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TWO-BODY QUANTUM NUMBER EXCHANGE FROM
MANY PARTICLE PRODUCTION DATA

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A B S T R A C T

An analysis of the charge flow distribution and charge correlations from multiparticle data leads, via unitarity considerations, to an understanding of the nature of charge exchange in two-body reactions.

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1. INTRODUCTION

At high energies, typical events have many particle final states. Such states will therefore give the dominating contribution in the unitarity relation, which we write symbolically as

$$\text{Im } A_{ab \rightarrow cd} = \sum_n A_{ab \rightarrow n} A_{cd \rightarrow n}^* \quad (1)$$

Experimentally, amplitudes for quantum number exchange processes are known to decrease rapidly with increasing energy, when compared to those for processes in which no quantum number exchange occurs. A possible explanation for this relative suppression of quantum number exchange is that, for $ab \neq cd$, the intermediate states in the unitarity sum of Eq. (1) may couple strongly either to ab or to cd , but not to both. Increasing the energy, the states dominating the unitarity sum become more and more complex and, as a result, it becomes increasingly likely that a mismatch occurs. Such a mismatch in the absolute values of the production amplitudes may be studied by examining data on the production cross-sections $|A_{ab \rightarrow n}|^2$ and $|A_{cd \rightarrow n}|^2$. Another possible source of suppression of quantum number exchange amplitudes is provided by the phases of the production amplitudes. For $ab \neq cd$, the amplitudes $A_{ab \rightarrow n}$ and $A_{cd \rightarrow n}$ may have different phases and this may suppress the overlap. Such phase effects are difficult to estimate since the phases of multiparticle amplitudes are extremely hard to measure and practically nothing is known about them.

In this letter we shall study Eq. (1) for charge exchange scattering with $q_a - q_c = q_d - q_b = \Delta q \neq 0$. We derive a bound on the intercept of the ρ trajectory, from a knowledge of the charge structure of the production cross-sections. An important feature of our analysis is that our estimates will be based directly on experimental data, with no intermediate model. In this and several other respects our approach resembles that of Krzywicki and Weingarten ¹⁾. Our results show that a large part of the energy dependence of charge exchange processes may be caused by a mismatch in the charge structure of the intermediate states of Eq. (1). This indicates that the role played by the phases of the multi-body amplitudes may be less important than is usually believed. When more detailed multi-body data become available, the techniques developed here will allow one to calculate a numerical bound for the actual size of the

charge-exchange cross-section. We discuss what kind of new data are needed. Finally, we comment on the problem of calculating the angular dependence of charge-exchange amplitudes.

2. SUPPRESSION DUE TO QUANTUM NUMBER MISMATCH

In order to take account elegantly of the mismatch connected with the exchange Δq , we use the quantum number transfers q and $q' = q + \Delta q$ as shown in Fig. 1. Thus for $ab \rightarrow n$ we order the n produced particles in rapidity and define the probability distribution $Q_{ab \rightarrow n}(q_k)$ for the k -th link to have a charge flow q_k . Likewise for $cd \rightarrow n$. Taking account of the mismatch of the moduli of the production amplitudes, we get a factor

$$\lambda_{\Delta q} = \sum_{q_k} (Q_{ab \rightarrow n}(q_k) Q_{cd \rightarrow n}(q_k + \Delta q))^{1/2} / \left(\sum_{q_k} Q_{ab \rightarrow n}(q_k) \sum_{q'_k} Q_{cd \rightarrow n}(q'_k) \right)^{1/2} \quad (2)$$

