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SIMPLE STRUCTURE OF CHARGED NEUTRAL CORRELATIONS

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A B S T R A C T

All available high energy data agree with the assumption that the charged versus neutral pion multiplicities are described by a binomial distribution. This distribution is obtained in a variety of models. One of these is the Mueller model, supplied with the assumption of absence of exotic contributions. In contrast to this, a binomial distribution cannot be obtained in any neutral cluster model.

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1. INTRODUCTION

Recent experiments at the NAL and ISR laboratories have revealed important dynamical short-range correlations between the particles produced. The existence of such correlations shows that, instead of being emitted independently, the particles produced tend to "stick together" or, in other words, to be produced in few particle clusters. Detailed experimental data are now available on the charge dependence of these correlations ¹⁾⁻³⁾.

In this letter we show that although particles are in general not produced statistically but are dynamically correlated, the distribution of charge among them might still happen statistically. More specifically, the present high energy data are in agreement, for fixed total multiplicity n of emitted pions, with the distribution of charged versus neutral ones being a binomial distribution with $\langle n_0 \rangle = \frac{1}{2} \langle n_{ch} \rangle = 1/3 n$:

$$\sigma_{n_{ch}, n_0} = \sigma_n \cdot \binom{n}{n_{ch}} \left(\frac{2}{3}\right)^{n_{ch}} \left(\frac{1}{3}\right)^{n_0} ; \quad n = n_{ch} + n_0 \quad (1)$$

This equation, or similar formulae, has already been suggested by several authors ⁴⁾⁻⁶⁾. Many of its predictions were also discussed in Ref. 7), although they were derived from the Mueller model, and the binomial formula was not explicitly mentioned ^{*)}.

To understand theoretically what kind of dynamics might give rise to Eq. (1), we investigate several theoretical models. We derive this formula in the Mueller model, assuming the absence of exotic contributions to all inclusive n pion distributions. The formula follows also in a general class of dual models, in which the isospin dependence of the exclusive n particle amplitudes are given by Chan-Paton factors ⁸⁾. Duality, however, is not a

^{*)} Eq. (1) allows n_{ch} to be odd as well as even. This does not, however, necessarily violate charge conservation since it should be valid for the pion multiplicity distribution only. Experimentally in roughly half of the collisions an incoming proton turns into a neutron, thus giving an excess of charge to the pionic system.

necessary ingredient for obtaining it. It is also a property of some models without any dual structure at all, such as the Cerulus statistical model. On the other hand, Eq. (1) cannot be obtained in any neutral cluster model. Finally, the importance of further experimental checks of the simple ansatz Eq. (1) is emphasized.

2. PREDICTIONS AND COMPARISON WITH DATA

The ansatz Eq. (1) relates the multiplicity distribution of neutral particles ^{*}) to that of charged ones. Thus, it provides unique predictions for all integrated correlation quantities involving charged and neutral particles, in terms of those involving charged particles only. The present high energy data involving neutral particles are not yet very detailed. Thus we shall be content with listing some of the simplest predictions of Eq. (1) and contrasting them with available data.

The following relations between the two-body correlation integrals can be derived in a straightforward way from Eq. (1), and charge conservation :

$$f_2^{00} = f_2^{0-} = f_2^{--} + \frac{1}{2} \langle n_- \rangle . \quad (2)$$

These are asymptotic predictions. At finite energies, the contributions due to the fragmentation regions will provide corrections, which at present can only be roughly estimated ^{**}).

The available data [for references to data, see Ref. (7)] on the second Mueller moment f_2^{0-} are shown in Fig. 1. The agreement of our relation (2) with data is seen to be good. Also shown are predictions of two simple neutral cluster models. These are the so-called ϵ model in which $I = 0$ clusters decay into two pions each, and the ω model in which each cluster decays into $\pi^+\pi^-\pi^0$. They predict $f_2^{0-} = f_2^{--}$ and $f_2^{0-} = f_2^{--} + \langle n_- \rangle$, respectively. To account for the experimental f_2^{--} , we assume correlations between clusters.

^{*}) For simplicity and lack of better data, we assume that all produced particles are pions.

^{**}) These are not corrections to Eq. (1) but follow from the fact that Eq. (1) refers to pions only while charge conservation involves also the leading proton.

No data on $\pi^0\pi^0$ correlations at NAL-ISR energies are yet available. At lower energies, a value of $f_2^{00} = 0.29 \pm 0.33$ at 12.4 GeV/c has been recently reported by Swanson et al. ⁹⁾, not in disagreement with Eq. (2). The only other measurement of f_2^{00} available is that at 69 GeV/c by Blumenfeld et al. ¹⁰⁾. Their reported value of $f_2^{00} = -2.0 \pm 1.0$ is at least two standard deviations below any model prediction we are aware of.

The data which have been most intensively studied to learn about charged-neutral correlations are those of the associated neutral multiplicities. These data are known to agree well with Eq. (1) ⁴⁾⁻⁷⁾. As an example, we show in Fig. 2 a comparison of the prediction of Eq. (1) with the data on the average number of neutral pions versus negative charged multiplicity at 303 GeV/c ¹¹⁾ *). As seen, the agreement is satisfactory **).

