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SU(6)_w AND DECAYS OF BARYON RESONANCES

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ABSTRACT

The decay rates of baryon resonances assigned to the $\{70, 1^-\}$ and $\{56, 2^+\}$ representations of $SU(6)_w \otimes O(3)$ are subjected to a detailed comparison with an $SU(6)_w$ model motivated by the recent work of Melosh. We conclude that $SU(6)_w$ is remarkably successful as an approximate symmetry for decay vertices.

1. INTRODUCTION

We have studied the decays of baryon resonances classified in the $\{70, 1^-\}$ and $\{56, 2^+\}$ representations of $SU(6)_w \otimes O(3)$ into the ground state $\{56, 0^+\}$ baryons and the pseudoscalar mesons. The recent work of Melosh⁽¹⁾ has provided a more theoretical foundation for the ℓ -broken form of $SU(6)_w$ used by Faiman and Plane⁽²⁾ in their analysis of the decay rates of the $\{70, 1^-\}$ resonances. Via PCAC and a simple assumption concerning the Melosh transformation between current and constituent quarks, an identical $SU(6)$ structure is obtained but different phase space and angular momentum barrier factors are suggested^(3,4).

Fits have been performed to 55 decay rates for the $\{70, 1^-\}$ and 26 for the $\{56, 2^+\}$, using two forms of barrier factor suggested by the use of PCAC,^(3,4) and a purely phenomenological $p^{2\ell}$ angular momentum barrier factor. Despite the obviously large mass splittings within the $SU(6)$ multiplets, very good fits are obtained, indicating that $SU(6)_w$ is a useful approximate symmetry to describe the decays. The phenomenological $p^{2\ell}$ barrier is preferred over the barriers based solely on a PCAC motivation, but the main features of all three types of fits are the same. We discuss the fits obtained using the phenomenological barrier factor and make predictions for the partial widths of the missing states in these multiplets. The $SU(6)_w$ formalism based on the Melosh transformation may be extended to include photoproduction of the N^* resonances^(3,5). We examine the consistency of our mixing angles and $N\pi$ amplitudes with the experimental photoproduction amplitudes. We emphasize that the $SU(6)_w$ scheme used here has the most general $SU(6)$ structure of any $SU(6)$ model^(*), (whether based on the Melosh transformation, dynamical quark models, quark pair creation, etc.), with the minimal dynamical assumptions.

Section 2 of the paper contains a brief summary of our theoretical approach to the $SU(6)_w$ model, section 3 details the fitting procedure, and section 4 reviews the data set that we have used. In section 5 the results of our fits are critically examined, and section 6 discusses their

(*) In this connection, see the recent paper of Lipkin, ref. 6. We exclude possible models with exotic $SU(6)$ representations.

implications and our predictions for the missing states. Our comparison with protoproduction amplitudes is described in sect. 7. In sect. 8 we compare the relative success of $SU(6)_w$ and $SU(3)$ and our conclusions are summarized in the final section.

2. $SU(6)_w$ AND PION DECAYS

2.1. Preamble

The experimental hadron spectrum and also resonance decay couplings possess an approximate $SU(3)$ symmetry. The situation is clouded by $SU(3)$ symmetry breaking effects - in the form of mass splittings, threshold factors and so on - but nevertheless $SU(3)$ is a useful approximate symmetry⁽⁷⁾. Since the experimental spectrum shows evidence for a higher symmetry than $SU(3)$, namely some form of $SU(6) \otimes O(3)$ ⁽⁸⁾, it is not unreasonable to search for a larger symmetry for resonance decays. At this point, one is faced with a decision as to how one should regard $SU(6)$ as an approximate symmetry. Treating $SU(6)$ as an approximate symmetry of the Hamiltonian in the manner of $SU(3)$ or isospin, leads to well-known difficulties⁽⁹⁾. One may distinguish at least two popular approaches to $SU(6)$ which avoid these problems^(*).

- (1) $SU(6)$ symmetry arises from the non-relativistic motion of spin 1/2 quarks⁽⁺⁾.
- (2) $SU(6)_w$ is a relativistic spin symmetry of single particle states in motion⁽¹⁵⁾.

The first approach lends itself more easily to the construction of "realistic" quark models in which one may construct the various operators of the theory - mass operator, electromagnetic current, pion emission operator and so on. From their postulated $SU(6)$ properties, together with $SU(6)$ wavefunctions for the resonant states, the algebraic consequences of this $SU(6)$ model for transition matrix elements may be derived. These

(*) There are many other approaches, ranging from phenomenological $SU(6)$ models (10,11), to relativistic bound state models⁽¹²⁾. The reviews of ref. 8 contain more complete references to the literature.

(+) Into this category would come non-relativistic harmonic oscillator quark models⁽¹³⁾ together with those that attempt to incorporate some relativistic corrections⁽¹⁴⁾.

models may make very powerful predictions - for example, the same quark parameters may appear both in pionic decays and photon transitions. Furthermore, the parameters for transition matrix elements taken between different multiplets may be simply related, e.g. the $\{70, 1^-\}$ pion decay parameters are related to those of the $\{56, 2^+\}$ decays.

In the second approach one derives the consequences of a relativistic $SU(6)_w$ symmetry of one particle states. Decay processes which involve two particle states must be approximated by single particle matrix elements of some current operator. This approach has received great impetus from the recent work of Melosh⁽¹⁾ who suggested a particularly simple structure for the transformation properties of current operators under the $SU(6)_w$ algebra that classifies the states.

These two approaches do not a priori lead to an identical $SU(6)$ structure, because of the W-S flip complication in the classification of mesons in the two schemes. Nevertheless, with the usual assumptions concerning the $SU(6)$ or $SU(6)_w$ transformation properties of the decay transition operator both schemes do in fact result in the same algebraic structure^(6,16). The results of the $SU(6)_w$ approach via the Melosh transformation, where one makes no attempt to construct a realistic bound state quark model, may therefore be regarded as minimal $SU(6)$ predictions common to both approaches.

2.2. $SU(6)_w$ Model

The $SU(6)_w$ model that we confront with the data has been described in detail in the literature^(3,4,16). Here we briefly review the essential elements of the theory.

From consideration of current algebra at infinite momentum, an $SU(6)_w$ of "good" charges emerges as a candidate for classifying one particle states. In technical language this algebra is generated by null plane charges $\hat{F}(\Lambda)$ which have a very simple form in the quark model. According to the current algebra postulate, physical currents transform

simply under this algebra - namely as members of a 35_w representation. This algebra is the so-called "current quark" $SU(6)_w$.

Resonant states on the other hand, also appear to be classified in $SU(6)_w$ multiplets. The generators of this "constituent quark" algebra we denote by $\hat{W}(\Lambda)$. The natural attempt to identify the $\hat{F}(\Lambda)$'s with the $\hat{W}(\Lambda)$'s that classify the states leads to serious contradictions⁽⁴⁾. In particular, the existence of the lowest 35 multiplet of mesons is incompatible with such an identification⁽¹⁷⁾! In view of these difficulties, Gell Mann suggested⁽¹⁸⁾ that some unitary transformation V links the two sets of generators

$$\hat{W}(\Lambda) = V \hat{F}(\Lambda) V^{-1}$$

Recently Melosh⁽¹⁾ attempted to motivate the form of V by a study of the free quark model and his suggestion is often called the Melosh transformation.

To apply these ideas to pionic decays, $A \rightarrow B + \pi$, since one has only a symmetry classification of single particle states, one must resort to using the PCAC hypothesis. With this assumption the decay matrix element is related to the matrix element of the axial charge

$$\langle B\pi | A \rangle \sim (M_B^2 - M_A^2) \langle B | Q_\pi^5 | A \rangle$$

Melosh's transformation must now be used to find how Q_π^5 transforms under $SU(6)_w$ constituents. His suggestion was that Q^5 remains in a 35_w but transforms as the sum of $\{35_w, L_z=0\}$ and $\{35_w, L_z=\pm 1\}$ representations. Although there is some confusion as to the theoretical status of Melosh's proposal⁽¹⁹⁾, it is clearly the simplest phenomenological expression of the fact that V cannot be unity.

Application of this model to decays of baryon resonances into the ground state multiplet, $\{56, 0^+\}$ is now straightforward. For baryon states there is no W-S flip, and the $SU(6)_w \otimes O(3)$ wavefunctions for states with total angular momentum J and helicity λ are constructed according to the usual LS coupling rules. The expected states in the $\{70, 1^-\}$ and $\{56, 2^+\}$ multiplets are shown in fig. 1. The helicity

amplitude g_λ for the process

$$A(J^A_\lambda: LS^A) \rightarrow B(\{56, 0^+\}: S^B_\lambda) + \pi$$

has the following structure

$$g_\lambda = (M_A^2 - M_B^2) \tilde{g}_\lambda$$

where

$$\tilde{g}_\lambda = \sum_{L_z} (L L_z S^A_{\lambda-L_z} | J^A_\lambda) (S^B_{\lambda} 1 - L_z | S^A_{\lambda} - L_z)$$

LS coupling W-spin

$$\left\{ \begin{matrix} 56 & 35 \\ (N^B, 2S^B+1) & (8, 3) \end{matrix} \right| \left\{ \begin{matrix} M^A \\ (N^A, 2S^A+1) \end{matrix} \right\}_\alpha \left\{ \begin{matrix} N^B & 8 \\ B & \pi \end{matrix} \right| \left\{ \begin{matrix} N^A \\ A \end{matrix} \right\}_\alpha \right\} \cdot a^{L_z}$$

SU(6) factor SU(3) factor

M^A, N^A and S^A denote the SU(6), SU(3) and SU(2) representations, respectively, of state A: N^B and S^B the SU(3) and SU(2) representations of state B. The SU(6) (SU(3)) factor is a Clebsch Gordan coefficient of SU(6) ⁽²⁰⁾ (SU(3)) and we have used the Baryon first convention in all these couplings. (The index α denotes a sum over F and D type couplings in the case of octets). The unpolarised decay rate is given by

$$\Gamma = \frac{p}{8\pi M_A^2} \frac{1}{(2J^A+1)} \sum_\lambda |g_\lambda|^2 \equiv \frac{p}{8\pi M_A^2} \cdot \bar{\Gamma}$$

where p is the CM 3- momentum of the decay and M_A the mass of state A.