by which charge exchange is suppressed compared to elastic scattering. If we are at high enough energies and in the central plateau, we expect $\lambda_{\Delta q}$ to be independent of the kind of state n and of the link number k . Also, we expect the averages $(\bar{q}_k)_{ab}$ and $(\bar{q}_k)_{cd}$ to tend to zero there, independently of the initial state. This fact, that the excess of charge likes to stay in the fragmentation regions is nicely illustrated by Fig. 2, in which the difference of the invariant spectra of positive and negative particles is plotted versus the rapidity y , for proton-proton scattering ^{2),3)}. Thus, since the averages $(\bar{q}_k)_{ab}$ and $(\bar{q}_k)_{cd}$ do not differ by just Δq units, $\lambda_{\Delta q}$ is indeed less than 1. To get a significant suppression, we need in addition that $Q_{ab \rightarrow n}(q)$ is not a very broad distribution. NAL data ⁴⁾ shown in Fig. 3 indicate that, again in the central region, the charge transfer distribution is indeed dominantly situated in $|q| \leq 1$.

Up to now, we have discussed only the suppression due to one particular link. The full suppression arises from the fact that the mismatch occurs at each link between any two adjacent final particles. A naïve estimate of the total suppression factor would be $\lambda_{\Delta q}^n$, for n links. The summation over n is easily done for a Poisson multiplicity distribution, and yields

$$R_{\Delta q} = \sum_n \lambda_{\Delta q}^n \frac{\langle n \rangle^n}{n!} e^{-\langle n \rangle} = e^{(\lambda_{\Delta q} - 1)\langle n \rangle} = S^{(\lambda_{\Delta q} - 1)} g^2, \quad (3)$$

where $\langle n \rangle = g^2 \log s$. Thus, in terms of a leading trajectory $\alpha_{\Delta q}$ for exchange of quantum number Δq , we have

$$\alpha_{\Delta q} \leq \alpha_{\Delta q=0} - (1 - \lambda_{\Delta q}) g^2. \quad (4)$$

Below we will estimate the suppression factor $\lambda_{\Delta q}$ for charge-exchange, thus obtaining an upper limit for the ρ intercept.

The above considerations of rapidity ordering and of the exponentiation of the suppression factor recall strongly the multiperipheral model. In particular, the models of Refs. 5)-7) advocate essentially the same mechanism to explain the difference between P and ρ trajectories. However, we emphasize that our estimates will be based directly on experimental data, with no intermediate model. This is indeed relevant in practice since the density in rapidity of produced particles is typically $\sim 3/\text{unit}$. At this density, the chance of a cross-over of particles in rapidity (a breakdown of the strong ordering assumption) is high. Another way of stating this is that the typical sub-energy between adjacent particles is so low ($< 1 \text{ GeV}$) that resonance effects are important. Instead of trying to untangle the produced particles, we appeal directly to the observed distributions themselves for our estimates.

3. EMPIRICAL ESTIMATES

The above discussion makes no mention of correlations between links. There are however, trivial correlations due to charge conservation - adjacent links cannot transfer charges differing by more than one unit. Many other effects may be included, but we exhibit here a simple estimate of the correlations which we believe illustrates well our approach.

As previously, only the ordering of particles in rapidity is considered. Any additional mismatch due to other properties is neglected. We denote the probability for two neighbouring links to have charges q_1, q_2 by $Q(q_1, q_2)$. In the central region, this should be independent of rapidity and of the incoming particles. $Q(q_1, q_2)$ is also equal to the probability $Q(q_1)$ that the first link has charge q_1 , and that the intervening emitted particle has charge $p = q_1 - q_2$ [probability $P(p)$]. Thus,

$$\sum_{q_2} Q(q_1, q_2) = Q(q_1)$$

and

$$\sum_{q_1} Q(q_1, q_1 - p) = P(p) \quad .$$

(5)

In the central region, $P(+) = P(0) = P(-) = 1/3$. Furthermore, data ⁴⁾ suggest $Q(\pm 1) = 1/4$ and $Q(0) = \frac{1}{2}$. Eqs. (5) then determine $Q(q_1, q_2)$ completely :

$$Q(q_1, q_2) = \begin{pmatrix} \frac{1}{2} & \frac{1}{6} & 0 \\ \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \\ 0 & \frac{1}{6} & \frac{1}{2} \end{pmatrix} \quad (q = +, 0, -). \quad (6)$$

