π .

Eq. (3.6) Delete "by a polynomial" and $\sum_N C_N$.

P. 14 $V_N \times V_D$ not $V_N + V_D$.

P. 15 $\phi_N \times \phi_D$ not $\phi_N + \phi_D$.

There is a possible confusion in our reference to the Melosh transformation. By this we refer to his first transformation (see J.S. Bell, CERN TH 1851) where $\theta = \theta(P_T)$ and is not a function of x . This is then discussed in Section III. The second Melosh transformation contains a dependence on the x variables as well and so comes under the framework of Section IV. I am indebted to Dr. H. Osborn for raising this question.

I. INTRODUCTION

During recent months there has been much interest ¹⁾ in the relation between constituent quarks ²⁾ and current quarks ³⁾ or figuratively speaking between quarks and quark partons ⁴⁾. Essentially the problem is that whereas the interaction of the quark partons with currents is by hypothesis known, the nucleon's representation in the parton basis is not known ⁵⁾. Conversely, although the nucleon's representation in the constituent quark basis is known ⁶⁾ the form of the interaction between these objects and currents is not known. In order to compute matrix elements it is necessary therefore to have some hypothesis for the form of the interaction of the constituent quarks with currents and then perform calculations in constituent space ⁷⁾⁻⁹⁾ or alternatively to have knowledge of the nucleon's representation in the current quark, or quark parton, basis.

At present most interest has focused on the operator V_B of Buccella et al. ¹⁰⁾ and of Melosh ¹¹⁾ which is a candidate for performing the transformation between these bases. This transformation has been applied to the current operators and the calculation of matrix elements performed in the constituent basis ⁷⁾⁻⁹⁾. In this manner successful applications to $\pi N \rightarrow N^*$ ⁷⁾ and $\gamma N \rightarrow N^*$ ⁸⁾ have been made and recently this programme has been extended to arbitrary current interactions ⁹⁾ and to deep inelastic scattering ^{9), 12)}.

In this work we investigate the complementary approach of applying the transformation operator V_B to the states and hence generate the nucleon's wave function in current space. Naturally the resulting matrix elements of the known current basis operators coincide with those obtained previously in the constituent space calculations ⁷⁾⁻⁹⁾; however, there are some advantages and insights in this current space approach that are hidden in the previous work. For example, one is able to relate electromagnetic and pionic interaction ^{*)} matrix elements and as a consequence resolve a paradox that was noted in Ref. 9) in connection with a sum rule of Bjorken ¹³⁾ for polarised electroproduction structure functions. Furthermore, the fact that the transformation V_B is simply an over-all rotation of the quark spins is made manifest. We also encounter some phenomenological difficulties with V_B and in the current basis it is clear how to discuss the effects of the more general transformation V that seems required empirically.

*) Via PCAC.

In addition to the above, the purpose of our work is two-fold :

First, we wish to discuss the general representation of the nucleon in the current quark basis and to see how matrix elements of current probes can yield information on the importance of the various possible configurations. In turn this will give information on the properties of the operator V and hopefully will ultimately lead to an understanding of the dynamical origin of this operator.

Secondly, there is a phenomenological motivation, namely to gain understanding and to make realistic estimates of the magnitudes of the polarised electroproduction structure functions ¹³⁾⁻¹⁵⁾. These are about to be measured for the first time ¹⁶⁾ and so it is important to have realistic estimates for use in planning experiments and for subsequent confrontation with the results of those experiments.

To motivate this work we briefly review in Section II the calculation of Kuti and Weisskopf which seems much favoured in making estimates of the polarisation asymmetries ¹⁷⁾. This work, however, took no account of the (crucial) difference between constituent and current quarks. Since this transformation V_B ^{10), 11)} is a spin rotation it will be expected to affect spin dependent calculations such as those discussed in Ref. 15). We also review the work of Ref. 9) which does take some steps in remedying this deficiency. This will bring us to the end of Section II and to our point of departure for the present work. The plan of this paper is therefore outlined at the end of that section.

For the reader familiar with previous work or only interested in the conclusions of this work, the summary table in Section V may be of some use.

II. PREVIOUS WORK IN QUARK PARTON MODELS AND A PARADOX IN THE BJORKEN SUM RULE

To aid the reader not familiar with previous work on polarisation dependence in the quark parton model, we briefly describe the calculation of the asymmetry

$$A^{\gamma N} \equiv \frac{\sigma_{1/2} - \sigma_{3/2}}{\sigma_{1/2} + \sigma_{3/2}} \quad (2.1)$$

in the model of Kuti and Weisskopf ¹⁵⁾ where $\sigma_{\frac{1}{2}, \frac{3}{2}}$ are the photoabsorption total cross-sections for the net photon plus nucleon value of $J_z = \frac{1}{2}$ and $\frac{3}{2}$, respectively. This review also acts as a point of departure for the subsequent discussion in the present paper.

In Ref. 15) the nucleon is treated as a three quark structure accompanied by a sea of quark-antiquark pairs

$$|N\rangle \rightarrow |qqq + \eta(q\bar{q})\rangle \quad (2.2)$$

The three "valence quarks" carry the internal quantum numbers of the nucleon whereas the "sea" has vacuum quantum numbers. The number η depends upon the frame of reference.

Motivated by ideas of two-component duality¹⁸⁾ the interaction of the photon with the valence quarks yields the non-diffractive contribution to cross-sections while interaction with the sea leads to diffractive behaviour^{19), 20)}. It was assumed in Ref. 15) that the difference $\sigma_{\frac{1}{2}} - \sigma_{\frac{3}{2}}$ receives contributions only from the interaction with the three valence quarks, the sum receiving contributions from both :

$$\sigma_T \sim \sigma_{1/2} + \sigma_{3/2} = \sigma_T^{ND} + \sigma_T^D \quad (2.3)$$

$$\sigma_T^{ND} \equiv \sigma_T^{\text{valence}} ; \quad \sigma_T^D \equiv \sigma_T^{\text{sea}}. \quad (2.4)$$

Consequently, one can calculate

$$A^{\gamma N} \equiv (A^{\gamma N})^{ND} \frac{\sigma_T^{ND}}{\sigma_T} = (A^{\gamma N})^{\text{valence}} R(\eta(x)) \quad (2.5)$$

$$(A^{\gamma N})^{\text{valence}} \equiv \left(\frac{\sigma_{1/2} - \sigma_{3/2}}{\sigma_{1/2} + \sigma_{3/2}} \right)^{\text{valence quarks}} \quad (2.6)$$

$$R(\eta(x)) = \left[\sigma_T^{\text{valence}} / \sigma_T \right] (x = Q^2/2M\nu). \quad (2.7)$$

This latter function of x (Eq. 2.7) presumably tends smoothly to zero as $x \rightarrow 0$ since in this limit one expects the diffractive contribution to dominate. In general $R(x)$ will be finite and $\leq 1^*)$. Throughout this paper we shall concern ourselves with the quantity $(A^{\gamma N})^{\text{valence}}$ which we shall write $A_V^{\gamma N}$ and it should always be remembered that the actual asymmetry is related to this quantity by the function $R[\eta(x)]$.

*) A comparison of unpolarised electron and neutrino data suggests that for $x > 0.2$ the valence quarks are dominant²¹⁾. In this region it will be an excellent approximation to set $R=1$ and hence $A_V^{\gamma N} = A^{\gamma N}$ here.

The essential assumption in the work of Ref. 15) was that the three quark system was in a $\underline{56}(8, 2)$ state of $SU_6[SU(3) \times SU(2)]$ and furthermore that there was no orbital angular momentum in the system. In constituent quark space there is good evidence for this assignment of the nucleon ⁶⁾ and so in absence of knowledge of the nucleon's representation in the current quark basis the work of Ref. 15) assumed this assignment to be good there also. The spin unitary spin wave function of three valence quarks describing a proton state with spin $+\frac{1}{2}$ in the Z direction and with the above $SU(6)$ representation reads ^{6), 22)}

$$\frac{1}{\sqrt{18}} \left\{ \begin{aligned} &2|p \uparrow n \downarrow p \uparrow\rangle + 2|p \uparrow p \uparrow n \downarrow\rangle + 2|n \downarrow p \uparrow p \uparrow\rangle \\ &- |p \uparrow p \downarrow n \uparrow\rangle - |p \uparrow n \uparrow p \downarrow\rangle - |p \downarrow n \uparrow p \uparrow\rangle \\ &- |n \uparrow p \downarrow p \uparrow\rangle - |n \uparrow p \uparrow p \downarrow\rangle - |p \downarrow p \uparrow n \uparrow\rangle \end{aligned} \right\} \quad (2.8)$$

The neutron wave function is immediately obtained by the replacement $p \rightleftharpoons n$.

The probabilities to find p or n quarks with spin along $\uparrow$ or opposite $\downarrow$ to the proton spin taken along its direction of motion are then

$$P_{p\uparrow} = \frac{5}{9}, \quad P_{p\downarrow} = \frac{1}{9}, \quad P_{n\uparrow} = \frac{1}{9}, \quad P_{n\downarrow} = \frac{2}{9} \quad (2.9)$$

If the transverse photon has helicity ± 1 and flips the quark spin on interaction, then one has

$$\begin{aligned} e_i S_+ |\uparrow\rangle &= 0 \\ e_i S_- |\uparrow\rangle &= e_i |\downarrow\rangle \end{aligned} \quad (2.10)$$

(and analogously for the $S_z = -\frac{1}{2}$ state) in an obvious notation where $S_{\pm}$ is the spin raising, lowering, operator describing the photon interaction and e_i is the charge of the (i^{th}) quark. The cross-sections for virtual photo-absorption by quarks with spins parallel, antiparallel, to the photon spin are then proportional to 0, e^2 , respectively. Hence if the cross-section on the nucleon is the sum of the individual quark cross-sections, one has immediately (including a factor 3 for the three quarks)

$$\sigma_{\frac{1}{2}}^{\gamma} = \sum_i e_i^2 \left[\sigma_{\gamma q \downarrow} P_{q \downarrow N \uparrow} + \sigma_{\gamma q \uparrow} P_{q \uparrow N \uparrow} \right] \sim \frac{2}{9} \quad (2.11)$$

$$\sigma_{\frac{1}{2}}^{\gamma} = \sum_i e_i^2 \left[\sigma_{\gamma q \uparrow} P_{q \uparrow N \uparrow} + \sigma_{\gamma q \downarrow} P_{q \downarrow N \uparrow} \right] \sim \frac{7}{9}$$

and hence from Eq. (2.6) one has

$$A_{\gamma}^{\gamma P} = 5/9 \quad (2.12)$$

Similarly for the neutron one finds

$$A_r^{\delta N} = 0 \quad (2.13)$$

These results (2.12 and 2.13) are identical with Kuti and Weisskopf ¹⁵⁾.

