Archives

CERN LIBRARIES, GENEVA

CM-P00060469

Ref.TH.1882-CERN

A THEORETICAL ARGUMENT

FOR SOMETHING LIKE THE SECOND MELOSH TRANSFORMATION

J.S. Bell

and

A.J.G. Hey

CERN -- Geneva

A B S T R A C T

Identification of $SU(6)_W$ currents with
 $SU(6)_W$ constituents is shown to exclude the exist-
ence of the $\underline{35}(L_z=0)$ mesons.

Ref.TH.1882-CERN

The search for a relativistic $SU(6)$ has centred for some time on the $SU(6)_W$ of light plane charges $(1), (2), (3), (4), (5)$. The attempt to regard this as a symmetry group, i.e., to identify this $SU(6)_{W, \text{currents}}$ with $SU(6)_{W, \text{constituents}}$, runs into a striking contradiction with experiment as regards the nucleon anomalous magnetic moments - they would have to be zero $(6), (4)$. Now Melosh ⁷⁾ has given indications that such symmetry would not even be theoretically possible, because of conflict with requirements of ordinary rotation invariance. Unfortunately these indications are based on the simulation of bound states by free quark and antiquark wave packets, so that not everyone has been convinced. Here we give a serious and quite simple demonstration that this hypothetical symmetry, when combined with ordinary rotation and Lorentz invariance, would indeed exclude the existence of at least the simplest composite system, the 35 ($I_z = 0$) mesons.

We need only the W spin subgroup of $SU(6)_{W, \text{currents}}$, generated by ^{*)}

$$W_{x,y,z} = \sqrt{2} \int d^4x \delta(x^+) \psi^\dagger \left(\frac{1+\alpha_3}{2} \right) (\beta \sigma_x, \beta \sigma_y, \sigma_z) \left(\frac{1+\alpha_3}{2} \right) \psi$$

We need to remember that these operators commute with the boost operators,

$$E_1 = K_1 + J_2 \quad ; \quad E_2 = K_2 - J_1 \quad ; \quad K_3$$

(where the J_i are the usual angular momenta, and the K_i the usual Lorentz boost operators), and with the "good" components of vector currents (for example, electromagnetic)

$$J^+ = \frac{1}{\sqrt{2}} (J^0 + J^3)$$

Finally we recall that for particles at rest the $W_z (= J_z) = 0$ ρ^+ meson is supposed to be a W spin singlet, while together with the π^+ the $W_z (= J_z) = \pm 1$ states form a W spin triplet. It then follows that, for matrix elements of the current between ρ^+ states, the symmetry under consideration would require

*) We use the notation $a^\mu = (a^+, a^1, a^2, a^-)$ with $a^\pm = 1/\sqrt{2} (a^0 \pm a^3)$, and a scalar product is written as $a \cdot b = a^+ b^- + a^- b^+ - \underline{a}_1 \cdot \underline{b}_1$.

$$\langle p', \lambda' | J^+ | p, \lambda \rangle \propto \delta_{\lambda' \lambda} \quad (1)$$

with

$$| p, \lambda \rangle = e^{-i \underline{\omega} \cdot \underline{E}_1} e^{-i \omega_3 K_3} | p_0, \lambda \rangle \quad (2)$$

where P_0 is the four-momentum of the rest state, λ the eigenvalue of W_z (and J_z) for that state, and

$$P^+ = \frac{M}{\sqrt{2}} e^{\omega_3} = \gamma$$

$$\underline{P}_1 = \underline{\omega} P^+ = \underline{\omega} \gamma$$

Consider now the requirements of Lorentz invariance. The matrix element (1) is characterized by three form factors ^{*)}

$$\langle p', \lambda' | J^\mu | p, \lambda \rangle = I_1^\mu F_1(t) + I_2^\mu F_2(t) + I_3^\mu F_3(t)$$

where $t = (p' - p)^2 = q^2$, and the kinematic functions are

$$I_1^\mu = -(p + p')^\mu \epsilon'^* \cdot \epsilon$$

$$I_2^\mu = (\epsilon'^* \cdot q) \epsilon^\mu - (\epsilon \cdot q) \epsilon'^*{}^\mu$$

$$I_3^\mu = (p + p')^\mu (\epsilon'^* \cdot q) (\epsilon \cdot q)$$

^{*)} This follows from Lorentz invariance, time reversal and current conservation. See, for example, M. Gourdin, "Diffusion des électrons de haute énergie" (Masson et Cie, 1966), p. 105 ff.

With the construction (2) of moving states the polarization vectors are

$$\epsilon^\mu(p, \lambda=+1) = \frac{-1}{\sqrt{2}} (0, 1, i, (p'_1 + i p'_2)/\eta)$$

$$\epsilon^\mu(p, \lambda=0) = \frac{1}{M} (\eta, p'_1, p'_2, (p_1^2 - M^2)/2\eta)$$

$$\epsilon^\mu(p, \lambda=-1) = \frac{1}{\sqrt{2}} (0, 1, -i, (p'_1 - i p'_2)/\eta)$$

From the vanishing of the $\lambda' = -1$ to $\lambda = +1$ matrix element, with $\underline{p}_1 \neq \underline{p}'_1 \neq 0$, we then have for $t \neq 0$

$$F_3(t) = 0$$

From the vanishing of the $\lambda' = 0$ to $\lambda = +1$ matrix element (with $F_3 = 0$)

$$F_1(t)(1 + \eta/\eta') + F_2(t) = 0$$

for $t \neq 0$. But η/η' is not determined by t :

$$t = M^2(2 - \eta/\eta' - \eta'/\eta) - \left(\frac{\eta}{\eta'} p_1'^2 - 2 \underline{p}'_1 \cdot \underline{p}_1 + \frac{\eta'}{\eta} p_1^2 \right)$$

Thus for $t \neq 0$,

$$F_1(t) = F_2(t) = F_3(t) = 0$$

A quite minimal analyticity assumption would then require the form factors to vanish also at $t=0$, where, however, we have $F_1(0)=1$ from the charge. With this contradiction the symmetry hypothesis in question would exclude the existence of such states.

ACKNOWLEDGEMENTS

One of us (TH) would like to thank Professor Speiser for his hospitality at Louvain-la-Neuve where part of this work was performed.

REFERENCES

- 1) H.J. Lipkin and S. Meshkov, Phys.Rev.Letters 14, 670 (1965) ;
See also K.J. Barnes, P. Carruthers and F. von Hippel, Phys.Rev.Letters 14, 82 (1965).
- 2) R. Dashen and M. Gell-Mann, Phys.Letters 17, 142 (1965) ;
S. Fubini and G. Furlan, Physics 1, 229 (1965).
- 3) J. Jersak and J. Stern, Nuclear Phys. B7, 413 (1968) ;
K. Bardakçi and M.B. Halpern, Phys.Rev. 176, 1686 (1968) ;
H. Leutwyler, Springer Tracts in Modern Physics 50, 29 (1969).
- 4) M. Gell-Mann, Schladming 1972, Acta Physica Austriaca, Suppl. IX,
733 (1972).
- 5) The lectures of J.S. Bell, Schladming 1974, CERN preprint TH.1851, review
these matters.
- 6) R. Dashen and M. Gell-Mann, "Symmetry principles at high energy", Coral
Gables Conference 1966 (Freeman, New York, 1966).
- 7) H.J. Melosh, Phys.Rev. D9, 1095 (1974).