With such a two-link correlation one can generalize the estimate of the unitarity suppression due to exchange of Δq . Neglecting end effects, the simplest ansatz for the multi-link probability compatible with charge conservation but with no correlations beyond nearest neighbours, is

$$Q(q_1, \dots, q_n) = \tilde{Q}(q_1, q_2) \tilde{Q}(q_2, q_3) \dots \tilde{Q}(q_{n-1}, q_n) \quad (7)$$

where

$$\tilde{Q}(q_1, q_2) = Q(q_1, q_2) / \sqrt{Q(q_1) Q(q_2)} \quad .$$

The suppression factor will then be

$$R_{\Delta q}^{(n)} = \sum_{q_1 \dots q_n} \prod_{r=1}^n \left[\tilde{Q}(q_r, q_{r+1}) \cdot \tilde{Q}(q_r + \Delta q, q_{r+1} + \Delta q) \right]^{1/2}. \quad (8)$$

Defining the matrix

$$(S_{\Delta q})_{q_r, q_{r+1}} = \tilde{Q}^{1/2}(q_r, q_{r+1}) \tilde{Q}^{1/2}(q_r + \Delta q, q_{r+1} + \Delta q),$$

we get $R_{\Delta q}^{(n)} \approx (\lambda_{\Delta q})^n$, where $\lambda_{\Delta q}$ is the largest eigenvalue of $S_{\Delta q}$. For the example of Eq. (6) this leads to

$$\begin{aligned}\lambda_0 &= 1 \\ \lambda_1 &= \frac{\sqrt{2}+1}{3} \\ \lambda_2 &= 1/3 .\end{aligned}\tag{9}$$

Substituting in Eq. (4) this gives $\alpha_{\Delta q=1} = 1 - g^2(1-\lambda_1) = 0.41$ with ^{*}
 $g^2 = 3$.

This intercept for charge exchange is very realistic and shows that the simple approach described above is capable of explaining the different energy behaviour of charge exchange and elastic scattering. Many refinements can be introduced but most require experimental data which are straightforward to evaluate but not available at present.

One simple extension that should be considered is to include charge 2 transfers which occur experimentally for $\approx 10\%$ of all links in the central region ⁴⁾. An easy estimate can be made if we neglect any information on neutral produced particles. Then for produced charged particles $P(+) = P(-) = \frac{1}{2}$, and $Q(q_1, q_2)$ will have zeros in the leading diagonal. With $Q(\pm 2) = \eta/4$, charge and probability conservation determine $Q(q_1, q_2)$ completely. Proceeding as above, we find

$$\lambda_1 = \frac{1}{2} \sqrt{\frac{1-\eta}{2}} \left(1 + \sqrt{1 + 4 \sqrt{\frac{2\eta}{1-\eta}}} \right)\tag{10}$$

and

$$\alpha_{\Delta q=1} \leq 1 - \frac{2}{3} g^2 (1-\lambda_1) ,\tag{11}$$

since $\langle n \rangle$ has to be replaced by $\langle n_{ch} \rangle = 2/3 g^2 \log s$. With $\eta = 0$ this gives exactly the same result as above, while with $\eta/4 = 0.05$ a substantial enhancement occurs giving $\alpha_p \leq 0.87$.

^{*}) This value arises from a fit to NAL and ISR multiplicity data.

At first sight this result - which is essentially the same as found in Ref. 1) - seems to suggest that the bound for α_p is very sensitive to the amount of $q = 2$ transfer. This, however, need not be the case. There are at least two important effects which were not taken into account in the above calculation and which would tend to cancel substantially the enhancement of the bound due to $q = 2$. Firstly, links with $q = \pm 2$ will in general have much shorter lengths in rapidity than those with $|q| < 2$, and thus their overlap with the latter will be rather small. Secondly, $q = 2$ transfers occur preferentially in events with very high multiplicity³⁾. Charge exchange is, however, built up mainly by low-multiplicity states, due to the weighting factor $\lambda_{\Delta q}^n$.