Of the assumptions that entered into the calculation there is one that is manifestly unreasonable, namely the $SU(6)$ assignment of the nucleon in the current quark basis [Eq. (2.8)]. The problems that arise from such an assignment are well known and discussed in Ref. 1). A phenomenological construction has been made by Buccella et al. ¹⁰⁾ and by Melosh ¹¹⁾ for a unitary operator V which transforms between the current space and constituent space pictures and hence enables one to compute matrix elements of current operators.

To illustrate the procedure we use subscripts C and Q to refer to current space and constituent space, respectively, the wave functions in these spaces being ϕ_C and ψ_Q . Then if we wish to compute the matrix element

$$\langle \phi_c | \hat{O}_c | \phi_c \rangle \quad (2.14)$$

then since $\hat{O}_C$ is a known current space operator one can either explicitly compute ϕ_C with the assumed form for V :

$$|\phi_c\rangle = V |\psi_Q\rangle \quad (2.15)$$

or instead one can transform the operator

$$\langle \phi_c | \hat{O}_c | \phi_c \rangle = \langle \psi_Q | V^{-1} \hat{O}_c V | \psi_Q \rangle = \langle \psi_Q | \hat{O}_Q | \psi_Q \rangle \quad (2.16)$$

and from the resulting transformation properties of $\hat{O}_Q$ plus the known ψ_Q representation the matrix element can again be computed. This latter approach (transforming the operator) was adopted in Ref. 9) to calculate the polarisation asymmetry where the form for V was assumed to have the transformation properties of that in Refs. 10) and 11). The result is that while the nucleon is of course still described by the $SU(6)$ wave function [Eq. (2.8)] (since we are working now in constituent space consistently) the electromagnetic interaction operator is no longer just a quark spin flip but now contains four pieces A, B, C, D , which can affect L_z as well as S_z . Specifically the electromagnetic interaction

operator in the constituent basis can be formally written ^{*)} for a photon with positive helicity

$$A v_+ + B s_+ + C v_+ s_z + D v_+ v_+ s_- \quad (2.17)$$

where $v_{\pm}$ are orbital angular momentum raising, lowering, operators according to their index. The calculation of Ref. 15) is reproduced if all but B are zero.

The effect of this operator on quark states is [cf. Eq. (2.10)]

$$(A v_+ + B s_+ + C v_+ s_z + D v_+ v_+ s_-) q \begin{bmatrix} \downarrow \\ \uparrow \end{bmatrix} = \begin{bmatrix} (A-C) q(\downarrow)_1 + B q(\uparrow)_0 \\ (A+C) q(\uparrow)_1 + D q(\downarrow)_2 \end{bmatrix} \quad (2.18)$$

where the subscripts denote the magnitude of L_z generated between the quark and the core. As a consequence

$$\sigma_{\downarrow\uparrow} \equiv \sigma_{\uparrow\downarrow} = e_i^2 [(A-C)^2 + B^2] \equiv e_i^2 z \quad (2.19)$$

$$\sigma_{\downarrow\downarrow} \equiv \sigma_{\uparrow\uparrow} = e_i^2 [(A+C)^2 + D^2] \equiv e_i^2 y$$

and so (cf. 2.11)

$$\sigma_{3/2}^V(\gamma P) = \frac{7}{9} y + \frac{2}{9} z \quad ; \quad \sigma_{1/2}^V(\gamma P) = \frac{2}{9} y + \frac{7}{9} z \quad (2.20)$$

which yields

$$A_{\gamma}^{\gamma P} = \frac{5}{9} \frac{z-y}{z+y} \leq \frac{5}{9}. \quad (2.21)$$

Similarly one finds $A_V^{\gamma N} = 0$. If the transformation had not been taken into account ^{15), 19)}, then $y=0$ and the previous result ¹⁵⁾ of $A_V^{\gamma P} = \frac{5}{9}$ would obtain.

A similar discussion applied to the unpolarised structure functions yielded

$$\frac{\sigma^{\gamma N}}{\sigma^{\gamma P}} = \frac{2}{3} \quad (2.22)$$

and so the results of Refs. 15) and 19) were unaltered.

*) For discussion see Refs. 8) and 9). Heuristically the current space operator for $\Delta J_z = +1$ had $\Delta S_z = +1$. The transformation is a spin space rotation which leaves the unitary spin quantum numbers unaffected. Hence one generates the $\Delta S_z = -1, 0, +1$ terms. No $\Delta S_z \geq 1$ occur since these would correspond to non 35 photon (an explicit assumption in those references).

We make here a few comments on the above and hence clarify the point of departure at which we have now arrived.

We have seen that predictions for the relative magnitudes of the inelastic structure functions can be modified by taking account of a non-trivial operator V . The particular V of Refs. 10), 11) is a rotation of the quark spins and is symmetric in the quarks. Therefore one can heuristically imagine its effect as no more than rotating the spin projection of the photon-nucleon system^{*)} and hence the polarisation asymmetry is "diluted" from its maximum value of $\frac{5}{9}$, the unpolarised results being unaffected. In the next section we shall explicitly consider the transformation of the wave function induced by the particular V and manifestly see this spin rotation effect. In fact it could a priori be such a great rotation that the asymmetry could be negative. This result would be paradoxical in the light of Bjorken's sum rule¹³⁾ which we can write

$$\int_0^1 \frac{dx}{x} \left(A^{\gamma P}(x) F_2^{\gamma P}(x) - A^{\gamma N}(x) F_2^{\gamma N}(x) \right) = \frac{1}{3} \left| \frac{g_A}{g_V} \right| \quad (2.23)$$

and so, since we have $A^{\gamma N} = 0$, $A^{\gamma P}$ cannot be negative for all x yet from Eq. (2.21) there seems to be nothing preventing $y > z$. The origin and resolution of this paradox will become clear in Section III when we transform

*) In dynamical models^{22,23,27)} the nucleon is thought of as a state of three (constituent) quarks in its rest frame. On boosting to a frame with $P_z \rightarrow \infty$ there is a Wigner rotation of the quark spins. The operator V represents this spin rotation and so in these models the distinction between current and constituent quarks then loses much of its significance. The transformation V is also then understood to be a Foldy-Wouthuysen transformation and contact is made with the old quark model calculations^{22),23)}. Indeed the forms of the current operators in those constituent quark models are the same as those generated in the Melosh philosophy. The apparent difference between explicit quark models and the work of Refs. 7)-9) is due to the large quark mass assumption (nuclear physics analogy) of the previous work²³⁾ or neglect of quark momentum in the initial state in Ref. 22). Relax these constraints and the transformation properties of the πN and also the electromagnetic interaction are identical in the two approaches. The dynamical approach to the current constituent quark relationship is nicely discussed in Ref. 24) with some impressive quantitative results.

the nucleon wave function to the current basis by the action of the operator V_B and then perform the above calculation in the current basis consistently instead of in the constituent basis as above.

Since the operator V_B is merely a spin rotation then it is also clear why the spin independent ratio σ^N/σ^P was unaltered from the value of $\frac{2}{3}$. However, the fact that this ratio falls to less than $\frac{2}{3}$ (possibly $\rightarrow \frac{1}{4}$ as $x \rightarrow 1$) suggests that the transformation between the two bases is empirically more complicated than the V_B operator of Refs. 10), 11). The predictive power that remains when less specific transformations V are employed will be discussed in Section IV.

A critical survey of predictions for polarised electroproduction behaviour is then made in Section V and some estimates of rates presented.

III. CURRENT AND CONSTITUENT QUARKS : THE EXPLICIT TRANSFORMATION OF BUCCELLA et al. AND OF MELOSH

We have discussed the work of Kuti and Weisskopf¹⁵⁾ which took no account of the difference between current and constituent quarks. We also mentioned the work of Ref. 9) where the transformation V_B of Buccella et al.¹⁰⁾ and of Melosh¹¹⁾ was applied in transforming the current operators to constituent space. As a result we noted that some of the results of Ref. 15) were vitiated.

In this section we shall again employ the transformation V_B but now by applying it to the states, we shall obtain the nucleon's current basis representation as induced by this operator. Then we shall compute matrix elements by sandwiching the known current space operators between these wave functions. This is useful to complement the previous calculations performed in constituent space and gives new insight into the origin of some of those results. In particular the spin rotation aspect of V_B and the resolution of the apparent paradox that arose in connection with the Bjorken sum rule are illuminating.

A) The current space picture of the nucleon :

The particular transformation operator introduced by Buccella et al.¹⁰⁾ and more recently discussed by Melosh¹¹⁾ can be formally written

$$V_B = \sum_{i=1}^3 V_B^{(i)} \quad (3.1)$$

with

$$V_B^{(i)} \sim (1 + \lambda (\sigma_+ v_- - \sigma_- v_+))^{(i)} \quad (3.2)$$

where $\sigma_{\pm} = \frac{1}{2} (\sigma_x \pm i\sigma_y)$ are the spin raising/lowering operators in quark spin space while $v_{\pm}$ act in the (L, L_z) space according to their index and λ is a number normalised such that V_B is unitary.

In a constituent quark basis the nucleon is a member of the representation $(\underline{8}, \underline{2})$ $\underline{56}$ ($L = 0$) of $[SU(3), SU(2)] \in SU(6)[0(3)]$. For a nucleon with spin projection $+\frac{1}{2}$ along the z axis then its wave function may be written^{*)}

$$\psi^{(0)} \sim \left[|8\rangle_{\alpha} \left| \frac{1}{2} \uparrow \right\rangle_{\alpha} + |8\rangle_{\beta} \left| \frac{1}{2} \uparrow \right\rangle_{\beta} \right]_0 \quad (3.3)$$

where the subscript after the bracket denotes the value of L_z .

