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HIGHER ORDER TERMS IN THE ϵ EXPANSION
OF THE POMERON PROPAGATOR

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A B S T R A C T

The indices η , ν and γ which characterize the high energy behaviour of the elastic scattering amplitude in the presence of a non-vanishing triple Pomeron interaction are calculated to order ϵ^2 where $4-\epsilon$ is the dimension of the space of transverse momenta. At the physical value $\epsilon=2$ these corrections to η and ν are larger than the ϵ terms calculated by Migdal, Polyakov and Ter-Martirosyan and Abarbanel and Bronzan.

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Migdal, Polyakov and Ter-Martirosyan ¹⁾ and Abarbanel and Bronzan ²⁾ have recently found solutions of the equations of Gribov's ³⁾ Reggeon calculus for the Pomeron propagator $G(\omega, \vec{k})$ in the limit $\omega \equiv j-1 \rightarrow 0$, $k \rightarrow 0$ (j is the complex angular momentum of the Pomeron and $\vec{k}$ is its two-dimensional momentum vector).

They find

$$G(\omega, \vec{k}) \rightarrow \omega^{-\eta-1} f(k^2/\omega^\nu) \quad (1)$$

as $\omega^\nu \sim k^2 \rightarrow 0$

where η and ν are constants which do not depend upon the value of the triple Regge coupling constant r . In the same limit, the triple Regge vertex $\Gamma(\omega_1, \vec{k}_1; \omega_2, \vec{k}_2; \omega, \vec{k})$, where $\omega = \omega_1 + \omega_2$, $\vec{k} = \vec{k}_1 + \vec{k}_2$, has the behaviour

$$\Gamma \rightarrow \omega^{+\gamma} F\left(\frac{k_1^2}{\omega_1^\nu}, \frac{k_2^2}{\omega_2^\nu}, \frac{k^2}{\omega^\nu}; \frac{\omega_1}{\omega}\right) \quad (2)$$

as $\omega_i^\nu \sim k_i^2 \rightarrow 0$, where γ is a constant which does not depend on r .

The possible existence of solutions of the form (1) and (2) had been previously pointed out by Gribov and Migdal ⁴⁾. They showed that all terms in the Schwinger-Dyson equations for G and Γ have the same singular behaviour if G and Γ have the forms (1) and (2), provided that η , ν and γ satisfy the equation of scaling

$$2\gamma - 3\eta + d = 2 \quad (3)$$

The authors of Refs. 1) and 2) obtained solutions (1) and (2) by considering a more general problem in which the dimension of the Pomeron momentum vector is given the arbitrary value $d = 4 - \epsilon$. The condition (3) generalizes to the equation

$$2\gamma - 3\eta + \frac{\nu d}{2} = 2 \quad (4)$$

They then showed that the indices η , ν and γ can be written as a series in ϵ and found to first order in ϵ

$$\eta = \epsilon/12, \quad \nu = 1 + \epsilon/24, \quad \gamma = \epsilon/3 \quad (5)$$

which for the physical value $\epsilon = 2$ gives $\eta = 1/6$, $\nu = 13/12$, $\gamma = 2/3$.

We have calculated the indices to order ϵ^2 and have found the following results :

$$\begin{aligned} \eta &= \frac{\epsilon}{12} \left[1 + \frac{\epsilon (514 \log^{4/3} + 37)}{288} \right] \\ &\approx \frac{\epsilon}{12} [1 + .64 \epsilon] \end{aligned} \quad (6)$$

$$\begin{aligned} \nu - 1 &= \frac{\epsilon}{24} \left[1 + \frac{\epsilon (310 \log^{4/3} + 79)}{288} \right] \\ &\approx \frac{\epsilon}{24} [1 + .58 \epsilon] \end{aligned} \quad (7)$$

$$\gamma = \frac{\epsilon}{3} \left[1 + \frac{\epsilon (154 \log^{4/3} + 13)}{288} \right] \approx \frac{\epsilon}{3} [1 + .2 \epsilon] \quad (8)$$

We see that for $\epsilon = 2$, η and $\nu - 1$ are changed by more than 100% and hence the usefulness of the ϵ expansion is in doubt.

The general technique of calculation will be described in a separate publication. The remainder of this note will be devoted to a description of the mechanism which produces these large corrections and to a discussion of possible implications these results may have on higher order terms.