These effects can easily be included in our formalism, provided we know the mean rapidity lengths of links carrying $q = 0, \pm 1$ and ± 2 , and if we neglect rapidity correlations. With reasonable values for the parameters, we found indeed that α_p was reduced.

It is a general advantage of our approach that straightforward modifications can take into account additional experimental data, when they become available. Very interesting pieces of information would be the dependence of the mean link length on the charge transferred, and also the link rapidity length distribution itself (in a naïve multiperipheral model it would be an exponential: $P(y) \propto e^{-y \cdot \text{const}}$). A further refinement would be to consider 3 link correlations which are related to correlations between neighbouring particles. Finally, in a careful analysis, only non-diffractive events should be included in the unitarity sum for charge exchange. Diffractive events contain one very long link with $q = 0$, which cannot match with a link carrying $q' = \Delta q$.

4. CONCLUSIONS

A knowledge of the experimental charge flow and charge correlation provides us with an upper bound for the two-body charge exchange scattering, via the unitarity equation. This bound is difficult to estimate numerically because of end effects in the chain and because of lack of suitable data (for $\pi^- p \rightarrow \pi^0 n$, for example, we would need data on $\pi^0 n \rightarrow \text{all}$, as well as $\pi^- p \rightarrow \text{all}$). Its energy dependence, however, can be estimated on the basis of our general knowledge of multi-body scattering, and it turns out to be fairly realistic. This indicates that the phase mismatch does not increase with

energy as rapidly as the charge mismatch, so that the latter dominates the energy dependence. It was pointed out what kind of further data is needed to improve our estimates.

A basic feature of our analysis is that quantum number exchange arises predominantly from the overlap of events with lower multiplicity than those contributing to σ_{tot} ^{*)}. The fact that charge exchange seems experimentally to be peripheral would then suggest that low-multiplicity events are more peripheral than high-multiplicity ones, in contradiction to what one finds in naïve multiperipheral calculations. The origin of this is presumably that at high multiplicities the particle density is too large, so that the multiperipheral model cannot be applied ^{**)}. One can even go a step further: since the states which contribute to charge exchange have low density, the multiperipheral model should hold for them. Thus their mean impact parameter should increase with energy like $\langle b \rangle \sim \langle n \rangle^{\frac{1}{2}} \sim (\log s)^{\frac{1}{2}}$. This might lead to a rather strong shrinkage of charge exchange, but a much weaker one for elastic scattering. An additional effect is that phase incoherence should become more important for charge exchange, if one goes away from the forward direction ⁷⁾. In conclusion, charge exchange amplitudes are built up by lower multiplicity intermediate states than diffractive amplitudes. This may provide an understanding of their different angular dependence and we believe it would be worth while examining this problem further.

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*) This is a common property of all the analyses of Refs. 1), 5)-7).

**) For a discussion of this and related problems, see Ref. 8).

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FIGURE CAPTIONS

1. Illustration of the unitarity equation. Intermediate state particles are ordered in rapidity.
2. Difference of positive (p, π^+, K^+) and negative ($\bar{p}, \pi^-, K^-$) particle spectra ^{2),3)} at $p_{\perp} = 0.4$ GeV/c. The figure shows that the net charge tends to remain in the fragmentation region. The limiting distribution indicated by the shaded area is obtained by extrapolation. Integrating these distributions over $p_{\perp}$ and from $y_{\text{lab}} = 0$ to y gives the average charge flow $\bar{q}$ at that energy and rapidity y .
3. Charge transfer distribution at 205 GeV/c at $y_{\text{c.m.}} = 0$ for pp interactions ⁴⁾. Assuming that the length of the rapidity gaps is independent of charge transfer q , this quantity is equal to the link probability distribution $Q(q)$ defined in the text.

FIG.1

FIG. 2

FIG. 3