It is convenient to express the width in terms of partial wave amplitudes g_ℓ such that

$$\bar{\Gamma} = \frac{1}{2J^A+1} \sum_\lambda |g_\lambda|^2 = \frac{1}{2J^A+1} \sum_\ell |g_\ell|^2$$

$$\equiv \sum_\ell \bar{\Gamma}_\ell$$

For the $\{70, 1^-\}$ and $\{56, 2^+\}$ transitions we may define linear combinations of the two independent reduced matrix elements (a^{L_z} 's) for each transition - S and D, and P and F respectively - to obtain the results displayed in Tables I and II. S, D, P and F are related to the amplitudes for the decay of the resonances in the corresponding partial waves.

This is the basic $SU(6)_w$ model that we use in this paper. However, before it can be confronted with the data some decisions must be made as to the way in which some obvious (mainly $SU(3)$) symmetry breaking effects are to be handled. Our prescriptions are described in the next section.

3. SYMMETRY BREAKING, EXTENSION TO K and η DECAYS AND DETAILED FITTING PROCEDURE

We wish to extend our $SU(6)_w$ model for π decays to include both K and η decays. Here we encounter some arbitrariness in how best to do this. The situation is analagous to the $SU(3)$ case where symmetry breaking is manifested by mixing (eg. singlet-octet) and by uncertainty in the choice of angular momentum barriers which are needed to account for the large mass differences (eg. π and η) of the decay product. Our choices are detailed below but it should be understood that they are not the only possibilities. The range of possibilities reflects the ambiguity in using "approximate symmetries".

3.1. Barrier Factors for π Decays

We consider three possibilities

- i) No attempt is made to account for the approximate nature of PCAC, but some attempt is made to incorporate symmetry breaking effects by using the physical particle masses in the PCAC (ΔM^2) factor and in the phase space 3-momentum. This is the approach of Gilman, Kugler and Meshkov⁽³⁾ (GKM) in which the same barrier factor is used for decays with different partial waves.
- ii) The PCAC result is regarded as exact in a world with massless pions. Using $M_\pi=0$ we obtain $\Gamma_\ell^{(0)}$

$$\Gamma_\ell^{(0)} = \frac{p^{(0)}}{8\pi M_A^2} \cdot \Gamma_\ell^-$$

To incorporate some correction for $M_\pi \neq 0$, a possible prescription is⁽⁴⁾

$$\Gamma_\ell = \left(\frac{p}{p^{(0)}} \right)^{2\ell+1} \Gamma_\ell^{(0)}$$

In practice this "modified GKM" approach gives very similar results to (i), although it is somewhat better for resonance decays near threshold.

- iii) One may follow the procedure used in the SU(3) case⁽⁷⁾ and extract an angular momentum barrier factor, $B_\ell(p,r)$, for each partial wave according to the prescription^(*) $^\ell$

$$\Gamma_\ell = \frac{M_N}{M_A} \cdot B_\ell(p,r) \cdot \bar{\Gamma}_\ell$$

(Where M_N is the nucleon mass). We have investigated the dependence of the fit on the radius of interaction r in the barrier factor $B_\ell(p,r)$. We reached similar conclusions to those of Faiman and Plane that the best fits were obtained for small r . In practice the "zero radius form" $p^{2\ell}$ was found to be as good as any other and this form was used in the final fitting.

For decays into the broad $\Delta(1230)$ and $\Sigma(1385)$ an effective barrier was obtained by integrating over the width of the decay resonance.

3.2. Mixing

States with the same isospin and hypercharge are in general expected to mix - both at the SU(3) level and at the SU(6) level. The possibilities are depicted in Fig. 1, (adapted from ref. 2). Our conventions for the $\{70, 1^-\}$ are as follows

- i) 2-way mixing between N^* 's

We define the mixing matrix as follows

$$\begin{pmatrix} N_1 \\ N_2 \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} 4_8 \\ 2_8 \end{pmatrix}$$

where the order of the physical states N_1 and N_2 is given in table 5. There are two mixing angles; θ_d for the D13 states and θ_s for the S11 states.

(*) One may or may not keep the (Δm^2) factor from PCAC in $\bar{\Gamma}_\ell$: in practice we have chosen not to keep it. Its inclusion leads to worse fits.

ii) 3-way mixing of the Y^* 's

For the Λ^* 's and Σ^* 's 3-way mixing is possible. We define our matrices as follows

$$\begin{pmatrix} \Lambda_1 \\ \Lambda_2 \\ \Lambda_3 \end{pmatrix} = \begin{pmatrix} M_{11} & M_{12} & M_{13} \\ M_{21} & M_{22} & M_{23} \\ M_{31} & M_{32} & M_{33} \end{pmatrix} \begin{pmatrix} \Lambda(4_8) \\ \Lambda(2_8) \\ \Lambda(2_1) \end{pmatrix}$$

and

$$\begin{pmatrix} \Sigma_1 \\ \Sigma_2 \\ \Sigma_3 \end{pmatrix} = \begin{pmatrix} M_{11} & M_{12} & M_{13} \\ M_{21} & M_{22} & M_{23} \\ M_{31} & M_{32} & M_{33} \end{pmatrix} \begin{pmatrix} \Sigma(4_8) \\ \Sigma(2_8) \\ \Sigma(2_{10}) \end{pmatrix}$$

The order of the physical states is given in table V. The mixing matrices are characterized by the usual three Euler angles⁽²¹⁾ (α, β, γ) and the elements M_{ij} are just a convenient shorthand. There are therefore 12 mixing angles for the Y^* 's

3.3. F/D Ratios and SU(3) Conversions

The $SU(6)_w$ model (as we have motivated it) involves the use of PCAC to treat π -decays. There is therefore some choice as to how the model may best be extended to include K and η decays. One alternative is to use K and η PCAC - although these are not expected to be good to the same accuracy as pion PCAC. Instead, we prefer to take the $SU(6)_w$ predictions for the π decays of the N^* 's and Y^* 's and use the "standard" $SU(3)$ ⁽⁷⁾ procedure to obtain predictions for the K and η decay modes. For octet states, the $SU(6)_w$ model predicts the F/D ratios of the pure quark spin $SU(6)$ members

a)	$\{70, 1^-\}$	decays to $\{56, 0^+\}$
	2_8 :	F/D = 5/3
	4_8 :	F/D = - 1/3
b)	$56, 2^+$	decays to $\{56, 0^+\}$
	2_8 :	F/D = 2/3

If we assume the SU(6) values for these ratios, the SU(3) conversion would then be straightforward were it not for the fact that the physical states are mixtures of the pure SU(6)_w states and there are various choices for the barrier factors in the π decay. We adopt the following procedure. With a given π barrier factor (GKM etc.), and mixing matrix, one predicts $\sqrt{\Gamma}$ for each π decay mode. If we extract from these $\sqrt{\Gamma}$'s, the π SU(3) barrier factor (the same as the π barrier only in case (iii)) and undo the mixing matrix, we obtain reduced SU(3) couplings for the pure SU(6) quark spin states. We now use the SU(6)_w F/D ratios for these reduced couplings to convert to the reduced coupling of the different modes (K, η). Then we put back the mixing matrix and multiply by the appropriate (K, η) SU(3) barrier factors to obtain predictions of $\sqrt{\Gamma}$ for these modes. (In the case of the SU(3) barrier factor for the pion decays (case (iii)), the model is essentially the same as that of Faiman and Plane).

3.4. Treatment of Amplitudes and Total Widths

We have adopted the method of Plane et al⁽⁷⁾ in fitting the amplitudes at resonance ($\sqrt{xx'}$) rather than the partial widths, since, for a given resonance, the latter are all dependent on the measurement of the elasticity whereas the $\sqrt{xx'}$ are independent. However, rather than fit the quantity $\Gamma.(\sqrt{xx'})$ for each measured amplitude, as was done by Faiman and Plane, we have taken the total width of each resonance as an extra parameter in the fits and added a contribution to the χ^2 for the difference between the measured and fitted widths. This procedure has the advantage that the large uncertainty on the width enters only once for each resonance instead of in each amplitude. This accounts for some of the much higher χ^2 that we obtain compared with that obtained in ref. 2. The difference between the total width and the sum of the partial widths predicted corresponds to decays into states other than the pseudoscalar mesons, which are outside the scope of our model.

4. Data

It is well known that data on resonance masses, widths and amplitudes from partial wave analyses contain large systematic errors depending on the

detailed assumptions of each analysis. This makes selection of data for an analysis of resonant amplitudes difficult and dangerous.

We have included in this analysis only those states whose existence we consider to be reasonably certain. This has excluded a number of reported resonances that have been seen in only one analysis or for which there are disagreements between the various analyses. Our results include predictions for the parameters of missing resonances. These can be compared with the possible candidates and may help to reinforce or discredit them. Fig. 1 shows the states required to fill the $\{70, 1^-\}$ and $\{56, 2^+\}$ multiplets, together with the mixing possibilities for states of the same isospin and hypercharge. Tables III, IV show the experimental states assigned to these multiplets.