We now transform this wave function to the current basis by acting with the operator V_B and obtain immediately (Appendix B)

$$\psi^{(1)} \equiv V_B \psi^{(0)} \rightarrow \psi^{(0)} + \lambda \delta\psi \quad (3.4)$$

$$\text{where } \delta\psi \sim \left[|8\rangle_{\beta} \left| \frac{1}{2} \downarrow \right\rangle_{\beta} - \frac{1}{3} |8\rangle_{\alpha} \left| \frac{1}{2} \downarrow \right\rangle_{\alpha} + \frac{\sqrt{2}}{3} |8\rangle_{\alpha} \left| \frac{3}{2} \downarrow \right\rangle_S \right]_1 + \left[\sqrt{\frac{2}{3}} |8\rangle_{\alpha} \left| \frac{3}{2} \uparrow \right\rangle_S \right]_{-1} \quad (3.5)$$

We can generalise the considerations of this section to include any operator V which is related to V_B by a polynomial, i.e.,

$$V = \sum_N c_N V_B^N \quad (3.6)$$

since one finds that

$$V_B \delta\psi \rightarrow -\lambda \psi^{(0)} \quad (3.7)$$

then it follows that

$$\psi^{(N)} \equiv V_B^N \psi^{(0)} \rightarrow A_N \psi^{(0)} + B_N \delta\psi. \quad (3.8)$$

Therefore, for any V of the above class [Eq. (3.6)] suitably normalised one obtains the following form for the transformed nucleon wave function :

$$\begin{aligned} |\phi_c\rangle &= \cos\theta |\psi\rangle + \sin\theta |\delta\psi\rangle \\ &\equiv \frac{\cos\theta}{\sqrt{2}} \left[|8\rangle_{\alpha} \left| \frac{1}{2} \uparrow \right\rangle_{\alpha} + |8\rangle_{\beta} \left| \frac{1}{2} \uparrow \right\rangle_{\beta} \right] \\ &\quad + \frac{\sin\theta}{\sqrt{2}} \left[\left(|8\rangle_{\beta} \left| \frac{1}{2} \downarrow \right\rangle_{\beta} - \frac{1}{3} |8\rangle_{\alpha} \left| \frac{1}{2} \downarrow \right\rangle_{\alpha} + \frac{\sqrt{2}}{3} |8\rangle_{\alpha} \left| \frac{3}{2} \downarrow \right\rangle_S \right)_1 + \left(\sqrt{\frac{2}{3}} |8\rangle_{\alpha} \left| \frac{3}{2} \uparrow \right\rangle_S \right)_{-1} \right] \end{aligned} \quad (3.9)$$

*) Our notation follows that of Ref. 22) - see also Appendix B

It is easily checked that this is identical with the chiral $SU(3) \times SU(3)$ wave function Eq.(13) of Buccella et al., Ref. 10a).

B) Matrix elements in current space :

We can now sandwich current operators between the nucleon wave functions [Eq.(3.9)] and hence compute matrix elements. From the matrix elements of the charge operator,

$$\begin{aligned} \langle \alpha | e | \alpha \rangle &= 0 (P) , \quad 1 (N) \\ \langle \beta | e | \beta \rangle &= 2 (P) , \quad -1 (N) \end{aligned} \quad (3.10)$$

we immediately find that the neutron and proton charges are 0 and 1, respectively. The contribution of the three quark state to the inelastic structure functions νW_2 is proportional to the matrix elements of the charge squared operator ^{*}). The matrix elements of this operator between various unitary spin states are for proton (P) and neutron (N)

$$\begin{aligned} \langle \alpha | e^2 | \alpha \rangle &= \frac{2}{3} (P) , \quad 1 (N) \\ \langle \beta | e^2 | \beta \rangle &= \frac{4}{3} (P) , \quad \frac{1}{3} (N) \end{aligned} \quad (3.11)$$

Hence for the proton

$$\langle \phi_c | e^2 | \phi_c \rangle_p = \cos^2 \theta + \sin^2 \theta \quad (3.12)$$

while for the neutron

$$\langle \phi_c | e^2 | \phi_c \rangle_n = \frac{2}{3} (\cos^2 \theta + \sin^2 \theta) \quad (3.13)$$

and so

$$\left(\sigma_N / \sigma_P \right)_{ND} = 2/3 \quad (3.14)$$

where we have appended the subscript ND to remind us that we have only considered the contribution from these three valence quarks, i.e., have examined the non-diffractive component.

This result is identical with that found in Ref. 9) which applied the operator V to the current operators and sandwiched the resulting

*) Alternatively a sum over the s channel resonance couplings can be performed and exotic t channel states excluded. The same results are obtained, see Ref. 9).

constituent space) operator between the constituent wave function. In the current basis approach we see clearly why this (bad) result is obtained. It is because the operator V applied to the initial constituent space wave function has merely rotated the quark spin

$$V \left[\left| \frac{1}{2} \uparrow \right\rangle_{\beta} \right]_0 \rightarrow \left[\left| \frac{1}{2} \downarrow \right\rangle_{\beta} \right]_1 \quad (3.15)$$

$$V \left[\left| \frac{1}{2} \uparrow \right\rangle_{\alpha} \right]_0 \rightarrow \left[\frac{\sqrt{2}}{3} \left| \frac{3}{2} \downarrow \right\rangle_s - \frac{1}{3} \left| \frac{1}{2} \downarrow \right\rangle_{\alpha} \right]_1 + \left[\frac{\sqrt{2}}{3} \left| \frac{3}{2} \uparrow \right\rangle_s \right]_{-1} \quad (3.16)$$

so the spin normalisation is untouched and hence the relative abundance of isoscalar ($|8 >_{\beta}$) and isovector ($|8 >_{\alpha}$) core states is unaltered. Consequently spin independent matrix elements are unchanged from the old results of Refs. 15) and 19). Spin dependent quantities like the polarisation asymmetries we would expect to be affected by the action of V due to its spin rotation property.

The probabilities to find quarks with spins parallel/antiparallel to the nucleon spin in various spin space configurations $|\frac{1}{2} >_{\alpha, \beta}$ $|\frac{3}{2} >_s$ can be straightforwardly obtained from the wave functions in Ref. 22). Upon weighing with the squared charges of the quarks one finds after some algebra (see also the more general discussion of Section IV) that

$$A_{\nu}^{\gamma N} = 0 ; \quad A_{\nu}^{\gamma P} = \frac{5}{9} (\cos^2 \theta - \sin^2 \theta) = \frac{5}{9} \cos 2\theta \quad (3.17)$$

This agrees with the results of Ref. 9). The deviation of the proton asymmetry from the old Kuti-Weisskopf prediction of $\frac{5}{9}$ is seen to be a measure of the "spin rotation angle" θ which confirms our intuitive expectation.

C) The Bjorken sum rule paradox and angular momentum in the nucleon

As outlined in Section II, the prediction that $A^{\gamma N} = 0$ combined with the Bjorken sum rule

$$\int_0^1 \frac{dx}{x} (A^{\gamma P}(x) F_2^{\gamma P}(x) - A^{\gamma N}(x) F_2^{\gamma N}(x)) = \frac{1}{3} \frac{g_A}{g_V} \quad (3.18)$$

requires that $A^{\gamma P}(x)$ be positive for some significant range of x . However, nothing in our work to date appears to prevent $\frac{3\pi}{4} > \theta > \frac{\pi}{4}$ for which values one has $A^{\gamma P} < 0$ and the sum rule would be in difficulty. In the old model 15)

one had $A^{\gamma P} = \frac{5}{9}$ and $g_A/g_V = \frac{5}{3}$ ^{22),23)}, so the sum rule was "satisfied".

Now, empirically ²⁴⁾ $g_A/g_V \neq \frac{5}{3}$ (this was one of the reasons that led to the introduction of operators like V in the first place). With our current space wave function [Eq. (3.9)] we can compute g_A/g_V by taking the ratio of the matrix elements of $S_z \tau^+$ and τ^+ (τ^+ being the isospin raising operator) between neutron and proton. This simple calculation yields the result

$$\frac{g_A}{g_V} = \frac{5}{3} (\cos^2 \theta - \sin^2 \theta) = \frac{5}{3} \cos 2\theta \quad (3.19)$$

Therefore, we have

$$A^{\gamma P} = \frac{1}{3} \frac{g_A}{g_V} \quad (3.20)$$

(and the Bjorken sum rule is again realised) and also ^{5),7),9),10)}

$$\frac{g_A}{g_V} \leq \frac{5}{3} \quad (3.21)$$

(which is indeed the case empirically). Furthermore we can now obtain the magnitude of the unknown rotation angle θ and since empirically $g_A/g_V \simeq \frac{5}{4}$ then $\cos 2\theta \simeq \frac{3}{4}$ or

$$\tan \theta = 1/\sqrt{7} \quad (3.22)$$

This latter quantity measures the amplitude to find $L_z=0$ and $L_z \neq 0$ in the nucleon state in current space [see Eq. (3.9)]. We therefore conclude that the state generated by the operator V has 40% of its amplitude with $L_z \neq 0$. This result is identical to that found by Seghal ²⁵⁾ in the parton model though his approach is rather different to that employed here ²⁶⁾. It is interesting, and suggestive, to note the fact that the deviation of g_A/g_V from $\frac{5}{3}$ has been related to the presence of orbital angular momentum in the nucleon by the authors of Ref. 28). In their model, the basic quarks have $g_A/g_V = 1$ (i.e., our current quarks) and their interaction determines the structure of the composite particles (constituent quarks). The relativistic motion of the quarks then generates angular momentum and in turn a deviation from $\frac{5}{3}$ for g_A/g_V nucleon. Subsequent related treatments of the dynamical origin of the relation between "current and "constituent" quarks are discussed in Refs. 27) and 29). These works highlight the Wigner spin rotation aspect of the transformation connected with boosting from the lab. to $P_z \rightarrow \infty$ frames.

D) Behaviour as $x \rightarrow 1$:

In this limit the data on the unpolarised cross-sections show that the neutron-proton ratio is less than $\frac{2}{3}$ and possibly tending to $\frac{1}{4}$. This latter ratio would be obtained if only the $|8 >_{\beta}$ states were contributing [cf. Eq. (3.11)] whereas the wave function so far studied [Eq. (3.9)] had equal probability for both $|8 >_{\alpha}$ and $|8 >_{\beta}$, hence the $\frac{2}{3}$ ratio was found. One ad hoc approach ^{*)} might be to suppose that in Nature the operator V_B has induced the correct spin structure in the nucleon wave function, but that the relative importance of $|8 >_{\alpha}$ and $|8 >_{\beta}$ states is x dependent. Then for $x \gtrsim 0.2$ (where it seems that the quark-antiquark sea is unimportant) the wave function reads

$$|\phi_c(x)\rangle = \sin\phi(x) |8\rangle_{\alpha} \left\{ \cos\theta \left[\left| \frac{1}{2} \uparrow \right\rangle_{\alpha} \right] + \sin\theta \left(\left[\frac{\sqrt{2}}{3} \left| \frac{3}{2} \downarrow \right\rangle_s - \frac{1}{3} \left| \frac{1}{2} \downarrow \right\rangle_{\alpha} \right] + \sqrt{\frac{2}{3}} \left[\left| \frac{3}{2} \uparrow \right\rangle_s \right] \right) \right\} \\ + \cos\phi(x) |8\rangle_{\beta} \left\{ \cos\theta \left[\left| \frac{1}{2} \uparrow \right\rangle_{\beta} \right] + \sin\theta \left[\left| \frac{1}{2} \downarrow \right\rangle_{\beta} \right] \right\} \quad (3.23)$$

$$\tan^2\phi(x) \equiv \frac{1-4R(x)}{2R(x)-3} ; R(x) \equiv F_2^{en}(x)/F_2^{ep}(x) \quad (3.24)$$

(the wave function Eq. (3.9) corresponding to $\phi(x) = \frac{\pi}{4}$ for which $R(x) = \frac{2}{3}$).

