A THEORETICAL ARGUMENT
FOR SOMETHING LIKE THE SECOND MELOSH TRANSFORMATION

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ABSTRACT

Identification of $\text{SU}(6)_w$ currents with $\text{SU}(6)_w$ constituents is shown to exclude the existence of the $\Delta^5(L_2 = 0)$ mesons.
The search for a relativistic $SU(6)$ has centred for some time on the $SU(6)_W$ of light plane charges 1,2,3,4,5. The attempt to regard this as a symmetry group, i.e., to identify this $SU(6)_W$,currents with $SU(6)_W$,constituents’ runs into a striking contradiction with experiment as regards the nucleon anomalous magnetic moments - they would have to be zero 6,4. Now Melosh 7) has given indications that such symmetry would not even be theoretically possible, because of conflict with requirements of ordinary rotation invariance. Unfortunately these indications are based on the simulation of bound states by free quark and antiquark wave packets, so that not everyone has been convinced. Here we give a serious and quite simple demonstration that this hypothetical symmetry, when combined with ordinary rotation and Lorentz invariance, would indeed exclude the existence of at least the simplest composite system, the $35\ (L_z=0)$ mesons.

We need only the $W$ spin subgroup of $SU(6)_W$,currents, generated by

$$W_{x,y,z} = \sqrt{2} \int d^4x \delta(x^+) \gamma^+ \left( \frac{1 + \gamma_z}{2} \right) \beta_s \gamma_x \beta_3 \gamma_y \gamma^+$$

We need to remember that these operators commute with the boost operators,

$$E_1 = K_1 + J_2 ; \quad E_2 = K_2 - J_1 ; \quad K_3$$

(where the $J_i$ are the usual angular momenta, and the $K_i$ the usual Lorentz boost operators), and with the "good" components of vector currents (for example, electromagnetic)

$$J^+ = \frac{1}{\sqrt{2}} (J^0 + J^z)$$

Finally we recall that for particles at rest the $W_{z}(=J_z)=0$ $\rho^+$ meson is supposed to be a $W$ spin singlet, while together with the $\pi^+$ the $W_{z}(=J_z)=\pm 1$ states form a $W$ spin triplet. It then follows that, for matrix elements of the current between $\rho^+$ states, the symmetry under consideration would require

* We use the notation $a^\mu = (a^+, a_1, a_2, a^-)$ with $a^\pm = 1/\sqrt{2} (a_0 \pm a_3)$, and a scalar product is written as $a \cdot b = a^\mu b^\mu - a^- b^-$. 
\[ \langle p', \lambda' | J^+ | p, \lambda \rangle \propto \delta_{\lambda' \lambda} \]  \hspace{1cm} (1)

with

\[ | p, \lambda \rangle = e^{-i\omega_3 K_3} e^{-i\omega_4 E_4} | p_0, \lambda \rangle \]  \hspace{1cm} (2)

where \( p_0 \) is the four-momentum of the rest state, \( \lambda \) the eigenvalue of \( W_z \) (and \( J_z \)) for that state, and

\[ p^+ = \frac{M}{\sqrt{2}} e^{i\omega_3} = \gamma \]

\[ p_\perp = \omega p^+ = \omega \gamma \]

Consider now the requirements of Lorentz invariance. The matrix element (1) is characterized by three form factors *

\[ \langle p', \lambda' | J^M | p, \lambda \rangle = I_1^M F_1(t) + I_2^M F_2(t) + I_3^M F_3(t) \]

where \( t = (p' - p)^2 = q^2 \), and the kinematic functions are

\[ I_1^M = -(p + p')^M \epsilon' \ast \epsilon \]
\[ I_2^M = (\epsilon' \ast q) \epsilon^M - (\epsilon \ast q) \epsilon' \ast \epsilon \]
\[ I_3^M = (p + p')^M (\epsilon' \ast q) (\epsilon \ast q) \]

*) This follows from Lorentz invariance, time reversal and current conservation. See, for example, M. Gourdin, "Diffusion des électrons de haute énergie" (Masson et Cie, 1966), p. 105 ff.
With the construction (2) of moving states the polarization vectors are

\[ \epsilon'(p, \lambda=+1) = -\frac{1}{\sqrt{2}} \left( 0, 1, i, (p' + ip^2)/\gamma \right) \]
\[ \epsilon'(p, \lambda=0) = \frac{1}{M} \left( \gamma, p', p^2, (p'^2 - M^2)/2\gamma \right) \]
\[ \epsilon'(p, \lambda=-1) = \frac{1}{\sqrt{2}} \left( 0, 1, -i, (p' - ip^2)/\gamma \right) \]

From the vanishing of the \( \lambda' = -1 \) to \( \lambda = +1 \) matrix element, with \( p_1 \neq p'_1 \neq 0 \) we then have for \( t \neq 0 \)

\[ F_3(t) = 0 \]

From the vanishing of the \( \lambda' = 0 \) to \( \lambda = +1 \) matrix element (with \( F_3 = 0 \))

\[ F_1(t)(1 + \gamma/\gamma') + F_2(t) = 0 \]

for \( t \neq 0 \). But \( \gamma/\gamma' \) is not determined by \( t \):

\[ t = m^2(2 - \gamma/\gamma' - \gamma'/\gamma') - \left( \frac{\gamma'}{\gamma} p'^2 - 2 p'.p + \frac{\gamma'}{\gamma} p^2 \right) \]

Thus for \( t \neq 0 \),

\[ F_1(t) = F_2(t) = F_3(t) = 0 \]

A quite minimal analyticity assumption would then require the form factors to vanish also at \( t = 0 \), where, however, we have \( F_1(0) = 1 \) from the charge. With this contradiction the symmetry hypothesis in question would exclude the existence of such states.
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