Where several analyses have measured the same quantity we have made a judgement, essentially by eye, of the mean value and have taken the spread of the results as an indication of the error. Where only one analysis is available, in particular the case for most of the $\Delta\pi$ and $\Sigma(1385)\pi$ results, we are forced to use the quoted errors provided they are not ridiculously small. We consider this to be a better procedure than ignoring altogether the estimated error of the analysis and arbitrarily assigning a fixed error for all measurements as has frequently been done in the past⁽⁷⁾. At least the relative magnitude of the errors within a given analysis probably reflects the amount of confidence to be placed in the results. Despite the arbitrariness of this procedure, we believe it is the best available. The quantity of the data is now so large and the number of parameters sufficiently small that the value and error of a single datum point do not significantly effect the fit (apart from the more poorly defined three-way mixing matrices such as that of the $1/2^- \Sigma$'s). Thus points with large χ^2 may either be due to a breakdown of the theory or to a bad determination of their value or error.

Table III shows the data used in the fit for the $\{70, 1^-\}$ comprising 54 measurements of $\sqrt{xx'}$ and one partial width (that of $\Lambda(1405)$ into $\Sigma\pi$)^(*). Table IV gives the data for the $\{56, 2^+\}$, 27 measurements of $\sqrt{xx'}$ ⁽²²⁻²⁶⁾. Each inelastic

(*) The sign of the virtual process $\bar{K}N \rightarrow \Lambda(1405) \rightarrow \Sigma\pi$ has been measured to be + ve⁽²⁷⁾ and was used as a consistency check in our analysis.

channel has an arbitrary overall + or - sign which has been chosen to give the best overall fit. Of course, within a given channel the signs of different resonant amplitudes are generally related by the analyses and these relative signs must be preserved. In some cases isolated analyses have not measured signs relative to other resonances. This is particularly true of the threshold analyses of $N\eta$ and $\Lambda\eta$ etc. In these cases the sign is placed in brackets to indicate that it is deduced from the fit and not measured.

It is of interest to note the major changes in the data since the somewhat similar analysis of the $\{70, 1^-\}$ by Faiman and Plane in 1972⁽²⁾.

- 1) The Slac-Berkeley analysis⁽²³⁾ of $\pi N \rightarrow \pi\pi N$ has provided the $\pi\Delta(1230)$ amplitudes for most of the N^* and Δ resonances.
- 2) The $\Sigma(1385)\pi$ analysis of the CHS collaboration⁽²⁴⁾ has been revised giving rather different results from the preliminary analysis used by Faiman and Plane.
- 3) The $3/2^- N^*(1710)$ has been established in both $N\pi$ ⁽²⁵⁾ and $\Delta\pi$ ⁽²³⁾ channels thus completing the nucleon resonances required for this multiplet.
- 4) Two new $3/2^- \Sigma^*$'s have been added, the $\Sigma(1580)$ ⁽²⁶⁾ and $\Sigma(1940)$.

In the $\{70, 1^-\}$ the only $S=0$ and -1 resonances that are missing or whose parameters are in doubt are a $3/2^-$ and $1/2^- \Lambda^*$ and two $1/2^- \Sigma^*$: only 4 out of the 21 states required for this multiplet.

In the $\{56, 2^+\}$ a $5/2^+ \Sigma^*$, two $3/2^+ \Sigma^*$ and a $1/2^+ \Sigma^*$ are missing: 4 out of the required 14 states.

5. RESULTS

a) $\{56, 2^+\}$

Figure 1 shows that the $\{56, 2^+\}$ is much simpler than the $\{70, 1^-\}$ because only 2 mixing angles are involved. In fact, because both of the $3/2^+$ and one of the $5/2^+ \Sigma^*$'s are missing, the fits were made with no mixing. There are thus only 2 free parameters to be varied, together with the widths of the resonances, and 27 $\sqrt{xx'}$ are measured.

Fits were tried with the three types of barrier factor. The "GKM" factor gave a χ^2 of 55, the "modified GKM" factor 52 and the $p^{2\ell}$ factor 34, all for 25 degrees of freedom. The more theoretically motivated barrier factors are quite strongly disfavoured relative to the somewhat arbitrary non-relativistic $p^{2\ell}$ barrier, which gives a satisfactory χ^2/NDF despite the uncertainties in the data and its errors. Below we will discuss only the best fit, the overall conclusions from the other fits are all similar. The values of the P and F coupling constants are given in Table V and the fitted values of the widths and $\sqrt{x x'}$ in Table IV. The positive sign of P relative to F is determined by the p wave decays of the $5/2^+ \Lambda(1680)$ into $\Delta\pi$ and the $5/2^+ \Lambda(1820)$ into $\Sigma(1385)\pi$.

The worst features of the analysis are the rather high predicted value of the $\Sigma\pi$ amplitude of the $5/2^+ \Lambda(1820)$ and the very small $\bar{K}N$ amplitude for the $5/2^+ \Sigma(1915)$. This latter feature has been noticed several times previously in SU(3) analyses^(7, 28).

The fact that the f wave $\pi\Delta$ decay of the $5/2^+ \Lambda(1860)$ is measured to be larger than the p wave $\pi\Delta$ decay which is not observed, has been interpreted as evidence for mixing with a $\{70, 2^+\}$ multiplet⁽²⁹⁾. The predicted rates for the two decays are approximately equal but smaller than the measured value so that, within the errors, the data agree with the predictions and the need for this mixing is not compelling.

b) $\{70, 1^-\}$

As shown in Fig. 1 most of the states in this multiplet can in principle be mixed, the $1/2^-$ and $3/2^-$ Σ 's and Λ 's having 3 way mixing. Thus the 55 data points have to determine 16 parameters.

Fits were obtained with the three barrier factors and again the simple $p^{2\ell}$ barrier was favoured: the decrease in χ^2 was 90 relative to the "modified GKM" barrier and 113 relative to the "GKM" barrier. As for the $\{56, 2^+\}$ we will only discuss results obtained from our best fit; the features of the others are similar. The final fit

had a χ^2 of 172 for 39 degrees of freedom, considerably worse than the fit to the $\{56, 2^{+ (+)}\}$. However, considering the uncertainties in the data and the known breaking of SU(6), this is not an unreasonable figure to expect.

The values of the fitted quantities are given in Table V. As well as the mixing angles, the mixing matrices which show directly the composition of the physical states in terms of the basic SU(6) states are given. In this multiplet the sign of the D wave parameter is opposite to the S wave.

i) The unmixed states

The $5/2^-$ states and the two Δ 's are unmixed and are perhaps the least ambiguous test of the basic SU(6)_w model. They are in fact the worst fitted part of the multiplet, a χ^2 of 119 (out of 172 total) arising from just these 5 states. Whether this indicates that for these states the symmetry is anomalously badly broken or that mixing, while allowing for some symmetry breaking, also covers the inadequacy of the model elsewhere is an open question. Its resolution will have to wait on completion of the mixed multiplets or more precise data.

Clear evidence for some breaking of the SU(6)_w symmetry is given by the $5/2^- \Lambda(1830)$: as an unmixed member of a 4^4_8 SU(6) state, it is predicted to be completely decoupled from $\bar{K}N$ and thus should not appear in formation experiments. It is, however, seen quite strongly in the $\Sigma\pi$ channel⁽²²⁾ and the χ^2 of 36 for the four measurements compared with the zero predictions probably gives the scale at which the symmetry breaking will be reflected in the χ^2 . Despite this large χ^2 , the breaking is small, the elasticity being less than 10%. The measured partial widths into $\Sigma\pi$ and $\Sigma(1385)\pi$ are compatible with those predicted, though as has been previously noted the measured sign of the $\Sigma(1385)\pi$ amplitude disagrees with theoretical expectations⁽³⁰⁾ even at the level of SU(3) within the $5/2^-$ octet.

(+) The errors quoted on the $\Sigma(1385)\pi$ resonant amplitudes appear to us to be unrealistically small. We have made fits with these errors doubled. The χ^2 was reduced by ~ 40 although all parameters remained the same within their errors.

The large $\Sigma\pi$ prediction for the $5/2^- \Sigma(1770)$ was also found in the analysis of Faiman and Plane and is difficult to reconcile with the data. The large χ^2 for the $\Sigma(1385)\pi$ mode may just reflect the rather optimistically small error given by the authors of ref. 24.

Both the $5/2^- N^*(1670)$ and $1/2^- \Delta(1610)$ suffer from small predicted partial widths into $N\pi$ and $\Delta\pi$ which when taken with the measured total widths give low predictions for $\sqrt{xx'}$. These could be improved by reducing drastically the total widths, e.g. a width of 50 MeV gives a reasonable fit for the $1/2^- \Delta$ compared with the measured width of 150 MeV. However, reducing the width of the $5/2^- N^*$ would produce an embarrassingly large $\sqrt{xx'}$ for $N\eta$ which is not observed. Moreover, the width of this leading resonance must be considered well measured.

ii) The mixed nucleon resonances

The $3/2^- N^*$'s are not strongly mixed, θ_D being of the order of 10° and both give good fits. In contrast the $1/2^- N^*$'s have a mixing angle of approximately 30° . That the mixing is necessary can be seen from the fact that the $N^*(1665)$, predominantly assigned to the 4_8 , has an appreciable ΛK decay mode which is forbidden for a pure 4_8 . The signs of the $N\eta$ and ΛK amplitudes which are not determined in the experiments have been chosen to give the best fit. The mixing angle is not significantly affected by this choice, the same value but higher χ^2 being obtained for other combinations. The magnitudes of the measured amplitudes for these two decay modes, together with that for the $1/2^-(1670)$ to $\Lambda\eta$, which are all very close to threshold, are high compared to the theoretical predictions possibly indicating that our barrier function is inadequate for very low centre of mass momenta.

iii) Mixed Y^* 's

The mixing matrices for our best fit are given in Table V. The fits to the data are generally good.