Given this wave function, we can compute the x dependence of the polarisation asymmetries and the result is plotted in Fig. 1. As $x \rightarrow 1$ the data $R(x)$ have been assumed to continue their trend to $\frac{1}{4}$ and hence $\phi(x) \rightarrow 0$ with the result that in this limit

$$A^{\gamma P} \rightarrow A^{\gamma N} \rightarrow \cos 2\theta \approx \frac{3}{4} .$$

If we set $\theta=0$ (i.e., ignore the transformation between current and constituent quarks), we recover the results of Refs. 30), 31) as $x \rightarrow 1$. In general, the asymmetries are

$$A^{\gamma N}(x) = \cos 2\theta \left\{ \frac{2}{5R(x)} - \frac{3}{5} \right\} \quad (3.25)$$

$$A^{\gamma P}(x) = \cos 2\theta \left\{ \frac{19}{15} - \frac{16R(x)}{15} \right\} \quad (3.26)$$

and the Fig. 1 assumed that no diffractive behaviour is present in $\nu W_2^{N,P}$ for $x > 0.1$.

^{*)} Actually the magnetic moment discussion in Section IV.C) suggests that the wave function (3.23) may be not far from the truth, i.e., $\mu_N/\mu_P = -\frac{2}{3}$ suggests that the operator has induced the right spin structure.

The quantity measured directly is the cross-section for lepton and hadron spins parallel and antiparallel. The relation between this and the asymmetry plotted here is ³²⁾

$$\frac{d\sigma^{\uparrow\uparrow} - d\sigma^{\uparrow\downarrow}}{d\sigma^{\uparrow\uparrow} + d\sigma^{\uparrow\downarrow}} = \sqrt{1 - \varepsilon^2} A \quad (3.27)$$

where

$$\varepsilon = \left(1 + 2 \frac{Q^2 + \nu^2}{Q^2} \tan^2 \frac{\theta}{2}\right)^{-1}$$

is the familiar photon polarisation factor for a lepton with energy E scattering through angle θ in the lab. frame and producing a virtual photon with energy $\nu = E - E'$ and mass squared $Q^2 = 4EE' \sin^2 \frac{\theta}{2}$.

IV) THE GENERAL CURRENT SPACE NUCLEON REPRESENTATION

We have discussed in Section III the observable effects of the transformation between constituent and current quark bases induced by operators of the type $V = \sum_N C_N V_B^N$. The phenomenological consequences of operators of this class have been extensively discussed in the literature ⁷⁾⁻¹⁰⁾ and some empirical successes have resulted. A problem that arose in Section III and Ref. 9) was that the unpolarised photoabsorption cross-section ratio for neutron and proton targets was forbidden to fall below $\frac{2}{3}$ in contradiction with data. An ad hoc approach to this problem was to consider some more general $V(x)$ which allowed the weights of $|8\rangle_\alpha$ and $|8\rangle_\beta$ configurations to vary with x but which had the same effect in spin space as the V of Buccella et al. and of Melosh. It is of course possible that the V of Nature is even more complicated than this and so we now finally consider the effect of a general V constrained only by the demand that it be an $SU(3)$ singlet. We also will separate it into two pieces

$$V \sim V_N + V_D$$

where V_N acts on the three quark constituent state and does not change quark number, any change in quark number being contained in the piece V_D ^{*)}. We shall concentrate here upon the action of V_N . The effect on the state is then

*) The picture we have in mind is that V describes the effect on the internal quark wave function upon boosting it from rest to infinite momentum. The absence of z diagrams in this limit suggests that the action of V_D is related to the binding interaction and would vanish in the free quark case.

$$V\psi \sim (V_N + V_D)\psi = \phi_N + \phi_D$$

where ϕ_N contains only three current quarks. We then also impose ideas borrowed from two component duality¹⁸⁾ and demand that V_D creates quark-antiquark pairs in unitary spin singlet states and these contribute only to the diffractive scattering, the non-diffractive component arising from interaction with the valence quarks in ϕ_N ^{15),19),20),33)}.

We shall first of all classify the possible representations in the state ϕ_N , then examine matrix elements of operators to see what predictive power remains and also make comparison with the results obtained so far with the specific transformation of Section III^{*)}.

A) SU(6) and SU(3)×SU(2) representations of three quark states :

Two quarks with spin $\frac{1}{2}$ can couple their spins to a total of spin 0 (antisymmetric in the spins) or spin 1 (symmetric). Such a diquark system will be referred to as the "core" in what follows. The core then couples with the spin $\frac{1}{2}$ of the remaining quark in any of three ways, the resulting states having spin $\frac{1}{2}$ or $\frac{3}{2}$ and the symmetry properties of the wave functions labelled α, β, S as below :

$$|\frac{1}{2}\rangle_{\beta} = |0\rangle_A \otimes |\frac{1}{2}\rangle \quad (4.1)$$

$$|\frac{1}{2}\rangle_{\alpha} = |1\rangle_S \otimes |\frac{1}{2}\rangle \quad (4.2)$$

$$|\frac{3}{2}\rangle_S = |1\rangle_S \otimes |\frac{1}{2}\rangle \quad (4.3)$$

*) It is a matter of taste whether one regards the general wave function as generated by an unspecified unitary transformation V or whether it is a reflection of the fact that relativistic motion of quarks within the nucleon cause the naive non-relativistic configuration ($\underline{56} \ L=0$) to be "effectively" altered. Specific models of this latter type^{25),28),29)} generate, in the present language, particular values for the $X, Y \dots$ unknowns with varying degrees of success. We here adopt the attitude that it is fruitful to start with the general state and attempt to constrain it by its observable matrix elements. Data will then yield the magnitudes of the $X, Y \dots$ and model builders should use these as their boundary conditions.

The states with labels α, β, S will be referred to in their symmetry properties as mixed symmetric, mixed antisymmetric and symmetric, respectively.

Similarly in isospin space the strangeness zero isospin $\frac{1}{2}(8)$ states are labelled α, β and the $I = \frac{3}{2}$ states (10) labelled S in direct analogy to the above due to coupling of three quarks each with $I = \frac{1}{2}$.

A state of three quarks with total spin projection $J_z = \frac{1}{2}$ and the same unitary spin quantum numbers as the nucleon can then exist in any of sixteen states of $SU(3) \times SU(2)$ with $S_z + L_z = J_z = \frac{1}{2}$. Hence the most general three quark state for this system can be written (where $\uparrow\uparrow, \uparrow, \downarrow, \downarrow\downarrow$ denote the quarks states with $S_z = \frac{3}{2}, \frac{1}{2}, -\frac{1}{2}, -\frac{3}{2}$ and the subscript will denote L_z)

$$|J_z = +\frac{1}{2}\rangle = \left\{ X|8_\alpha|\frac{1}{2}_\alpha + Y|8_\beta|\frac{1}{2}_\beta + Z|8_\alpha|\frac{3}{2}_S + P|8_\alpha|\frac{1}{2}_\beta + Q|8_\beta|\frac{1}{2}_\alpha + R|8_\beta|\frac{3}{2}_S \right\}^{\uparrow\uparrow}_0 + \left\{ \dots \right\}_1^{\uparrow} + \left\{ Z|8_\alpha|\frac{3}{2}_S + R|8_\beta|\frac{3}{2}_S \right\}_{-1}^{\uparrow\uparrow} + \left\{ \dots \right\}_2^{\downarrow\downarrow}, \quad (4.4)$$

where the $X, Y, Z \dots$ are a priori arbitrary parameters whose squares measure the probability that the three quark system is in the cited $[SU(3) \times SU(2)]_{L_z}$ states.

The $SU(3) \times SU(2)$ representations $\alpha\alpha, \beta\beta, \alpha S$ arise in the symmetric $\underline{56}$ and mixed symmetric $\underline{70}^\alpha$ representations of $SU(6)$ as follows $(6), (22), (23)$

$$\underline{56}(\underline{8}, \underline{2}) \sim \frac{1}{\sqrt{2}}(\alpha\alpha + \beta\beta) \quad (4.5)$$

$$\underline{70}^\alpha(\underline{8}, \underline{2}) \sim \frac{1}{\sqrt{2}}(-\alpha\alpha + \beta\beta) \quad (4.6)$$

$$\underline{70}^\alpha(\underline{8}, \underline{4}) \sim \alpha S. \quad (4.7)$$

The representations $\alpha\beta, \beta\alpha, \beta S$ arise in the antisymmetric $\underline{20}$ and mixed antisymmetric $\underline{70}^\beta$

$$\underline{20}(\underline{8}, \underline{2}) \sim \frac{1}{\sqrt{2}}(\beta\alpha - \alpha\beta) \quad (4.8)$$

$$\underline{70}^\beta(\underline{8}, \underline{2}) \sim \frac{1}{\sqrt{2}}(\beta\alpha + \alpha\beta) \quad (4.9)$$

$$\underline{70}^\beta(\underline{8}, \underline{4}) \sim \beta S \quad (4.10)$$

and the wave function Eq. (4.4) could then be rewritten in terms of these states if desired.