The essential idea behind Eq. (1) is that, while the probability of emitting pions is not purely statistical, whether an emitted pion will be charged or neutral, is statistical. Thus, according to this idea, also the normalized rapidity correlations should be independent of whether one considers charged or neutral pions or any combination. To test this prediction, we display in Fig. 3 the difference $R_{\gamma, ch}(0, y) - R_{ch, ch}(0, y)$ taken from Refs. 2) and 3). The experimental values are well consistent with being zero. We also show the predictions of the ϵ and ω models, assuming the short-range parts of the correlations to be Gaussians of unit width ¹²⁾,

$$R_{\gamma, ch}(0, y) - R_{ch, ch}(0, y) = \pm \frac{e^{-y^2/2}}{\rho_{ch}(0)\sqrt{2\pi}} ; \begin{cases} + & \text{for } \omega\text{-model} \\ - & \text{for } \epsilon\text{-model} \end{cases}$$

We find indeed strong support for our statistical idea of charge distribution.

*) For the techniques of calculating $\langle n_0 \rangle_{n_-}$ and other quantities from Eq. (1), see Ref. 13). A useful intermediate result is that the generating function $G(z_0, z_{ch})$ corresponding to Eq. (1) is a function of the one variable $z_0 + 2z_{ch}$ only.

**) The behaviour of the associated neutral multiplicity is closely related to the values of the correlation integrals [see Ref. 7)]. Thus, the results shown in Figs. 1 and 2 are not independent.

Finally, Eq. (1) can be applied even to exclusive cross-sections. We have checked the predictions of this equation against the data on several exclusive cross-sections and found no disagreement.

In summary, the formula given in Eq. (1) agrees with all the available data on charged-neutral correlations. This suggests that the charge structure of multiple production might well turn out to be very simple. It will be very important to carry out further experimental checks on the validity of the predictions of Eq. (1).

3. MUELLER MODEL

We now turn to the study of models, in order to find theoretical justification for the binomial distribution. We first discuss the Mueller model. According to the generalized optical theorem the inclusive n pion cross-section $\rho_{c_1, \dots, c_n}(p_1, \dots, p_n)$ is related to the discontinuity of the forward amplitude of the elastic $n + 2$ body process $abc_1 \dots c_n \rightarrow abc_1 \dots c_n$. This is illustrated in Fig. 4. Assume now that $I = 0$ and $I = 1$ exchanges alone are important and that $I \geq 2$ exchanges may be neglected. No other assumption about the nature of the singularities exchanged is made. In particular, no factorization in kinematic variables is assumed. This is important since the model will be applied not only in the Regge limit of large subenergies but also in the case of an average particle density. Then, for any inclusive n particle distribution function with a pion of charge c_k produced at momentum $\vec{p}_k$, one gets the following four contributions

$$\begin{aligned} \rho_{..c_k..}(\dots \vec{p}_k \dots) &= \rho_{..c_k..}^{(0,0)}(\dots \vec{p}_k \dots) + \rho_{..c_k..}^{(0,1)}(\dots \vec{p}_k \dots) \\ &+ \rho_{..c_k..}^{(1,0)}(\dots \vec{p}_k \dots) + \rho_{..c_k..}^{(1,1)}(\dots \vec{p}_k \dots), \end{aligned} \quad (5)$$

corresponding to $I = 0$ or 1 immediately before and after particle k . More specifically, one gets (suppressing the momenta)

$$\rho_{..+..} = \rho_{..+..}^{(0,0)} \pm \rho_{..+..}^{(0,1)} \pm \rho_{..+..}^{(1,0)} + \rho_{..+..}^{(1,1)} \quad (6a)$$

$$\varrho_{..o..} = \varrho_{..+..}^{(0,0)} + \varrho_{..o..}^{(1,1)} \quad (6b)$$

Adding the distribution functions $\rho_{...+...}$ and $\rho_{...-...}$, one gets the distribution for finding any charged pion at $\vec{p}_k$

$$\begin{aligned} \varrho_{..c_k = ch..} &= \varrho_{..+..} + \varrho_{..-..} \\ &= 2\varrho_{..o..} + 2\left(\varrho_{..+..}^{(1,1)} - \varrho_{..o..}^{(1,1)}\right). \end{aligned} \quad (7)$$

This means that up to terms with $I = 1$ exchange before and after particle k , the probability for the k -th pion to be charged is twice that for it being neutral, independent of other particles produced. Continuing with what we did for the k -th particle, we end up with the production of n pions each of which is labelled as "charged" or "neutral" only, but where positive pions are not distinguished from negative ones. There remain then two contributions only : one with $I = 0$ exchange throughout the whole diagram, and another with $I = 1$. At high energies the second graph is suppressed by a factor $s^{-\frac{1}{2}}$ compared to the first one. Thus one obtains the following important result : if one distinguishes only between charged and neutral pions, but not between positively and negatively charged pions, then only contributions of $I = 0$ trajectories appear in the Mueller amplitudes, up to $O(s^{-\frac{1}{2}})$. Since any $I = 0$ trajectory cannot distinguish between coupling of charged and neutral particles, we see already that their relative distribution is purely statistical, q.e.d.