The matrices are very approximately diagonal indicating that the resonances are predominantly pure SU(6) states with only small mixing of the other states.

The $3/2^-$ Λ 's fit well with mixing only between the predominantly $^2_8 \Lambda(1690)$ and predominantly $^2_1 \Lambda(1520)$. The unobserved 4_8 is practically unmixed. This is different from the result of Faiman and Plane who invoked strong mixing between all states to explain the then measured amplitudes for $\Lambda(1690)$ and $\Lambda(1520)$ into $\Sigma(1385)\pi$. The revised values used here no longer require this mixing.

The recent observations of the $\Sigma(1580)$ and $\Sigma(1940)$ have completed the $3/2^-$ Σ 's. The three states fit well, the only embarrassment being the rather high predicted elasticity of the $\Sigma(1940)$ which has not been observed in the $N\bar{K}$ channel. Again the mixing between the 4_8 and 2_8 is small in agreement with the $3/2^-$ Λ 's and N^* 's.

The two $1/2^-$ Λ 's also fit well. The mixing is slightly larger, consistent with the greater mixing observed in the $1/2^-$ N^* 's. There has been some discussion as to whether the $\Lambda(1405)$ should be included in quark model fits or whether it is a dynamic effect⁽³¹⁾. The predicted partial width into $\Sigma\pi$ is very close to the measured value and similar fits are obtained with or without the resonance. Thus if the $\Lambda(1405)$ is not the quark model resonance there presumably must be another state with very similar properties close by.

The $1/2^-$ Σ 's are the worst determined part of the multiplet. There is strong evidence only for the $\Sigma(1740)$ and, even for this, there are large disagreements as to the resonant amplitudes. We have taken the large elasticity (0.8) of LMMO⁽³²⁾ and Kim⁽³³⁾ with a large error, but the fits cannot reproduce this measurement. Although the row of the mixing matrix corresponding to the $\Sigma(1740)$ is well determined, using only the measurements for this resonance a number of different solutions for the mixing angles can be found which give

completely different predictions for the two unobserved states. We thus only quote the $\Sigma(1740)$ row of the matrix in Table V.

6. DISCUSSION OF THE RESULTS

It is interesting that the magnitudes of the S and D coupling constants in the $\{70, 1^-\}$ are remarkably close to the P and F constants in the $\{56, 2^+\}$. This suggests that the two multiplets may be intimately connected in some way, outside the scope of our purely algebraic theory.

While part of the much worse χ^2/NDF of the [70] with respect to the [56] clearly comes from the breaking of the $\text{SU}(6)_w$ symmetry, some of it may be due to lack of adequate barrier factors which are much more important for the lower lying [70]. This uncertainty in barrier factors reflects our inability to treat even $\text{SU}(3)$ symmetry breaking in a quantitative manner.

The very good fit to the $\{56, 2^+\}$ without any mixing is surprising in view of the quark model prediction of the presence of a $\{70, 2^+\}$ more or less degenerate in mass⁽³⁴⁾. This contains many states which would be expected to mix with those observed here. Thus our result casts doubt on the existence of this multiplet. Two possible candidates for this multiplet, the $F_{17}N(2050)$ and the $F_{07}\Lambda(2100)$, can plausibly be assigned to a $\{56, 4^+\}$ since there is evidence at relatively low masses, for an $H_{19}N(2250)$ and a possible $H_{3,11}\Delta(2400)$ which cannot belong to the $\{70, 2^+\}$.

Comparison of our mixing matrices for the $3/2^-$ and $1/2^-$ Λ 's with those of Faiman and Plane show broad agreement with their solution 2. Solution 1 which predicts a positive sign for the amplitude $N\bar{K} \rightarrow \Lambda(1670) \rightarrow \Lambda\eta$ is disfavoured by a χ^2 difference of approximately 40 and is thus ruled out by our fit. The detailed differences are probably explained by the different $\Sigma(1385)\pi$ data used here, as discussed in the previous section.

A comparison with the mixing matrices found by Horgan⁽³⁴⁾ in a fit to the mass spectrum using a symmetric quark model is less satisfactory. His Λ and Σ states are considerably more mixed than ours and it is difficult

to find any significant points of agreement. Also while his result of no mixing for the $3/2^- N^*$ is consistent with ours his similar result for the $1/2^- N^*$ is at variance with the observed $N(1700)$ decay to ΛK which requires some admixture of 2_8 into the 4_8 .

As our mixing matrices indicate that the physical states can be considered to be predominately pure basic SU(6) states we can construct Table VI which shows the assignment of the various resonances to the SU(6) multiplets. Of course the residual mixing will shift the resonances from the basic SU(6) state masses but hopefully the shift should not be too great. The table gives the somewhat surprising result that the highest mass states are the $3/2^-$ members of the 4_8 whereas the $3/2^- \Sigma$ belonging to the $^2_{10}$ is lighter than its Λ partner.

In fact since two $3/2^- \Sigma$'s have masses below 1700 MeV, the empirical rule that the strange member of a multiplet should be 100-200 MeV higher in mass than the nucleon member must be broken whatever assignment to multiplets is made. This feature is reflected in the fit of Horgan where the lowest Σ is even below the $N^*(1510)$ in mass.

Provided that we have an approximate estimate of their masses we may make predictions for the partial widths of the unknown resonances as well as predicting the unmeasured amplitudes for the known resonances. However, without a knowledge of their total widths, (which must of course be greater than the sum of the partial widths given here by an unknown factor due to their decay into other modes such as Λp , $\Lambda(1520)\pi$ etc.) we cannot predict the resonant amplitudes. We have taken as a guide the masses to be near those given by Horgan for the $\{70, 1^-\}$ and in the vicinity of possible resonances for the $\{56, 2^+\}$. The predictions are given in Table VII.

In the [70] the $^4_8 3/2^- \Lambda$ is essentially unmixed and thus is decoupled from the $N\bar{K}$ channel as is the $5/2^- \Lambda(1830)$. It is therefore not surprising that it has not been seen. It does however have a very strong coupling to $\Sigma(1385)\pi$ and its presence could explain the problem with the sign of the $\Lambda(1830)$ amplitude into $\Sigma(1385)\pi$. The $1/2^- \Lambda(1850)$ is predicted to be

rather broad, 235 MeV being contributed by the channels considered here and thus may easily have been overlooked as background. It has strong $\Sigma\pi$ and ΞK modes and should be most obvious in these channels. Nothing definite may be said about the $1/2^- \Sigma$'s because the several solutions found for the mixing angles give different predictions for the missing states.

In the [56] the $5/2^+ \Sigma$ probably has a mass above the range of present partial wave analyses but should be visible in several channels when the range is extended. The $3/2^+ \Sigma$ predictions are given assuming no mixing (which of course may be a bad assumption). However, both they and the $1/2^+ \Sigma$ are predicted to be rather narrow.

7. PHOTOPRODUCTION OF THE $\{70, 1^-\}$

Having determined the parameters (S, D and the mixing angles) for the hadronic decays of the members of the $\{70, 1^-\}$ we now see if these results are consistent with the data on photoproduction of the $S = 0$ resonances of this multiplet^(35,36).

In order to do this we have to introduce a prescription for calculating the electromagnetic transitions involved. We again use a model motivated by Melosh's transformation which has been developed by a number of authors^(37,38). In particular we use the results of Ref. (37). If we then combine these results with helicity decay amplitudes of Table 8 we obtain the following expressions for the A and B amplitudes of Walker⁽³⁵⁾.

$$\left. \begin{array}{l} A_{n+} \\ A_{(n+1)-} \end{array} \right\} = -\sqrt{2} \alpha F_- g_{\frac{1}{2}}^N$$

$$\left. \begin{array}{l} B_{n+} \\ B_{(n+1)-} \end{array} \right\} = 2\sqrt{\frac{2}{n(n+2)}} \times (-1)^{J-3/2} F_+ g_{\frac{1}{2}}^N$$

where $n = j - \frac{1}{2}$

F_-, F_+ are the amplitudes of Ref. (37)
 $g_{\frac{1}{2}}^N$ are the helicity $\frac{1}{2}$ decay amplitudes to $N\pi$
 α is a factor which accounts for a change in the construction of states between this paper and Ref. (37). (We use LS coupling whereas S-L coupling was used in Ref. (37)).

The remaining factors are necessary to change from the helicity amplitudes to the A and B forms for comparison with analyses of the experimental data. The amplitudes, F_- and F_+ , are dependent in general on three amplitudes A, B and C. In simple quark models^(13,14) A and B correspond to orbital and spin flip terms while C is set to zero⁽³⁷⁾. Finally in order to make numerical comparisons we have to introduce kinematic factors, which again account for different definitions⁽³⁵⁾, giving

$$\left. \begin{matrix} A(W_o) \\ B(W_o) \end{matrix} \right\} = \left. \begin{matrix} A \\ B \end{matrix} \right\} C^I \frac{k_o q_o^L}{(2J+1)\Gamma} \frac{M_N}{M_A}$$

where $L = 0, 2$ for S, D wave resonances in the πN system and $k_o, (q_o)$ is the photon (pion) momentum at resonance. Γ is the total resonance width from the fit and C^I is the isospin Clebsch Gorden coefficient corresponding the $N\pi$ final state. We have assumed a barrier factor for the γN system equivalent to $L = 1$ ⁽³⁵⁾.

In making the fits we only allow A, B and C to vary, the other parameters (θ_S, θ_D, S, D) being taken from our analysis of the strong decays of the $\{70, 1^-\}$. In tables 9(a) and 9(b) we present the results of the fits to the recent analysis of Walker and Metcalf⁽³⁵⁾. We have fitted the amplitudes for all resonances in the $\{70, 1^-\}$.