In a constituent quark basis the nucleon is well described as three quarks in an s state coupled to total spin $\frac{1}{2}$ and in a $\underline{56}$ of $SU(6)$. Hence in the terminology of Eq. (4.4) the constituent space nucleon wave function has non-zero elements

$$X_0 = Y_0 = \frac{1}{\sqrt{2}}. \quad (4.11)$$

Notice that the probabilities for the core to be in $I=0$ or $I=1$ states are equal. As a second example consider the wave function ϕ_c generated by the operator $V = \sum_N C_N V_B^N$ in Section II which has non-zero elements

$$(i) \quad X_0 = Y_0$$

$$(ii) \quad \begin{bmatrix} X_1 \\ Z_1 \\ Z_{-1} \end{bmatrix} = \begin{bmatrix} -1/3 \\ \sqrt{2}/3 \\ \sqrt{2/3} \end{bmatrix} Y_1 \quad (4.12)$$

where $Y_0 \equiv \frac{1}{\sqrt{2}} \cos \theta$ and $Y_1 \equiv \frac{1}{\sqrt{2}} \sin \theta$. That the coefficients P, Q, R did not occur there, is a consequence of the fact that the operator V_B is symmetric in the three quarks and so can only connect the initial $\underline{56}$ (symmetric) state to $\underline{56}$ or $\underline{70}^\alpha$ (symmetric and mixed symmetric) representations. Again, notice that the probabilities for the core to be in $I=0$ or $I=1$ are still equal. This is because the operator V employed is symmetric in the three quarks and an $SU(3)$ singlet, hence the relative abundances of isoscalar and isovector cores are not altered from their values in the initial wave function [Eq. (3.3) or (4.5)]. As we shall see, it is for this reason that the (bad) result

$$\sigma_N / \sigma_P = 2/3$$

obtained.

The question that we wish to consider is what is the description of the nucleon wave function in the current quark (or quark parton) basis, and in particular how matrix elements of bilinear operators (polarised and unpolarised inelastic electroproduction) yield information on this. Equivalently, if there is a unitary operator V that transforms the constituent space wave function $|\psi_Q\rangle$ into the current space $|\phi_c\rangle$

$$|\phi_c\rangle \sim V |\psi_Q\rangle \quad (4.13)$$

then what are the properties of V ?

B) Matrix elements for the general case :

In section II we saw that in the particularly simple case where all but $X_0=Y_0$ were zero [Eq. (2.8) or (4.4)] one obtained predictions for the polarised and unpolarised cross-sections on neutron or proton targets. We have just shown that a priori the valence quark content of the nucleon can exist in sixteen different $[SU(3) \times SU(2)]_{L_z}$ states in arbitrary amounts. Therefore, one might ask whether there is any predictive power remaining if one does not make ad hoc assumptions as to the relative importances of the sixteen unknowns. In this section, we show how measurements of polarised and unpolarised structure functions can shed light on the current basis representation content of the nucleon and see what restrictions obtain for the polarisation asymmetries in absence of a priori information on the nucleon's current space representation.

It turns out that of these sixteen unknowns there are four combinations that are probed by the above structure functions, and which are related to the four degrees of freedom of $(F,D)_{\text{vector}}$, axial couplings. We shall therefore define ^{*)}

$$C_0^V(x) = (\gamma^2 + p^2 + R^2)_0 + (\dots)_1 + R_{-1}^2 + R_2^2 \quad (4.14)$$

$$C_1^V(x) = (x^2 + Q^2 + Z^2)_0 + (\dots)_1 + Z_{-1}^2 + Z_2^2 \quad (4.15)$$

$$C_0^A(x) = (\gamma^2 - \frac{1}{3}p^2 + \frac{1}{3}R^2)_0 - (\dots)_1 + R_{-1}^2 - R_2^2 + \frac{4\sqrt{2}}{3}(p_0 R_0 + p_1 R_1) \quad (4.16)$$

$$C_1^A(x) = (Q^2 - \frac{1}{3}x^2 + \frac{1}{3}Z^2)_0 - (\dots)_1 + Z_{-1}^2 - Z_2^2 + \frac{4\sqrt{2}}{3}(x_0 Z_0 + x_1 Z_1) \quad (4.17)$$

where the subscripts C_i denote the isospin of the diquark core in the three quark system. For the particular example considered in Section III the wave function ϕ_c had non-vanishing elements given by Eq.(4.12) and so for that case

$$C_0^V = C_1^V = \frac{1}{2}(\gamma_0^2 + \gamma_1^2) = \frac{1}{2} \quad (4.18)$$

$$C_0^A = 3C_1^A = \frac{1}{2}(\gamma_0^2 - \gamma_1^2) = \frac{1}{2} \cos 2\theta \quad (4.19)$$

^{*)} In an infinite momentum frame $P_z \rightarrow \infty$ the core carries fraction X of the target momentum in the z direction. Compare with Ref. 34).

Substitution of these expressions in our general results, Eqs. (4.20, 21) and (4.26, 27) will recover the results Eq. (3.14) and Eq. (3.17) of section III.

The unpolarised structure function $\nu W_2(x)/x$ probes the expectation value of the charge squared operator ^{4),34)} sandwiched between the nucleon states and we find (for the valence quark component)

$$\frac{\nu W_2^{\text{p}}(x)}{x} = \frac{1}{3} (2C_1^{\text{v}}(x) + 4C_0^{\text{v}}(x)) \quad (4.20)$$

$$\frac{\nu W_2^{\text{n}}(x)}{x} = \frac{1}{3} (3C_1^{\text{v}}(x) + C_0^{\text{v}}(x)). \quad (4.21)$$

Clearly one has the bounds ³⁵⁾

$$\frac{1}{4} \leq \nu W_2^{\text{n}}(x) / \nu W_2^{\text{p}}(x) \leq \frac{3}{2} \quad (4.22)$$

and the result $\frac{2}{3}$ for this ratio [Eq. (2.22)] only obtains if

$$C_0^{\text{v}}(x) = C_1^{\text{v}}(x) \quad (4.23)$$

i.e., equal probabilities for the core to be isoscalar or isovector. Empirically the ratio falls below $\frac{2}{3}$, possibly tending to $\frac{1}{4}$ as $x \rightarrow 1$ ³⁶⁾. If so, then in this limit

$$\frac{C_1^{\text{v}}(x)}{C_0^{\text{v}}(x)} \xrightarrow{(x \rightarrow 1)} 0 \quad (4.24)$$

and so in general $V(x)$ is more complicated than that discussed by Buccella et al. and by Melosh since as noted in Sections II and III the ratio $\frac{2}{3}$ still obtained after their transformation was employed ^{*)}, this being due to the $|8 >_{\alpha}$ and $|8 >_{\beta}$ states (the $I=1,0$ core states) having equal probability.

For the spin dependent structure functions ^{**) $G_1^{\text{p}}, G_1^{\text{n}}(x)$} we find

$$G_1^{\text{p}}(x) = \frac{1}{3} (2C_1^{\text{A}}(x) + 4C_0^{\text{A}}(x)) \quad (4.25)$$

$$G_1^{\text{n}}(x) = \frac{1}{3} (3C_1^{\text{A}}(x) + C_0^{\text{A}}(x)) \quad (4.26)$$

^{*)} We are always assuming the incoherent impulse approximation to be valid. If one relaxes this and allows t channel exotic representations to be exchanged in the forward virtual Compton amplitude, then the ratio can fall to zero. See Ref. 9).

^{**) See Refs. 13)-15). For detailed kinematics see Ref. 37) and for the relation between the definitions of these various authors see Ref. 38).}

The photoproduction asymmetries Eqs. (2.5), (2.6) are given by

$$A(x) \equiv \frac{x G_1(x)}{2W_2(x)}$$

(assuming the Callan-Gross relation ³⁹⁾ to be valid between W_1 and νW_2)

and so

$$A_{\nu}^{\gamma N}(x) = \frac{3C_1^A(x) + C_0^A(x)}{3C_1^V(x) + C_0^V(x)} \quad (4.27)$$

$$A_{\nu}^{\gamma P}(x) = \frac{2C_1^A(x) + 4C_0^A(x)}{2C_1^V(x) + 4C_0^V(x)} \quad (4.28)$$

Unlike the work of Ref. 15), we are unable to constrain $A^{\gamma N, \gamma P}$ other than the trivial result $-1 \leq A \leq 1$ arising from the fact that

$$\left| \frac{C_{0,1}^A}{C_{0,1}^V} \right| \leq 1.$$

The reason for this impotence is easily understood : the calculation of the polarisation asymmetry requires knowledge of the (spin) transformation properties of both the electromagnetic operator and also of the states between which it is sandwiched. Only if the states are specified ¹⁵⁾ can the asymmetries be calculated and conversely if the state is completely arbitrary then one cannot constrain the asymmetries.

However, the state is not completely arbitrary as we know, for example, the charges of the nucleons. Taking the expectation value of the charge operator between proton and neutron states yields ^{*)}

$$1 = 2 \int_0^1 dx C_0^V(x) \quad (4.29)$$

$$0 = \int_0^1 dx (C_1^V(x) - C_0^V(x)) \quad (4.30)$$

and so we find the sum rules

$$\int_0^1 dx C_0^V(x) = \int_0^1 dx C_1^V(x) = \frac{1}{2} \quad (4.31)$$

*) Since the sea is isoscalar and neutral it does not contribute to the charge nor to the calculation of g_A/g_V that follows. Further, by the duality assumption it does not contribute to the difference of $\nu W_2^{P,N}$ nor to the polarised structure functions G_1 . Therefore, the present discussion only requires the valence quark state and is exact.

Combined with Eqs. (4.20), (4.21) this yields immediately the familiar sum rule^{15),19),40)}

$$\int_0^1 \frac{dx}{x} \left(\nu W_2^{\gamma P}(x) - \nu W_2^{\gamma N}(x) \right) = \frac{1}{3} \quad (4.32)$$

which empirically seems to be satisfactory³⁶⁾.

We can also calculate

$$\left(\frac{g_A}{g_V} \right)_{PN} \equiv \frac{\langle P | \tau^+ S_z | N \rangle}{\langle P | \tau^+ | N \rangle} \quad (4.33)$$

and for this quantity one finds

$$\left(\frac{g_A}{g_V} \right)_{PN} = \frac{\int_0^1 dx (3C_0^A(x) - C_1^A(x))}{\int_0^1 dx (3C_0^V(x) - C_1^V(x))}. \quad (4.34)$$

The integrand in the denominator is unity [sum rules Eq. (4.31)] while the numerator is related to $G_1^{\gamma P, \gamma N}(x)$ by Eqs. (4.25), (4.26). We therefore find the following constraint on the polarisation structure functions

$$\int_0^1 dx (G_1^{\gamma P}(x) - G_1^{\gamma N}(x)) = \frac{1}{3} \left(\frac{g_A}{g_V} \right) \quad (4.35)$$

which is the familiar sum rule of Bjorken¹³⁾. Hence from the $SU(6)$ transformation properties of the spin dependent bilinear current one can obtain an integrated constraint on G_1 .