To any finite order in perturbation theory G and Γ do not have the limiting forms of Eqs. (1) and (2) obtained by writing $\omega^{-\eta}$, $\omega^{-\nu}$ and ω^{γ} as $1 - \eta \log \omega + \dots$, $1 - \nu \log \omega + \dots$ and $1 + \gamma \log \omega + \dots$, respectively. Then comparison of the coefficients of the first power of $\log \omega$ in these expansions with the corresponding expansions of perturbation theory yields functions $\eta(r^2)$, $\nu(r^2)$ and $\gamma(r^2)$ of the coupling constant r . The values of η , ν and γ are then equal to

$$\eta = \eta(\kappa_i^2), \quad \nu = \nu(\kappa_i^2), \quad \gamma = \gamma(\kappa_i^2) \quad (9)$$

where r_1^2 is chosen so that the condition of scaling (4) is satisfied. We can write this equation as

$$\Phi(\kappa_i^2) = \frac{\epsilon}{2} \quad (10)$$

where

$$\Phi(\kappa_i^2) = 2\gamma(\kappa_i^2) - 3\eta(\kappa_i^2) - (1 - \nu(\kappa_i^2))(2 - \epsilon/2) \quad (11)$$

Equation (10) is easily seen to be equivalent to the equation $\Psi(r_1^2) = 0$, where $\Psi(r^2)$ is the Gell-Mann-Low function of the Reggeon field theory. [For $r^2 = r_1^2$ the solutions for G and Γ will have the scaling forms (1) and (2) for all values of $\vec{k}$ and ω .]

Let us write the perturbation expansion of $\Phi(r^2)$ as

$$\Phi(\kappa_i^2) = B(\epsilon)\kappa_i^2 + C(\epsilon)\kappa_i^4 + D(\epsilon)\kappa_i^6 + \dots \quad (12)$$

where we have made explicit the dependence of $\Phi(r^2)$ upon ϵ . $B(\epsilon)$ is positive because the vertex function contribution $2\gamma(r^2)$ to $\Phi(r^2)$ [Eq. (11)] dominates and, to order r^2 , $\gamma(r^2)$ is positive. Thus if we neglect the higher order terms in Eq. (12), Eq. (10) has the positive solution

$$\kappa_i^2 = \frac{\epsilon}{2B(\epsilon)} \xrightarrow{\epsilon \rightarrow 0} \frac{2\epsilon}{3} \quad (13)$$

Inserting this value into the lowest order perturbation expansions of $\eta(r^2)$, $\nu(r^2)$ and $\gamma(r^2)$ yields the previously obtained ^{1),2)} values of Eq. (5) for the indices valid in the limit $\epsilon \rightarrow 0$.

To calculate r_1^2 to order ϵ^2 we must include the r^4 term in Eq. (12). Equation (10) then becomes

$$B(\epsilon)\kappa_i^2 = \frac{\epsilon}{2} - C(\epsilon)\kappa_i^4 \quad (14)$$

We have calculated $C(\epsilon)$ at $\epsilon = 0$ and find that the contributions of the Pomeron propagator are very small. Including only the vertex contribution $2\gamma(r^2)$ to $\Phi(r^2)$ then gives a good approximation to $C(0)$ which turns out

to be negative. Then from Eq. (14) we see that the ϵ^2 contribution to r_1^2 is positive. The ϵ^2 contribution to the indices η , ν and γ is then obtained by inserting this value of r_1^2 into the fourth order expansions of $\eta(r_1^2)$, $\nu(r_1^2)$ and $\gamma(r_1^2)$. But since the fourth order terms in the expansions of $\eta(r^2)$ and $\nu(r^2)$ are very small, a very good approximation to the formulae (6) and (7) for η and ν is obtained by inserting the ϵ^2 value of r_1^2 into the lowest order expressions for $\eta(r^2)$ and $\nu(r^2)$. Thus the large values of the ϵ^2 corrections to η and $\nu-1$ reflect the correspondingly large increase of r_1^2 produced by the negative value of $C(0)$ in Eq. (14).

If the vertex graphs also dominate the higher order terms in $\Phi(r^2)$, we might suppose, because of the imaginary coupling constant, the signs of the coefficients in the expansion (12) of $\Phi(r^2)$ alternate in sign ^{*)}, i.e.,

$$B(\epsilon) > 0, C(\epsilon) < 0, D(\epsilon) > 0, \quad \text{etc.} \quad (15)$$

The coefficient of ϵ^3 in the expansion of r_1^2 obtained from

$$B(\epsilon)r_1^2 = \frac{\epsilon}{2} - C(\epsilon)r_1^4 - D(\epsilon)r_1^6 \quad (16)$$

would then be the sum of two contributions of opposite sign. The negative contribution arises from the $-D(\epsilon)r_1^6$ term in Eq. (16) with r_1^2 given by Eq. (13). The positive contribution arises from the $-C(\epsilon)r_1^4$ term with r_1^2 calculated to order from Eq. (14). In order to determine which term dominates we must calculate D . At present, however, we cannot rule out the possibility that these two terms compensate each other to a sufficient extent that the ϵ^3 contributions to η and ν are smaller than the ϵ^2 contributions. In any case, it is interesting to note that this possibility for cancellation arises for the first time to order ϵ^3 . As a final comment, we note that the contribution of the explicit ϵ expansions of the coefficient $B(\epsilon)$ to the higher order corrections to the indices η , ν and γ turns out to be rather small.

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*) Actually this alternation in sign is generally true only for the contributions of the irreducible vertex graphs since they contain only a single power of $\log \omega$. However, to fourth order at least the irreducible graphs give the main contribution.

R E F E R E N C E S

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