A number of comments are necessary.

- a) The fits are reasonable and demonstrate the consistency of photo-production with the hadronic decays of the $\{70, 1^-\}$ within the Melosh transformation approach.
- b) In order to achieve this result C has to be large which is not consistent with conventional quark model approaches. Table 9(b) contains the result when we constrain C to be zero. The fit is clearly unacceptable. Indeed the need for a large value of C is apparent in the data since the B amplitudes for excitation of the $D_{13}(1520)$ from proton and neutron are not equal in magnitude as would be required if C were zero. We also note that our best fit, while having C the same order of magnitude as A, does not demand their equality as predicted by a quark pair creation model⁽³⁹⁾.
- c) The observation of the excitation of the D15 resonance from protons (B_{2+}) (violating the Moorhouse selection rule⁽⁴¹⁾) again demonstrates the limit of validity of our $SU(6)_w$ model^(*).
- d) In our comparison, we have used the fitted value for the total width obtained from the meson decay analysis (which enables us to obtain reasonable agreement for the S31).
- e) Fit 3 is a fit just to Walker's well measured resonances D13(1520), S11(1510) and S31 (1610) and χ^2 total is obtained using these fitted parameters to calculate the χ^2 of the remaining resonances. As can be seen the results are very similar to Fit 1 to all resonances.

8. $SU(6)_w$ AND $SU(3)$

It is interesting to compare the results of our $SU(6)_w$ analysis of the $\{70, 1^-\}$ and $\{56, 2^+\}$ multiplets with more familiar $SU(3)$ analyses. In fig. 2a) we attempt to give an idea of the relative success of these two types of fits by plotting the ratio

$$R = \frac{g_{\text{expt}}^{SU(3)}}{g_{\text{model}}^{SU(3)}} \quad (40)$$

(*) In a recent paper Osborne⁽⁴⁰⁾ has suggested that such violations of the Moorhouse selection rule may be evidence for exotic (non -35 plet) contributions to the current operator. At our present level of understanding, it appears very difficult to disentangle such exotic contributions from symmetry breaking.

for members of both these multiplets. The coupling constants $g_{\text{Expt}}^{\text{SU}(3)}$ are the results obtained from independent analyses^(7,28,42) for the couplings of various SU(3) multiplets within our two SU(6) supermultiplets. The $g_{\text{model}}^{\text{SU}(3)}$ predictions are obtained from our SU(6) fit in the following way. For unmixed states ($5/2^-$, $5/2^+$ octets and $7/2^+$ decuplet) we use the theoretical F/D ratio (for octet couplings) and the parameters of our analysis to predict $g_{\text{model}}^{\text{SU}(3)}$. The $3/2^-$ states used in the SU(3) analyses are those classified predominantly as 2_8 and 2_1 in our analysis. Our fits show that mixing of these states with the 4_8 and $^2_{10}$ is small. Thus using our parameters and only the mixing between the 2_8 and 2_1 we may calculate approximate coupling constants to compare with the SU(3) values.

It can be seen from fig. 2a that the SU(6) predictions fare quite well. In general the ratios cluster around one but their spread is somewhat larger than the errors which are those quoted by the individual SU(3) measurements. Clearly these errors depend on the errors used on the decay rates analysed and these are probably underestimated.

However, it is evident from the fig. 2b that there is something seriously wrong with the analogous SU(6)_w comparison for the $\frac{1}{2}^-$ states assuming that the N^* , Σ^* and Λ^* states of the SU(3) analysis belong to the 2_8 and 2_1 representations. It has been noted before (42) that the F/D ratio for the $\frac{1}{2}^-$ octet is in total disagreement with that expected for the 2_8 state of the $[70, 1^-]$. Our results give an explanation of this failure. The states used in the SU(3) analysis, the $N^*(1520)$, $\Lambda^*(1670)$, $\Sigma^*(1740)$ are classified in our analysis as predominantly 2_8 , 4_8 and $^2_{10}$ respectively (Table V). Thus an SU(3) analysis of these states is inappropriate.

9. CONCLUSIONS

We have been able to obtain excellent descriptions of the hadronic decays of the $\{70, 1^-\}$ and $\{56, 2^+\}$ multiplets within the framework of an SU(6)_w symmetry scheme motivated by the Melosh transformation.

It must be emphasized, however, that $SU(6)_w$ is clearly a broken symmetry. Some idea of the extent of its validity may be gauged by a scrutiny of the unmixed $5/2^-$ and Δ states of the $\{70, 1^-\}$. In general these resonances are not as well fitted as the states which we have allowed to mix. It is possible therefore that the mixing is disguising the limitations of the model to some extent, although the mixing is itself an expression of symmetry breaking. Nevertheless, it is a pleasing feature of our results that to a reasonable approximation, the physical states may be associated with a definite quark spin member of an $SU(6)_w$ supermultiplet. Symmetry breaking also introduces ambiguity in the angular momentum barrier factors. We note that a simple phenomenological form of the type used successfully in $SU(3)$ analyses is preferred over factors incorporating merely a PCAC factor.

The $\{56, 2^+\}$ fits better than the $\{70, 1^-\}$ even though the mixing is much more limited. This may be due to the higher average mass of this multiplet which makes the form used for the angular momentum barrier less critical. The good fit is surprising in view of the prediction of a $\{70, 2^+\}$ degenerate in mass with the $\{56, 2^+\}$ which would contain many states which could mix with the states in the $\{56\}$. The states often tentatively assigned to the $\{70, 2^+\}$ could equally well be assigned to a higher lying $\{56, 4^+\}$ multiplet and we conclude that we find no evidence for the existence of a $\{70, 2^+\}$.

The fit for the $\{70, 1^-\}$ multiplet gave the ratio S/D negative corresponding to dominance of the $\{35, L_z = \pm 1\}$ terms in the Melosh transformed axial charge. For the $\{56, 2^+\}$ P/F is positive indicating that the $\{35, L_z = 0\}$ term dominates. Thus we agree with the qualitative conclusions of several previous authors^(3,43) but disagree with the predictions of a specific quark pair creation model⁽⁴⁴⁾ which always predicts the negative sign for these ratios. Rosner (ref. 8) has pointed out that some realistic quark models (ref. 14) also predict the negative sign for the ratios, but that it is possible to construct a model⁽⁴⁵⁾ in which the $L_z = \pm 1$ term dominates for odd L and the $L_z = 0$ term, for even L.

The parameters obtained for the $\{70, 1^-\}$ multiplet (mixing matrices, S and D) may be used in conjunction with an $SU(6)_W$ model to describe resonance photoproduction. Using a model for the electromagnetic transitions motivated by the Melosh approach, we find excellent agreement. However, this does require the presence of a term normally ignored in conventional quark model analyses of photoproduction. The necessity for such a term to obtain a reasonable quantitative fit, is evident from even a cursory glance at the data.

The overall success of our analysis allows a comparison of the validity of $SU(3)$ and $SU(6)_W$ (fig. 2a). It is clear that $SU(6)_W$ is not far inferior to $SU(3)$ in its predictions for coupling constants. Furthermore, we may use our $SU(6)_W$ knowledge to examine the $SU(3)$ fits. In particular, we suspect that the N^* , Σ^* and Λ^* states currently analysed as belonging to the $SU(3) \frac{1}{2}^-$ nonet are in fact misclassified.

Our results may be summarized by the statement that $SU(6)_W$ models provide an economical and successful framework for describing the pseudoscalar meson decays of baryon resonances and also pion photoproduction: There are at present no major discrepancies.

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TABLE I: $70 \rightarrow 56 + \pi$: g_ℓ in terms of S and D.

Resonance in $\{70, 1^-\}$	g_S^N	g_D^N	g_S^Δ	g_D^Δ
$2_{8_{3/2}}$	-	$+\frac{1}{3}\sqrt{\frac{2}{3}} \cdot D$	$+\frac{1}{3}\sqrt{\frac{2}{3}} \cdot S$	$-\frac{1}{3}\sqrt{\frac{2}{3}} \cdot D$
$2_{8_{1/2}}$	$+\frac{1}{3}\sqrt{\frac{1}{3}} \cdot S$	-	-	$-\frac{1}{3}\sqrt{\frac{2}{3}} \cdot D$
$4_{8_{5/2}}$	-	$+\frac{1}{2}\sqrt{\frac{1}{15}} \cdot D$	-	$+\sqrt{\frac{7}{30}} \cdot D$
$4_{8_{3/2}}$	-	$+\frac{1}{6}\sqrt{\frac{1}{15}} \cdot D$	$+\frac{1}{3}\sqrt{\frac{5}{3}} \cdot S$	$+\frac{4}{3}\sqrt{\frac{1}{15}} \cdot D$
$4_{8_{1/2}}$	$-\frac{1}{6}\sqrt{\frac{1}{3}} \cdot S$	-	-	$+\frac{1}{6}\sqrt{\frac{2}{3}} \cdot D$
$2_{10_{3/2}}$	-	$-\frac{1}{6}\sqrt{\frac{1}{3}} \cdot D$	$+\frac{1}{3}\sqrt{\frac{5}{6}} \cdot S$	$-\frac{1}{3}\sqrt{\frac{5}{6}} \cdot D$
$2_{10_{1/2}}$	$-\frac{1}{12}\sqrt{\frac{2}{3}} \cdot S$	-	-	$-\frac{1}{3}\sqrt{\frac{5}{6}} \cdot D$

Notation: $g^N (N^* \rightarrow N\pi)$ and $g^\Delta (N^* \rightarrow \Delta\pi)$

TABLE 1 (cont'd)

Resonance in $\{70, 1^-\}$	g_s^Σ	g_D^Σ
$^2_{13/2}$	-	$+\frac{1}{2}\sqrt{\frac{1}{2}}D$
$^2_{11/2}$	$+\frac{1}{4}S$	-

Notation: g^Σ ($\Lambda \rightarrow \Sigma\pi$)

SU(6) multiplet members are denoted by $(2S + 1)\eta_J$, where S is total quark spin, η the SU(3) representation and J the total angular momentum of the resonance. This notation is used throughout the tables.