We shall see in Section IV.C) that one can obtain a slightly stronger sum rule than that in Eq. (4.35). Apart from that, there is one other prediction that can be made in this general approach. If as $x \rightarrow 1$ the ratio of $\sigma^{\gamma N}/\sigma^{\gamma P}$ tends to $\frac{1}{4}$ then this constrains the wave function to be entirely in the C_0 modes. Hence

$$\frac{C_0^A}{C_1^A}(x \rightarrow 1) \longrightarrow 0 \quad (4.36)$$

with the consequence that

$$A^{\gamma N}(x \rightarrow 1) \longrightarrow A^{\gamma P}(x \rightarrow 1)$$

with equality when (if) the unpolarised ratio reaches $\frac{1}{4}$. We can only predict

the absolute magnitude of these asymmetries if C_0^A is known. In the examples of Section III, by our knowledge of the wave function we were able to write down C_0^A and hence specify the magnitudes of the asymmetries. This is no longer true in the general example. Data will eventually enable us to constrain the various combinations of the sixteen a priori unknown parameters.

C) Semi-leptonic decays of baryons :

With our wave function [Eq. (4.4)] for the nucleon, and in general the octet of baryons, we can calculate matrix elements between members of the octet and hence by comparison with data obtain some additional constraints on the configurations to which the nucleon belongs. We have already computed $(g_A/g_V)_{NP}$ which is proportional to $(F+D)_{\text{axial}}$. Similarly we can compute $(g_A)_{\Sigma\Lambda}$ which is $\sqrt{\frac{2}{3}}D_{\text{axial}}$ and $(g_A/g_V)_{\Xi^- \Xi^0}$ which is $(F-D)_{\text{axial}}$. Comparison with data then yields $(F/D)_{\text{axial}}$ and hence a constraint upon $C_{0,1}^A(x)$ that will enable us to write the sum rules for $G_1^{N,VP}$ separately.

Formally, the magnitude of g_A is given by the matrix element

$$g_A \sim \langle h=+\frac{1}{2} | \tau^+ S_z | h=+\frac{1}{2} \rangle \quad (4.37)$$

where τ^+ is the isospin raising operator, and h the baryon spin projection in the z direction.

Computing g_A we find

$$(g_A)_{NP} = \int_0^1 dx \left(3C_0^A(x) - C_1^A(x) \right) = F+D \quad (4.38)$$

$$(g_A)_{\Sigma\Lambda} = \sqrt{\frac{3}{2}} \int_0^1 dx \left(C_0^A(x) - C_1^A(x) \right) = \sqrt{\frac{2}{3}} D \quad (4.39)$$

$$(g_A)_{\Xi \Xi^0} = 2 \int_0^1 dx \zeta_1^A(x) = F - D \quad (4.40)$$

the relations between these quantities and the axial F,D couplings also being quoted. Substituting into (4.25) and (4.26) we obtain the two sum rules

$$\int_0^1 dx G_1^{\sigma_m}(x) = \left(\frac{1}{3} \frac{g_A}{g_v} \right) \frac{2}{3} \left[\frac{3F/D - 2}{F/D + 1} \right] \quad (4.41)$$

$$\int_0^1 dx G_1^{\sigma_p}(x) = \left(\frac{1}{3} \frac{g_A}{g_v} \right) \frac{1}{3} \left[\frac{9F/D - 1}{F/D + 1} \right] \quad (4.42)$$

At present only $(g_A/g_v)_{NP}$ and $(g_A)_{\Sigma\Lambda}$ of the three strangeness conserving axial transitions have been measured and the data are shown in Fig. 2²⁴⁾. The line $(F/D)_{\text{axial}} = 2/3$ is also indicated and coincides nicely with the data. Hence we would deduce that

$$\begin{aligned} D &= 0.74 \pm 0.01 \\ F &= 0.49 \pm 0.01 \end{aligned} \quad (4.43)$$

and so predict that

$$-(g_A)_{\Xi \Xi^0} = 0.25 \pm 0.01 \quad (4.44)$$

This would lead to the sum rules

$$\int_0^1 dx G_1^{\gamma_n}(x) = 0 \quad (4.45)$$

$$\int_0^1 dx G_1^{\gamma_P}(x) = \frac{1}{3} \frac{g_A}{g_V} \quad (4.46)$$

in contrast to the predictions of Ellis and Jaffe who used the value ^{*)} of $(F/D)_{\text{axial}} = 0.58 \pm 0.03$ which is a best fit including the strangeness changing decays, and assuming the validity of Cabibbo theory ²⁴⁾.

^{*)} This was also employed by Sehgal ²⁵⁾. De Alwis ¹²⁾ assumed $F=D$; the motivation for this is unclear to us.

The deviation from $F/D=2/3$ upon incorporation of $\Delta S \neq 0$ decays is not statistically significant ^{*)} and we suspect that, given the above results, the data will subsequently converge to the $2/3$ ratio for F/D . Failure to do so might reflect uncertainty in the magnitude of the Cabibbo angle, the possibility that this angle differs for vector and axial vector couplings or that the $\Delta S=0$ and $\neq 0$ processes are not related by the Cabibbo hypothesis or that the above picture is in some way deficient. It would clearly be of interest to have a measurement of the remaining $\Delta S=0$ transition to see if the result (4.44) (with $F/D=2/3$) obtains or whether the result

$$-(g_A)_{\Xi\Xi^0} = 0.33 \pm 0.01 \quad \left(\frac{F}{D} = 0.58 \pm 0.03 \right) \quad (4.47)$$

is found. The exploitation of the new high intensity hyperon beams will hopefully lead to better information on the above questions.

V) SUMMARY AND CONCLUSIONS

We have shown that the results of work which did not take account of the difference between current and constituent quark bases can be radically altered when a non-trivial transformation between these bases is taken into account. A digestible summary of our conclusions is presented in Table 1.

The transformation of Refs. 10), 11) was an $SU(3)$ singlet, acted on single quarks and was in effect a spin rotation operator. Hence matrix elements that are spin dependent are affected by the action of this transformation whereas spin independent ones are not. The empirical successes with resonance helicity couplings ^{7), 8)} suggest that the spin structure of this transformation is reasonable but application to deep inelastic structure functions showed its isospin properties to be inadequate if the incoherent impulse assumption of the parton model is valid.

*) The result $F/D=2/3$, together with the magnetic moment ratio $\mu_N/\mu_P = -2/3$ are often cited as the great successes of $SU(6)$. We see here that it is in fact somewhat fortuitous since in the absence of further information on the nucleon's configuration in current space these ratios are unconstrained. Conversely their value suggests that the spin space structure of the Melosh transformation is perhaps reasonable

The simplest generalisation of this operator to incorporate the desired isospin properties was discussed and applied to predicting the behaviour of polarised electroproduction structure functions. These predictions were exhibited in Fig. 1.

In general, the transformation could of course be still more complicated and so a more general example was discussed in Section IV. Matrix elements of charge and magnetic moments empirically constrained the isospin and spin space structure of the general wave function. However, for polarised electroproduction one now only found the sum rule constraints Eqs. (4.44), (4.45) and no predictions could be made for the actual x dependence, save that as $x \rightarrow 1$ the proton and neutron asymmetries become equal. The hierarchy of tests in polarised electroproduction is therefore as follows.

- 1) The weakest quark dependent result is the Bjorken sum rule [Eq. (4.35)] for the difference of proton and neutron structure functions.
- 2) The additional assumption that only the valence quarks contribute to the polarisation structure functions then allows one to obtain sum rules for the proton and neutron separately (4.41). The magnitude of the right-hand side of the sum rules is a function of the F/D ratio for the octet of axial currents. A best fit to the semi leptonic decays of baryons yields

$$\frac{F}{D} = 0.58 \pm 0.03$$

and the sum rules of Ellis and Jaffe ⁴¹⁾ obtained [Eq. (4.51)]. The strangeness conserving decays and also the baryon magnetic moments suggested that the F/D could be more like $2/3$ and hence the sum rule Eqs. (4.44), (4.45) were found. Therefore, it is necessary to learn whether the F/D for the baryon leptonic decays and the magnitude of the sum rules are consistent. In this regard data on $\Xi^- \rightarrow \Xi^0 e^- \bar{\nu}$ would be of use.

- 3) The explicit x dependence of the asymmetries [Eqs. (4.27), (4.28)] will yield detailed information on the spin structure of the transformation V and supplement the information on its isospin structure already available from unpolarised inelastic electroproduction. If the spin structure of V_B [Refs. 10), 11)] is correct then the results of Fig. 1 should be obtained.

One advantage of our general discussion in Section IV is that for any particular model, one need only describe the model's wave function in terms of the notation Eq. (4.4) and then matrix elements of observables can be immediately obtained, since we give the general forms for these quantities [Eqs. (4.13), (4.17), (4.20), (4.21), (4.25), (4.26), (4.34), (4.39)]. As an illustrative example, we discuss the wave function (3.23) in Appendix A, and in Appendix C some specific models are noted.

In conclusion, it seems clear that spin dependent predictions which do not take account of a non-trivial transformation V are likely to be in trouble empirically. Furthermore, the particular transformation $V_B^{10),11)}$ seems inadequate and hence the desire for polarised electroproduction and semi-leptonic decays to yield further information on its properties. In particular it seems clear that the restriction to single quark operators is not sufficient, or equivalently that non 35 contributions are present in the constituent space representation of the current ⁴²⁾.

Model	Current-quark wave function	$\frac{\nu W_2^{\gamma N}}{\nu W_2^{\gamma P}}$	Spin dependence	
a) Kuti-Weisskopf $V_N \sim 1$	$\underline{56}(L_z=0)$ [Eq. (2.8)]	$\geq \frac{2}{3}$	$A_V^{\gamma N}=0$ $A_V^{\gamma P}=\frac{5}{9}$	Section II Ref. 15)
b) Buccella-Melosh $V_N=V_B$ [Eq. (3.1)]	Specified mixture $\underline{56}, \underline{70}^\alpha$ [Eq. (3.9)]	$\geq \frac{2}{3}$	$A_V^{\gamma N}=0$ $A_V^{\gamma P}=\frac{1}{3} \frac{g_A}{g_V} = \frac{5}{9} x$ $\cos 2\theta$	Sections II, III Ref. 9)
c) $V_N=V_B(x)$	Same spin structure as b) plus x dependence [Eq. (3.23)]	$\geq \frac{1}{4}$	$A^{\gamma N}, A^{\gamma P}$, Fig. 1	Section III.D Appendix A
d) V_N arbitrary SU(3) singlet	Arbitrary mix of $\underline{56}, \underline{70}^{\alpha\beta}, \underline{20}$ [Eq. (4.4)]	$\geq \frac{1}{4}$	$\int dx (A_{F_2}^{\gamma P P} - A_{F_2}^{\gamma N N}) =$ $\frac{1}{3} \frac{g_A}{g_V}$ As $x \rightarrow 1$: $A^{\gamma N} = A^{\gamma P}$	Section IV.A,B
e) As d) plus constraint that $\frac{F_1}{D} = +\frac{2}{3}$		$\geq \frac{1}{4}$	$\int dx A_{F_2}^{\gamma P P} =$ $\frac{1}{3} \frac{g_A}{g_V}$ $\int dx A_{F_2}^{\gamma N N} = 0$	Section IV.C

Table 1

Table caption : Summary of the various hypotheses for the transformation V between current and constituent quark bases. The models in the literature ^{9),15)} have the bad result for $\nu W_2^{\gamma N}/\nu W_2^{\gamma P}$ [a) and b)] which raises questions as to their reliability for polarised predictions. The transformation c) is the simplest generalisation of the Melosh transformation that satisfies all presently available data. The transformation d) is the most general and e) is constrained empirically by $\mu_N/\mu_P = -\frac{2}{3}$.

APPENDIX A

THE PHENOMENOLOGICAL WAVE FUNCTION (3.23) FOR THE NUCLEON

The simplest generalisation of the Melosh transformation which satisfies all available data generated the wave function Eq. (3.23). We show here the empirical consequences, sum rules and matrix elements, that arise from this wave function and illustrate the general approach of Section IV by reference to this wave function.

Comparison of the "empirical" wave function (3.23) with the general form (4.4) shows that (3.23) is described by the following non-zero values of the sixteen parameters.

$$\begin{aligned}
 \chi_0 &= \sin \phi(x) \cos \theta & \gamma_0 &= \cos \phi(x) \cos \theta \\
 \chi_1 &= -\frac{1}{3} \sin \phi(x) \sin \theta & \gamma_1 &= \cos \phi(x) \sin \theta \\
 z_1 &= \frac{\sqrt{2}}{3} \sin \phi(x) \sin \theta \\
 z_{-1} &= \sqrt{\frac{2}{3}} \sin \phi(x) \sin \theta
 \end{aligned} \tag{A.1}$$

Consequently, the four quantities measured in the various matrix elements are [Eqs. (4.13)-(4.16)]

$$\begin{aligned}
 C_0^V(x) &= \cos^2 \phi(x) \\
 C_1^V(x) &= \sin^2 \phi(x) \\
 C_0^A(x) &= \cos^2 \phi(x) \cos 2\theta \\
 C_1^A(x) &= -\frac{1}{3} \sin^2 \phi(x) \cos 2\theta
 \end{aligned} \tag{A.2}$$

The non-diffractive contributions to νW_2 [Eqs. (4.19), (4.20)] become

$$\begin{aligned}
 \frac{\nu W_2^{\sigma N}}{x}(x) &= \frac{1}{3} (1 + 2 \sin^2 \phi(x)) \\
 \frac{\nu W_2^{\sigma P}}{x}(x) &= \frac{4}{3} (1 - \frac{1}{2} \sin^2 \phi(x))
 \end{aligned} \tag{A.3}$$

and so

$$\left(\frac{\nu W_2^{\pi N}}{\nu W_2^{\pi P}} \right)_{ND} = \frac{1 + 2 \sin^2 \phi(x)}{4 - 2 \sin^2 \phi(x)} \quad (A.4)$$

from which one can solve for $\phi(x)$ by employing the data for νW_2 at least in that region of x where the non-diffractive component is believed known.

The spin dependent structure functions [Eqs. (4.25), (4.26)] are then

$$\begin{aligned} G_1^{\pi N}(x) &= \frac{1}{3} \cos 2\theta \cos 2\phi(x) \\ G_1^{\pi P}(x) &= \frac{1}{3} \cos 2\theta \left(4 \cos^2 \phi(x) - \frac{2}{3} \sin^2 \phi(x) \right) \end{aligned} \quad (A.5)$$

and from the form for $\cos \phi(x)$, $\sin \phi(x)$ found from (A.4) the curves from the asymmetry can be computed via (A.5) and the result shown in Fig. 1.

It is this quantity (Fig. 1) that is this wave function's predictive power, and hence ultimate test for the validity of it as a description of Nature.

From Eq. (4.34) we obtain

$$\left(\frac{g_A}{g_v} \right)_{NP} = \frac{\int dx \left(3 \cos^2 \phi(x) + \frac{1}{3} \sin^2 \phi(x) \right) \cos 2\theta}{\int dx \left(3 \cos^2 \phi(x) - \sin^2 \phi(x) \right)} \quad (A.6)$$

while from (4.39)

$$\left(\frac{\mu_N}{\mu_P} \right) = - \frac{\int dx \left(\frac{1}{3} \sin^2 \phi(x) + \cos^2 \phi(x) \right)}{2 \int dx \cos^2 \phi(x)} \quad (A.7)$$

The sum rules (4.31) which arose from the matrix elements of the charge operator giving the nucleon charges read

$$\int dx \cos^2 \phi(x) = \int dx \sin^2 \phi(x) = \frac{1}{2}$$

and so on substituting into the results of this Appendix we find

$$\int dx G_1^{\gamma N}(x) = 0$$

$$\int dx G_1^{\gamma P}(x) = \frac{5}{9} \cos 2\theta = \frac{1}{3} \frac{g_A}{g_V}$$

$$\left(\frac{g_A}{g_V} \right)_{AN} = \frac{5}{3} \cos 2\theta$$

$$\left(\frac{\mu_N}{\mu_P} \right) = -\frac{2}{3}$$

This latter result suggests that the spin structure of the transformation is indeed satisfactory. The amount of the spin rotation (θ) is given by the empirical result for g_A/g_V . In any event $g_A/g_V < \frac{5}{3}$ is predicted (7), (9), (10). The sum rules for $G_1^{\gamma N, \gamma P}$ are as yet untested and will be the test of the wave functions validity. Furthermore we have the bounds

$$-\frac{1}{3} \leq \frac{A^{\gamma P}}{\cos 2\theta} \leq 1$$

and so an asymmetry less than $-\frac{1}{3}$ would rule out this picture.

APPENDIX B

MATRIX ELEMENTS OF THE SPIN OPERATOR

The three-quark spin space wave functions are spanned by the representations $|\frac{1}{2} >_{\alpha, \beta}$ and $|\frac{3}{2} >_S$ [Section IV and Ref. 22)]. From the explicit form of these wave functions $^{22)}$ one can compute the effect of operation with the cited spin operators.

For

$$\hat{O}|i\rangle = \sum_n c_n |n\rangle \quad (B.1)$$

where $n=\alpha, \beta, S$ then if the state i has spin $\frac{1}{2}$ we find by straightforward calculation the results of Table B1. Similarly for i with spin $\frac{3}{2}$ the results of Table B2 are found.

	$i = \alpha \downarrow$	$i = \alpha \uparrow$	$i = \beta \downarrow$	$i = \beta \uparrow$
S_-	$C_S = \frac{\sqrt{2}}{3}$	$C_\alpha = -\frac{1}{3}$ $C_\alpha = \frac{\sqrt{2}}{3}$	$C_n = 0$	$C_\beta = 1$
S_z	$C_\alpha = \frac{1}{3}$ $C_S = \frac{2\sqrt{2}}{3}$	$C_\alpha = -\frac{1}{3}$ $C_S = \frac{2\sqrt{2}}{3}$	$C_\beta = -1$	$C_\beta = 1$
S_+	$C_\alpha = -\frac{1}{3}$ $C_S = -\frac{\sqrt{2}}{3}$	$C_S = -\frac{\sqrt{2}}{3}$	$C_\beta = 1$	$C_n = 0$

Table B1. The operators $\hat{S}$ of left hand column act on the spin $\frac{1}{2}$ states i and the non-vanishing coefficients C_n of Eq. (B.1) are listed.

	$i = S \downarrow\downarrow$	$i = S \downarrow$	$i = S \uparrow$	$i = S \uparrow\uparrow$
S_-	0	$C_S = \frac{1}{\sqrt{3}}$	$C_\alpha = -\frac{\sqrt{2}}{3}$ $C_S = \frac{2}{3}$	$C_\alpha = -\frac{\sqrt{2}}{\sqrt{3}}$ $C_S = \frac{1}{\sqrt{3}}$
S_z	$C_S = -1$	$C_\alpha = \frac{2\sqrt{2}}{3}$ $C_S = -\frac{1}{3}$	$C_\alpha = \frac{2\sqrt{2}}{3}$ $C_S = \frac{1}{3}$	$C_S = +1$
S_+	$C_\alpha = \frac{\sqrt{2}}{\sqrt{3}}$ $C_S = \frac{1}{\sqrt{3}}$	$C_\alpha = \frac{\sqrt{2}}{3}$ $C_S = \frac{2}{3}$	$C_S = \frac{1}{\sqrt{3}}$	0

Table B2. As Table B1, but for $S=\frac{3}{2}$ initial states.

The effect of the operator V_B [Eqs. (3.1), (3.2) and (3.7)] on various states can now be written down at once and from the results of the Tables B1 and B2, defining

$$\begin{pmatrix} \alpha \uparrow \\ \beta \uparrow \\ S \uparrow \\ S \uparrow \uparrow \end{pmatrix} \rightarrow \cos \theta \begin{pmatrix} \alpha \uparrow \\ \beta \uparrow \\ S \uparrow \\ S \uparrow \uparrow \end{pmatrix} + \sin \theta \begin{pmatrix} \delta \alpha \uparrow \\ \delta \beta \uparrow \\ \delta S \uparrow \\ \delta S \uparrow \uparrow \end{pmatrix}$$

where $\delta \alpha \uparrow \equiv (v_+ S_- - v_- S_+) \alpha \uparrow$ etc., one has

$$\delta \alpha \uparrow = \left(\frac{\sqrt{2}}{3} S \downarrow - \frac{1}{3} \alpha \downarrow \right)_+ + \left(\frac{\sqrt{2}}{3} S \uparrow \right)_-$$

$$\delta \beta \uparrow = (\beta \downarrow)_+$$

$$\delta S \uparrow = \left(\frac{2}{3} S \downarrow - \frac{\sqrt{2}}{3} \alpha \downarrow \right)_+ - \left(\frac{1}{\sqrt{3}} S \uparrow \uparrow \right)_-$$

$$\delta S \uparrow \uparrow = \left(\frac{1}{\sqrt{3}} S \uparrow - \sqrt{\frac{2}{3}} \alpha \uparrow \right)_+$$

and similarly for the states with $S_z = -\frac{1}{2}, -\frac{3}{2}$.