TABLE II: $56 \rightarrow 56 + \pi$: g_ℓ in terms of P and F

Resonance in $\{56, 2^+\}$	g_P^N	g_F^N	g_P^Δ	g_F^Δ
$2_8^{5/2}$	-	$-\frac{1}{3} \cdot F$	$+\frac{4}{15}\sqrt{\frac{6}{5}} \cdot P$	$-\frac{8}{15}\sqrt{\frac{1}{5}} \cdot F$
$2_8^{3/2}$	$-\frac{1}{3}\sqrt{\frac{2}{3}} \cdot P$	-	$+\frac{8}{15}\sqrt{\frac{1}{30}} \cdot P$	$-\frac{8}{5}\sqrt{\frac{1}{30}} \cdot F$
$4_{10}^{7/2}$	-	$+\frac{4}{15}\sqrt{\frac{6}{7}} \cdot F$	-	$+\frac{2}{5}\sqrt{\frac{5}{7}} \cdot F$
$4_{10}^{5/2}$	-	$+\frac{4}{15}\sqrt{\frac{1}{7}} \cdot F$	$+\frac{7}{30}\sqrt{\frac{12}{7}} \cdot P$	$+\frac{8}{15}\sqrt{\frac{2}{7}} \cdot F$
$4_{10}^{3/2}$	$-\frac{4}{15}\sqrt{\frac{1}{3}} \cdot P$	-	$+\frac{4}{15}\sqrt{\frac{2}{3}} \cdot P$	$+\frac{1}{5}\sqrt{\frac{2}{3}} \cdot F$
$4_{10}^{1/2}$	$-\frac{4}{15}\sqrt{\frac{1}{3}} \cdot P$	-	$+\frac{1}{15}\sqrt{\frac{5}{3}} \cdot P$	-

Notation: $g^N(N^* \rightarrow N\pi)$ and $g^\Delta(N^* \rightarrow \Delta\pi)$

TABLE III: Decays of $\{70,1\}^-$

A: $S = 0$ Baryons

Resonance J^P	Mass	Total Width (MeV)			Decay Mode	Amplitude at Resonance			Partial Width Fit (MeV)
		Expt	Fit	χ^2		Expt	Fit	$\sqrt{\chi^2}$	
N (D13) $3/2^-$	1520	130 ± 20	134	0.0	$N\pi$	$+0.58 \pm .05$	0.58	0.0	78
					$(\Delta\pi)_{DD}$	$-0.27 \pm .07$	-0.16	2.5	6
					$(\Delta\pi)_{DS}$	$-0.25 \pm .07$	-0.31	0.6	21
					$N\eta$	$(+)0.02 \pm .05$	.02	0.0	<1
N (D15) $5/2^-$	1670	150 ± 10	132	3.3	$N\pi$	$0.43 \pm .05$	0.25	13.2	33
					$(\Delta\pi)_{DD}$	$0.45 \pm .07$	0.31	3.8	52
					$N\eta$	$0.0 \pm .05$	-0.09	3.0	4
					ΛK	$0.0 \pm .05$	0.0	0.0	0

Resonance J^P	Mass	Total Width (MeV)			Decay Mode	Amplitude at Resonance			Partial Width Fit (MeV)
						Expt	Fit	$\sqrt{\chi^2}$	
N (D13) $3/2^-$	1710	200 $\pm$ 100	312	1.3	$N\pi$	0.10 $\pm$.05	0.10	0.0	30
					$(\Delta\pi)_{DD}$	0.13 $\pm$.06	0.11	0.1	41
					$(\Delta\pi)_{DS}$	-0.18 $\pm$.06	-0.24	1.1	195
					$N\eta$	0.0	- .01		<1
					ΛK	0.0	- .01		<1
					ΣK	0.0	0.00		<1
D (D33) $3/2^-$	1700	230 $\pm$ 40	236	0.0	$N\pi$	0.18 $\pm$.05	0.14	0.7	33
					$(\Delta\pi)_D$	0.05 $\pm$.06	0.16	3.1	41
					$(\Delta\pi)_{DS}$	0.24 $\pm$.06	0.22	0.2	80
					ΣK	0.0	0.00		<1

Resonance J^P	Mass	Total Width (MeV)		Decay Mode	Amplitude at Resonance		$\sqrt{\chi^2}$	Partial Width Fit (MeV)
		Expt	Fit		Expt	Fit	χ^2	
N (S11) $1/2^-$	1510	75 $\pm$ 25	99	$N\pi$	0.27 $\pm$.07	0.36	1.5	35
				$(\Delta\pi)$ SD	0.06 $\pm$.06	0.10	0.4	3
				$N\eta$	(+)0.40 $\pm$.10	0.20	3.9	11
N (S11) $1/2^-$	1665	135 $\pm$ 20	171	$N\pi$	0.50 $\pm$.05	0.51	0.1	87
				$(\Delta\pi)$ SD	0.15 $\pm$.06	0.35	11.0	41
				$N\eta$	0.0	-0.09		2
				ΛK	(-)0.22 $\pm$.05	-0.12	3.9	5
Δ (S31) $1/2^-$	1610	150 $\pm$ 20	53	$N\pi$	0.35 $\pm$.05	0.23	6.1	12
				$(\Delta\pi)$ SD	-0.40 $\pm$.07	-0.37	0.1	33

B: Λ^* Resonances

Resonance J^P	Mass	Total Width (MeV)		Decay Mode	Amplitude at Resonance			Partial Width
		Expt	Fit		Expt	Fit	$\sqrt{\chi^2}$	Fit (MeV)
$\Lambda(D03) \ 3/2^-$	1690	60 ± 20	78	$\Sigma\pi$	$-0.32 \pm .05$	-0.32	0.0	39
				$(\Sigma^*\pi)_{DD}$	$0.0 \pm .04$	-0.08	3.9	2
				$(\Sigma^*\pi)_{DS}$	$-0.27 \pm .04$	-0.28	0.1	30
				$N\bar{K}$	$0.22 \pm .05$	0.20	0.1	16
				$\Lambda\eta$	0.0	0.00		<1
$\Lambda(D05) \ 5/2^-$	1830	90 ± 20	90	$\Sigma\pi$	$-0.16 \pm .05$	0.0	10.2	59
				$(\Sigma^*\pi)_{DD}$	$-0.13 \pm .03$	0.0	18.8	42
				$N\bar{K}$	$0.07 \pm .04$	0.0	3.1	0
				$\Lambda\eta$	$(-)0.04 \pm .02$	0.0	4.8	3
				ΞK	0.0	0.0		<1

Resonance J ^P	Mass	Total Width (MeV) Expt Fit χ^2	Decay Mode	Amplitude at Resonance Expt Fit $\sqrt{\chi^2}$			Partial Width Fit (MeV)
$\Lambda(1520) \ 3/2^-$	1520	15 $\pm$ 2 15 0.0	$\Sigma\pi$	0.43 $\pm$.02	0.42	0.4	6
			$(\Sigma^*\pi)_{DD}$	0.00 $\pm$.04	-0.02	0.3	0
			$(\Sigma^*\pi)_{DS}$	(-)0.19 $\pm$.04	-0.19	0.0	1
			$N\bar{K}$	0.45 $\pm$.02	0.46	0.3	7
$\Lambda(1670) \ 1/2^-$	1670	30 $\pm$ 5 30 0.0	$\Sigma\pi$	-0.27 $\pm$.04	-0.27	0.0	11
			$(\Sigma^*\pi)_{SD}$	0.18 $\pm$.05	0.08	4.3	1
			$N\bar{K}$	0.16 $\pm$.04	0.21	1.3	6
			$\Lambda\eta$	(-)0.25 $\pm$.05	-0.19	1.3	5
$\Lambda(1402) \ 1/2^-$	1402	38 $\pm$ 4	$\Sigma\pi$	Fit to Partial Width $\Gamma(\Sigma\pi)$ 38 $\pm$ 4 39 0.1			

C: Σ^* Resonances

Resonance J^P	Mass	Total Width (MeV)		Decay Mode	Amplitude at Resonance		$\sqrt{\chi^2}$	Partial Width
		Expt	Fit		Expt	Fit		Fit (MeV)
$\Sigma(D13) \ 3/2^-$	1665	45 ± 10	40	$\Sigma\pi$	$0.19 \pm .04$	0.21	0.2	38
				$\Lambda\pi$	$0.09 \pm .03$	0.04	2.3	2
				$(\Sigma^*\pi)_{DD}$	0.0	-0.04		1
				$(\Sigma^*\pi)_{DS}$	$-0.11 \pm .03$	-0.10	0.2	8
				$N\bar{K}$	$0.08 \pm .04$	0.05	0.8	2
$\Sigma(D15) \ 5/2^-$	1770	110 ± 10	119	$\Sigma\pi$	$0.07 \pm .03$	0.17	11.2	9
				$\Lambda\pi$	$-0.25 \pm .02$	-0.27	1.1	22
				$(\Sigma^*\pi)_{DD}$	$-0.20 \pm .02$	-0.13	13.3	5
				$N\bar{K}$	$0.40 \pm .04$	0.40	0.0	40
				$\Sigma\eta$	0.0	-0.01		<1