The general matrix elements in Section IV can all be computed from the Tables B1, B2 via lengthy but straightforward algebra.

APPENDIX C

MODEL WAVE FUNCTIONS IN THE NOTATION OF THE GENERAL WAVE FUNCTION

Given any model wave function for the nucleon in current quark space then once the coefficients $X, Y, \dots$ of Eqs. (4.4) are specified, all matrix elements can be immediately obtained following the particular example in Appendix A. We here tabulate the non-vanishing elements for some typical wave functions in the literature.

I. $\underline{56}(L=0)$ [Ref. 15)] : $X_0 = Y_0 = 1/\sqrt{2}$

II. Naive Melosh Transformation [Ref. 11)] :

$$X_0 = Y_0$$

$$\begin{bmatrix} X_1 \\ Z_1 \\ Z_{-1} \end{bmatrix} = \begin{bmatrix} -1/3 \\ \sqrt{2}/3 \\ \sqrt{2}/3 \end{bmatrix} Y_1$$

where $Y_0 = \frac{1}{\sqrt{2}} \cos \theta$, $Y_1 = \frac{1}{\sqrt{2}} \sin \theta$

III. "Empirical wave function" [Eq. (3.23)] :

$$X_0 = \cos \theta \sin \phi(x) \quad ; \quad Y_0 = \cos \theta \cos \phi(x)$$

$$\begin{bmatrix} X_1 \\ Z_1 \\ Z_{-1} \end{bmatrix} = \begin{bmatrix} -\frac{1}{3} \\ \frac{\sqrt{2}}{3} \\ \frac{\sqrt{2}}{3} \end{bmatrix} \sin \theta \sin \phi(x) \quad ; \quad Y_1 = \sin \theta \cos \phi(x)$$

IV. Ref. 43)

$$X_0 = \frac{1}{\sqrt{2}} \left(a + \frac{b}{\sqrt{3}} \right) \quad ; \quad Y_0 = \frac{1}{\sqrt{2}} \left(a - \frac{b}{\sqrt{3}} \right)$$

$$X_1 = \frac{b}{\sqrt{3}} \quad ; \quad Y_1 = -\frac{b}{\sqrt{3}}$$

$$|a|^2 + |b|^2 = 1$$

There is an additional x dependence in this model that describes the structure functions of the quarks in the above configuration.

REFERENCES

- 1) See the lectures of J. Weyers, CERN TH 1743 (1973) and F.J. Gilman in Proc. of 14th Scottish Universities Summer School in Physics (1973).
- 2) M. Gell-Mann, Phys.Letters 8, 214 (1964) ;
G. Zweig, CERN preprints TH 401, 412 (1964) (unpublished).
- 3) See, e.g., S. Adler and R. Dashen, "Current Algebra" (W.A. Benjamin, 1968).
- 4) R.P. Feynman, Phys.Rev.Letters 23, 1415 (1969) ;
J.D. Bjorken and E.A. Paschos, Phys.Rev. 185, 1975 (1969) ;
See also C.H. Llewellyn-Smith, Physics Reports 30, 264 (1972);
R.P. Feynman, "Photon-Hadron Interactions" (W.A. Benjamin, N.Y., 1972);
H.J. Lipkin, Physics Reports 80, 256 (1973).
- 5) R.F. Dashen and M. Gell-Mann in Proceedings of the Third Coral Gables Conference on Symmetry Principles at High Energies (W.H. Freeman and Co., San Francisco, 1966);
H. Harari, in Spectroscopic and Group Theoretical Methods in Physics (North-Holland, Amsterdam, 1968), p. 363.
- 6) See, e.g., R.H. Dalitz in Proceedings of the Second Hawaii Topical Conference in Particle Physics, S. Pakvasa and S.F. Tuan eds., (Univ. of Hawaii Press, 1968) ;
R.H. Dalitz, Invited paper at Low Energy Hadron Physics Meeting, Smolenice, Czechoslovakia ; Oxford University report (1974).
- 7) F.J. Gilman and M. Kugler, Phys.Rev.Letters 30, 518 (1973) ;
A.J.G. Hey and J. Weyers, Phys.Letters 44B, 263 (1973) ;
F.J. Gilman, M. Kugler and S. Meshkov, SLAC-PUB 1286 (1973).
- 8) A.J.G. Hey and J. Weyers, TH 1718 pub. Phys.Letter 48B, 69 (1974);
F.J. Gilman and I. Karliner, Phys.Letters 46B, 426 (1973).
- 9) F.E. Close, H. Osborn and A.M. Thomson, CERN TH 1818 (1974) (Nuclear Physics B to be published).
- 10a) F. Buccella et al., Nuovo Cimento, 69A, 133 (1970) ;
b) F. Buccella, F. Nicolo and A. Pugliese, Nuovo Cimento 12A, 640 (1972).

- 11) H.J. Melosh, Ph.D. thesis (Caltech) (unpublished) ; Enrico Fermi Institute preprint EFI 73/26 (1973).
- 12) S.P. de Alwis, Nuclear Phys. B55, 427 (1973).
- 13) J.D. Bjorken, Phys.Rev. 148, 1467 (1966).
- 14) H. Burkhardt and N. Cottingham, Ann.Phys. 56, 453 (1970) ;
C. Carlson and Wu-Ki Tung, Phys.Rev. D5, 721 (1972) ;
A.J.G. Hey and J. Mandula, Phys.Rev. D5, 2610 (1972) ;
D. Dicus, R. Jackiw and V. Teplitz, Phys.Rev. D4, 1733 (1971).
- 15) J. Kuti and V.F. Weisskopf, Phys.Rev. D4, 3418 (1971) ;
F.E. Close, F.J. Gilman and I. Karliner, Phys.Rev. D6, 2533 (1972).
- 16) V.F. Hughes, SLAC proposal.
- 17) R. McElhaney and S.F. Tuan, Phys.Rev. (to be published) ;
Muon SPS collaboration, (E. Gabathuler, personal communication).
- 18) H. Harari, Phys.Rev.Letters 20, 1395 (1968) ;
P.G.O. Freund, Phys.Rev.Letters 20, 235 (1968).
- 19) P.V. Landshoff and J.C. Polkinghorne, Nuclear Phys. B28, 240 (1970).
- 20) M. Chaichian et al., Nuclear Phys. B51, 221 (1973).
- 21) D. Myatt and D. Perkins, Oxford University Report (1971) ;
J. Cleymans and L. Sehgal, Nuclear Phys. B61, 479 (1973) ;
See also Ref. 4).
- 22) R.P. Feynman, M. Kislinger and F. Ravndal, Phys.Rev. D3, 2706 (1971).
- 23) C. Becchi and G. Morpurgo, Phys.Rev. 149, 1284 (1966) ; 140, B687 (1965) ;
D. Faiman and A. Hendry, Phys.Rev. 180, 1609 (1969) ;
L.A. Copley, G. Karl and E. Obryk, Nucl.Phys. B13, 303 (1969).
- 24) H. Ebenhöf et al., Z. für Physik 241, 473 (1971).
- 25) L.M. Sehgal, Aachen preprint (1974).
- 26) F.E. Close "Why polarised electroproduction is interesting", CERN TH 1843,
to be published in the Proceedings of the IX Rencontre de Moriond,
J. Tran Thanh Van editor (1974).

- 27) A. Le Youanc, L. Oliver, O. Pene and J. Raynal, Orsay report 73/15 (1973).
- 28) P.N. Bogoliubov, Ann.Inst. Henri Poincaré 8, 163 (1968).
- 29) B.H. Kellett, Glasgow University preprints (1974).
- 30) F.E. Close, Phys.Letters 43B, 422 (1973).
- 31) M. Chaichian and S. Kitakado, Nuclear Phys. B59, 285 (1973).
- 32) N. Dombey in "Hadronic Interactions of Electrons and Photons", (J. Cumming and H. Osborn editors, Academic Press, 1971).
- 33) H. Harari p. 560 of Ref. 32).
- 34) S.D. Drell, D. Levy and T.M. Yan, Phys.Rev. D1, 1035 (1970) ;
S.D. Drell, Erice lectures (1969) published in "Subnuclear Phenomena"
(ed. A. Zichichi, N.Y. Academic Press, 1970).
- 35) O. Nachtmann, Nuclear Phys. B38, 397 (1972) : Phys.Rev. D5, 686 (1972).
- 36) E.D. Bloom, Rapporteur's talk at the International Conference on Electron-Photon Interactions, Bonn 1973.
- 37) A.J.G. Hey, CERN TH 1841 to be published in the Proc. of IX Rencontre de Moriond, J. Tran Thanh Van editor (1974).
- 38) F.E. Close, p. 208 of "Physics with polarised targets at high energy", RHEL-73-088 (1973) (A.G. Clark and R. Worden eds.).
- 39) C. Callan and D. Gross, Phys.Rev.Letters 22, 156 (1969).
- 40) K. Gottfried, Phys.Rev.Letters 18, 1174 (1967).
- 41) J. Ellis and R.L. Jaffe, SLAC-PUB-1288 (1973).
- 42) This conclusion is also noted by H. Osborn "On the presence of non-additive contributions to constituent quark calculations of resonance photoproduction", Caltech preprint (1974).
- 43) G. Altarelli, N. Cabibbo, L. Maiani and R. Petronzio, CERN TH 1727 (1973).

FIGURE CAPTIONS

Fig. 1 The polarisation asymmetries on nucleon targets computed from the wave functions Eq. (3.23) which was motivated by the explicit transformation Eq. (3.1) between constituent and current quarks.

Fig. 2 Graphical representation of the F and D couplings for the $\Delta S=0$ semi-leptonic weak decays in the nucleon octet [Ref. 25)]. The $SU(6)$ prediction $F/D=2/3$ is also shown. The point A is the best fit to $\Delta S=0$ and $\Delta S=1$ decays of Ref. 24). The lines 1 and 2 are the predictions for Ξ decay in this work and that of Ref. 25) respectively.

FIG. 1

FIG. 2