Resonance J^P	Mass	Total Width (MeV)		Decay Mode	Amplitude at Resonance			Partial Width Fit (MeV)
		Expt	Fit		Expt	Fit	$\sqrt{\chi^2}$	
$\Sigma(D13) \ 3/2^-$	1580	12 ± 4	10	$\Sigma\pi$	$0.03 \pm .04$	0.01	0.3	<1
				$\Lambda\pi$	$0.10 \pm .02$	0.10	0.0	4
				$(\Sigma^*\pi)_{DD}$	0.0	0.02		<1
				$(\Sigma^*\pi)_{DS}$	0.0	0.10		4
				$N\bar{K}$	$0.03 \pm .01$	0.03	0.1	<1
$\Sigma(D13) \ 3/2^-$	1940	100 ± 30	109	$\Sigma\pi$	0.0	-0.04		1
				$\Lambda\pi$	$-0.05 \pm .03$	-0.06	0.2	1
				$(\Sigma^*\pi)_{DD}$	0.0	-0.05		1
				$(\Sigma^*\pi)_{DS}$	0.0	0.41		63
				$N\bar{K}$	0.0	0.30		32

Resonance J^P	Mass	Total Width (MeV)		Decay Mode	Amplitude at Resonance		Partial Width
		Expt	Fit χ^2		Expt	Fit	Fit (MeV)
Σ (D13) $3/2^-$				$\Sigma\eta$	0.0	-0.08	3
				EK	0.0	0.02	<1
Σ (S11) $1/2^-$	1750	60 ± 20	55 0.1	$\Sigma\pi$	$0.16 \pm .05$	0.17	3
				$\Lambda\pi$	$0.05 \pm .04$	0.04	<1
				$(\Sigma^*\pi)$ SD	$-0.18 \pm .15$	-0.33	12
				$N\bar{K}$	$0.80 \pm .20$	0.52	28
				$\Sigma\eta$	0.0	-0.14	2

* Σ^* represents the $\Sigma(1385)$ and Δ the $\Delta(1230)$. An entry of 0.0 in the Expt. column indicates that no measurement if the rate exists; in the fit column that the prediction is identically zero.

In a few cases $\Sigma \Gamma_{\text{partial}}$ is slightly larger than Γ_{total} derived from our fit. Since the excess is within the experimental errors, constraining the sum to be less than Γ_{total} would not effect the results significantly.

TABLE IV Decays of $\{56, 2^+\}$

A: S = 0 Baryons

Resonance J^P	Mass	Total Width (MeV)		Decay Mode	Amplitude at Resonance $\sqrt{x x'}$			Partial Width Fit (MeV)
		Expt	Fit χ^2		Expt	Fit	χ^2	
N (F15) $5/2^+$	1680	130 ± 15	114 1.1	$N\pi$	$0.60 \pm .05$	0.57	0.3	65
				$(\Delta\pi)_{FF}$	$0.05 \pm .06$	0.10	0.7	2
				$(\Delta\pi)_{FP}$	$-0.25 \pm .06$	-0.20	0.8	8
				$N\eta$	0.0	0.03		<1
				ΛK	0.00 ± 0.05	-0.01	0.1	<1
				ΣK	$0.00 \pm .04$	0.00	0.0	<1
Δ (F37) $7/2^+$	1925	230 ± 30	275 2.2	$N\pi$	$0.40 \pm .05$	0.48	2.8	133
				$(\Delta\pi)_{FF}$	$0.29 \pm .07$	0.26	0.3	37
				ΣK	$-0.09 \pm .04$	-0.08	0.1	4

Resonance J^P	Mass	Total Width (MeV)		Decay Mode	Amplitude at Resonance $\sqrt{x x'}$			Partial Width Fit (MeV)
		Expt	Fit χ^2		Expt	Fit	χ^2	
Δ (F35) $5/2^+$	1860	240 ± 30	275 2.2	$N\pi$	$0.16 \pm .05$	0.09	1.8	20
				$(\Delta\pi)_{FF}$	$0.17 \pm .06$	0.09	1.7	20
				$(\Delta\pi)_{FP}$	0.0	0.09		19
				ΣK	$-0.03 \pm .05$	-0.01	0.2	<1
N (P13) $3/2^+$	1730	130 ± 30	167 1.6	$N\pi$	$0.20 \pm .05$	0.26	1.6	44
				$(\Delta\pi)_{PF}$	0.0	0.11		8
				$(\Delta\pi)_{PP}$	0.0	-0.05		2
				$N\eta$	0.0	0.03		1
				ΛK	0.0	-0.06		2
				ΣK	0.0	-0.01		<1

Resonance J^P	Mass	Total Width (MeV)		Decay Mode	Amplitude at Resonance		$\sqrt{\chi^2}$	Partial Width
		Expt	Fit		Expt	Fit	χ^2	Fit (MeV)
Δ (P33) $3/2^+$	1900	200 ± 30	184	$N\pi$	$0.19 \pm .05$	0.12	2.2	21
				$(\Delta\pi) pF$	0.0	-0.09		14
				$(\Delta\pi) pP$	0.0	-0.10		17
				ΣK	0.0	-0.05		4
Δ (P31) $1/2^+$	1850	210 ± 50	202	$N\pi$	$0.24 \pm .10$	0.19	0.3	38
				$(\Delta\pi) pP$	0.0	-0.06		4
				ΣK	0.0	-0.07		5

B: Λ^* Resonances

Resonance J^P	Mass	Total Width (MeV)		Decay Mode	Amplitude at Resonance		$\sqrt{\chi^2}$	Partial Width Fit (MeV)
		Expt	Fit		Expt	Fit	χ^2	
Λ (FO5) $5/2^+$	1820	80 ± 20	57	$\Sigma\pi$	$-0.27 \pm .05$	-0.37	3.8	13
				$(\Sigma^*\pi)_{FF}$	0.0	0.11		1
				$(\Sigma^*\pi)_{FP}$	$-0.27 \pm .03$	-0.23	1.4	5
				$N\bar{K}$	$0.60 \pm .04$	0.58	0.3	33
				$\Lambda\eta$	$-0.10 \pm .04$	-0.06	0.9	<1
Λ (PO3) $3/2^+$	1890	100 ± 20	108	$\Sigma\pi$	0.0	-0.20		16
				$(\Sigma^*\pi)_{PF}$	0.0	0.13		7
				$(\Sigma^*\pi)_{PP}$	0.0	-0.06		1
				$N\bar{K}$	$0.25 \pm .04$	0.28	0.2	29

Resonance J^P	Mass	Total Width (MeV) Expt Fit χ^2	Decay Mode	Amplitude at Resonance Expt Fit $\sqrt{\chi^2}$	Partial Width Fit (MeV)
Λ (Po3) $3/2^+$			Λn	0.0 -0.08	2
			EK	0.0 0.02	<1

C: Σ^* Resonances

Resonances J^P	Mass	Total Width (MeV)		Decay Mode	Amplitude at Resonance			Partial Width Fit (MeV)
		Expt	Fit χ^2		Expt	Fit	$\sqrt{\chi^2}$	
Σ (F15) $5/2^+$	1905	75 ± 20	76 0.0	$\Sigma\pi$	$-0.10 \pm .04$	-0.11	0.0	25
				$\Lambda\pi$	$-0.08 \pm .02$	-0.09	0.1	17
				$(\Sigma^*\pi)_{FF}$	0.0	-0.02		1
				$(\Sigma^*\pi)_{FP}$	0.0	0.03		2
				$N\bar{K}$	$0.11 \pm .04$	0.03	3.6	3
				$\Sigma\eta$	0.0	-0.01		<1
				ΞK	0.0	0.01		<1
Σ (F17) $7/2^+$	2030	170 ± 20	195 1.5	$\Sigma\pi$	$-0.10 \pm .04$	-0.14	0.8	21
				$\Lambda\pi$	$0.19 \pm .02$	0.21	0.7	49
				$(\Sigma^*\pi)_{FF}$	(+) $0.10 \pm .03$	0.11	0.2	14

Resonance J^P	Mass	Total Width (MeV) Expt Fit χ^2	Decay Mode	Amplitude at Resonance Expt Fit $\sqrt{\chi^2}$	Partial Width Fit (MeV)
Σ (F17) $7/2^+$			$\bar{N}\bar{K}$	$0.18 \pm .03$ 0.17 0.1	33
			$\Sigma\eta$	0.0 -0.06	4
			ΞK	$(-)-0.03 \pm .03$ -0.03 0.0	1

* Σ represents the $\Sigma(1385)$ and $\Delta(1230)$. An entry of 0.0 in the Expt. column indicates that no measurement exists.

TABLE V

Values of $SU(6)_W$ Parameters

$$\begin{array}{ll} \{70\} \quad S = 4.1 & D = -19.6 \\ \{56\} \quad P = 4.3 & F = 17.8 \end{array}$$

Mixing Matrices for N^* 's

$$\begin{array}{ll} \underline{D13 \ 3/2^-} & \begin{bmatrix} N(1710) \\ N(1520) \end{bmatrix} = \begin{bmatrix} 0.98 & 0.18 \\ -0.18 & 0.98 \end{bmatrix} \begin{bmatrix} 4_8 \\ 2_8 \end{bmatrix} & \theta_d = 10.2^\circ \\ \underline{S11 \ 1/2^-} & \begin{bmatrix} N(1665) \\ N(1510) \end{bmatrix} = \begin{bmatrix} 0.87 & -0.49 \\ 0.49 & 0.87 \end{bmatrix} \begin{bmatrix} 4_8 \\ 2_8 \end{bmatrix} & \theta_s = -29.4^\circ \end{array}$$

Mixing Matrices for Λ^* 's

$$\begin{array}{ll} \underline{DO3 \ 3/2^-} & \begin{bmatrix} \Lambda(\quad) \\ \Lambda(1690) \\ \Lambda(1520) \end{bmatrix} = \begin{bmatrix} -0.99 & 0.01 & -0.06 \\ -0.04 & -0.91 & 0.42 \\ -0.05 & 0.42 & 0.91 \end{bmatrix} \begin{bmatrix} 4_8 \\ 2_8 \\ 2_1 \end{bmatrix} & \begin{array}{l} \alpha = 97.4^\circ \\ \beta = 25.0^\circ \\ \gamma = 81.2^\circ \end{array} \\ \underline{SO1 \ 1/2^-} & \begin{bmatrix} \Lambda(\quad) \\ \Lambda(1670) \\ \Lambda(1402) \end{bmatrix} = \begin{bmatrix} -0.19 & 0.88 & -0.43 \\ -0.95 & -0.05 & 0.32 \\ 0.26 & 0.47 & 0.85 \end{bmatrix} \begin{bmatrix} 4_8 \\ 2_8 \\ 2_1 \end{bmatrix} & \begin{array}{l} \alpha = 60.7^\circ \\ \beta = 32.3^\circ \\ \gamma = 36.6^\circ \end{array} \end{array}$$

Note: for the predictions mass values of the missing Λ^* resonances were taken as

$$\Lambda(DO3) = 1900 \text{ MeV} \quad \Lambda(SO1) = 1870 \text{ MeV}$$

Mixing Matrices for Σ^* 's

$$\begin{array}{ll} \underline{D13 \ 3/2^-} & \begin{bmatrix} \Sigma(1580) \\ \Sigma(1665) \\ \Sigma(1940) \end{bmatrix} = \begin{bmatrix} 0.37 & 0.26 & -0.89 \\ -0.18 & 0.96 & 0.21 \\ 0.91 & 0.08 & 0.40 \end{bmatrix} \begin{bmatrix} 4_8 \\ 2_8 \\ 2_{10} \end{bmatrix} & \begin{array}{l} \alpha = 5.2^\circ \\ \beta = 66.3^\circ \\ \gamma = 13.3^\circ \end{array} \\ \underline{S11 \ 1/2^-} & \begin{bmatrix} \Sigma(\quad) \\ \Sigma(1750) \\ \Sigma(\quad) \end{bmatrix} = \begin{bmatrix} & & \\ 0.45 & 0.19 & -0.87 \\ & & \end{bmatrix} \begin{bmatrix} 4_8 \\ 2_8 \\ 2_{10} \end{bmatrix} \end{array}$$

TABLE VI

4_8

$5/2^-$	N* (1670)	Σ (1765)	Λ (1830)
$3/2^-$	N* (1710)	Σ (1940)	Λ ()
$1/2^-$	N* (1660)	Σ ()	Λ (1670)

2_8

$3/2^-$	N* (1520)	Σ (1660)	Λ (1690)
$1/2^-$	N* (1510)	Σ ()	Λ ()

2_{10}

$3/2^-$	Δ (1700)	Σ (1580)
$1/2^-$	Δ (1610)	Σ (1740)

2_1

$3/2^-$	Λ (1520)
$1/2^-$	Λ (1405)

Classification of the states in the $\{70, 1^-\}$

TABLE VII

Predictions of Partial Width of missing resonances

Resonance J^P	Assumed Mass (MeV)	Decay Mode	Partial Width (MeV)	
{70, 1 ⁻ } $\Lambda(D03)$	1900	$\Sigma\pi$	7	-
		$(\Sigma^*\pi)_{DD}$	63	+
		$(\Sigma^*\pi)_{DS}$	123	-
		$\bar{N}\bar{K}$	1	+
		$\Lambda\eta$	2	-
{70, 1 ⁻ } $\Lambda(S01)$	1870	$\Sigma\pi$	107	-
		$(\Sigma^*\pi)_{SD}$	64	+
		$\bar{N}\bar{K}$	22	+
		$\Lambda\eta$	<1	+
		ΞK	29	+
{56, 2 ⁺ } $\Sigma(F15)$	2200	$\Sigma\pi$	12	-
		$\Lambda\pi$	26	+
		$(\Sigma^*\pi)_{FF}$	50	+
		$(\Sigma^*\pi)_{FP}$	18	+
		$\bar{N}\bar{K}$	18	+
		$\Sigma\eta$	5	-
		ΞK	2	-
{56, 2 ⁺ } $\Sigma(P13)$	1730	$\Sigma\pi$	7	-
		$\Lambda\pi$	3	-
		$(\Sigma^*\pi)_{PF}$	<1	-
		$(\Sigma^*\pi)_{PP}$	<1	+
		$\bar{N}\bar{K}$	<1	+

Resonance J^P	Assumed Mass (MeV)	Decay Mode	Partial Width (MeV)
{56, 2 ⁺ } $\Sigma(P13)$	2080	$\Sigma\pi$	5 +
		$\Lambda\pi$	9 -
		$(\Sigma^*\pi)_{PF}$	10 +
		$(\Sigma^*\pi)_{PP}$	9 +
		$N\bar{K}$	6 -
		$\Sigma\eta$	4 +
		ΞK	2 +
{56, 2 ⁺ } $\Sigma(P11)$	1950	$\Sigma\pi$	7 +
		$\Lambda\pi$	14 -
		$(\Sigma^*\pi)_{PP}$	2 +
		$N\bar{K}$	9 -
		$\Sigma\eta$	4 +
		ΞK	1 +

We have made no predictions for the missing S11 states of the {70, 1⁻} as the mixing is large and undetermined by the present data.

As a guide the masses are taken to be near those given by Horgan ⁽³⁴⁾ for the {70, 1⁻}, and in the vicinity of possible resonances for the {56, 2⁺}

After the partial width prediction, we also give the sign of the decay amplitude predicted by the model. The expected reaction amplitude sign is obtained by multiplying by the sign of the $\bar{K}N$ amplitude.

TABLE VIII

Helicity 1/2 matrix elements for the decay of the $\{70, 1^-\}$ to $N\pi$

Resonance	$g_{1/2}^N$
${}^2_8{}_{3/2}$	$-\frac{1}{3}\sqrt{\frac{1}{3}} D$
${}^2_8{}_{1/2}$	$\frac{1}{3}\sqrt{\frac{1}{6}} S$
${}^4_8{}_{5/2}$	$\frac{1}{4}\sqrt{\frac{2}{15}} D$
${}^4_8{}_{3/2}$	$-\frac{1}{12}\sqrt{\frac{2}{15}} D$
${}^4_8{}_{1/2}$	$-\frac{1}{6}\sqrt{\frac{1}{6}} S$
${}^2_{10}{}_{3/2}$	$\frac{1}{6}\sqrt{\frac{1}{6}} D$
${}^2_{10}{}_{1/2}$	$-\frac{1}{12}\sqrt{\frac{1}{3}} D$

TABLE IX (a)

Results of Fit to photoproduction of the $\{70, 1^-\}$

Resonance	Amplitude	Value	Prediction	Contribution to χ^2
D13 (1520)	A_{2-}^p	$-.05 \pm .05$	$-.02$	.4
	A_{2-}^n	$.53 \pm .08$	.33	5.9
	B_{2-}^p	$-1.51 \pm .1$	-1.45	.4
	B_{2-}^n	$-1.08 \pm .12$	-1.12	.1
D15 (1670)	A_{2+}^p	$-.05 \pm .06$	0	.7
	A_{2+}^n	$.02 \pm .07$	.12	2.1
	B_{2+}^p	$.14 \pm .08$	0	3.1
	B_{2+}^n	$.03 \pm .10$	.12	.8
D13 (1710)	A_{2-}^p	$0.0 \pm .10$	0.0	.0
	A_{2-}^n	$0.0 \pm .10$	$-.03$	.1
	B_{2-}^p	$0.0 \pm .10$	$-.03$	.1
	B_{2-}^n	$0.0 \pm .15$	$-.01$	.0
D33 (1700)	A_{2-}^p	$0.0 \pm .10$	.16	2.7
	B_{2-}^p	$0.0 \pm .10$	$-.20$	4.1
S11 (1665)	A_{0+}^p	$-.11 \pm .13$	$-.26$	1.4
	A_{0+}^n	$-.17 \pm .20$	$-.38$	1.1
S11 (1510)	A_{0+}^p	$-.60 \pm .12$	$-.47$	1.1
	A_{0+}^n	$-.48 \pm .20$	$-.55$	0.1
S31 (1610)	A_{0+}^p	$-.50 \pm .18$	$-.68$	1.0
<div style="border: 1px solid black; padding: 10px; display: flex; justify-content: space-between;"> $\chi^2 = 25.4$ $A = 8.25$ $B = 2.15$ $C = 4.24$ </div>				

TABLE IX (b)

Results of different fits to the photoproduction of the $\{70, 1^-\}$

	χ^2 total	χ_{3*}^2	A	B	C
1. To all resonances	25.4	9.0	8.25	2.15	4.24
2. To all resonances constraining C = 0	73.4	46.1	8.86	3.92	0.0
3. To D13 (1520), S11 (1560), S31 (1610)	26.9	7.7	8.83	2.70	3.96

* χ^2 for the 3 resonances D13 (1520), S11 (1510), S31 (1610)

FIGURE CAPTIONS

Fig. 1 States required to make up the $\{70, 1^-\}$ and $\{56, 2^+\}$. The mixing possibilities are shown by the dotted lines. (With acknowledgements to Ref. 2).

Fig. 2 Comparison of the $SU(6)_w$ predictions for the $SU(3)$ coupling constants with the measured values

- a) For the $7/2^+$ decuplet, $5/2^+$ octet, $5/2^-$ octet and $3/2^-$ nonet where the agreement is very good.
- b) For the $1/2^-$ nonet showing the bad disagreement due to misassignment of states.

The vertical lines define a corridor within which we would expect the ratio to lie.

fig. 1

fig. 2