4. OTHER MODELS

A crucial ingredient of the above derivation was the assumption of absence of $I \geq 2$ Mueller amplitudes. Since general multiperipheral models with $I = 0$ and $I = 1$ exchanges generate not only $I = 0$ and 1 but also $I = 2$ output singularities, it is easy to guess that Eq. (1) is not a general property of such models. This is the case, for instance, in the A and I models of Caneschi and Schwimmer¹⁴⁾, which both yield two-body correlations rather different from Eq. (2)¹⁵⁾. A third example of a multiperipheral model which does not yield Eq. (1) is the MR model of Ref. 16).

A specific class of models satisfying Eq. (1) is of course given by those in which all Mueller diagrams are given by quark diagrams, i.e., where all inclusive cross-sections are multiplied by Chan-Paton factors ⁸⁾. Thus, the following might not be surprising : a binomial distribution is found in a general class of dual models in which the isospin dependence of the exclusive n particle amplitudes is given by Chan-Paton factors ^{*}). An example of such a model is a multiperipheral model with exchange degenerate $I = 0$ and $I = 1$ trajectories of equal coupling strength [the M model of Ref. 16)]. Another example is provided by the dual statistical bootstrap model of Ref. 17).

In addition to the models discussed above, Eq. (1) is a property of several non-dual statistical models. Thus, duality is not a necessary ingredient for obtaining it. An example is provided by the fireball model of Ref. 18). There, the fireballs are allowed to have any isospin. Thus, this model is more or less opposite to dual models. Another example is provided by the Cerulus statistical model ¹⁹⁾. There, the final state charge distribution is obtained by assuming that when the isospins of the produced particles are coupled together, all possible isospin states occur with equal probabilities. This is again opposite to dual models (and does not take into account that exotic resonances do not exist).

Finally, let us make some remarks about the nowadays very popular cluster models. It is well known that all correlation and charge transfer data available are compatible with the emission of neutral clusters of finite size. It is thus interesting to see that no such cluster model can lead to a binomial distribution.

To prove this statement, we first notice that in cluster models of the above type, the generating function is given by [see, e.g., Ref. 13)]

$$G(z_d, z_o) = \tilde{G}(g(z_d, z_o)) , \quad (9)$$

^{*}) This result is easily derived by using the techniques of Wambach ¹⁷⁾.

where $\tilde{G}(z)$ describes the multiplicity distribution of the clusters, while $g(z_{ch}, z_o)$ describes their decay. From Eq. (1) one finds that any generating function describing a binomial distribution must be a function of $2z_{ch} + z_o$ only. Combined with Eq. (9), this means that also g is a function of $2z_{ch} + z_o$ only, and thus the decay of each single cluster leads to a binomial distribution. This, however, implies that a finite fraction of the clusters will decay into an odd number of charged pions, which of course contradicts the assumption that all clusters are neutral. As examples of this general statement we just mention the ϵ and ω models of Section 2. These examples show also that neutral cluster models cannot be excluded (or preferred) on the basis of present correlation data. The pure ϵ and ω models could be excluded. But consider a model in which 60% of the clusters decay by the " ϵ " mode and 40% by the " ω " mode. This would yield two-particle correlations satisfying Eq. (2) and would deviate only in the (essentially unknown) three-particle correlations from the predictions of Eq. (1).

It is very interesting that this negative statement about cluster models can be turned into a positive one : if one assumes clusters of charge ≤ 1 only, Eq. (1) fixes the number of charged clusters to be equal to the number of neutral clusters ^{*)}. Thus, no predictive power would be lost compared to neutral cluster models.

5. CONCLUSION

Charge conservation fixes up the total charge of pions produced in high energy collisions. Thus, the charge state of pions produced, let us say, at a certain rapidity, cannot be random and independent of the other produced pions. The message of the present letter is that the available data suggest the next simplest logical possibility, which is indeed found in theoretically well founded models : what seems to be random is not the charge itself but rather whether a pion is charged or neutral. It will be very interesting to see - when better data become available - how well this simple statistical picture of the charge structure of multiple production will stand under a detailed scrutiny.

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*) This holds strictly only for large clusters.

REFERENCES

- 1) C. Bromberg et al., Phys.Rev. D9, 1864 (1974) ;
R. Singer et al., Argonne preprint ANL/HEP 7368 (1973).
- 2) S.R. Amendolia et al., Phys.Letters 48B, 359 (1974).
- 3) H. Dibon et al., Phys.Letters 44B, 313 (1973).
- 4) E.L. Berger, D. Horn and G.H. Thomas, Phys.Rev. D7, 1412 (1973) ;
D. Horn and A. Schwimmer, Nuclear Phys. B52, 627 (1973) ;
D. Drijard and S. Pokorski, Phys.Letters 43B, 509 (1973) ;
A. Arneodo and J. Kubar-Andre, Nice preprint N TH 74/1 (1974).
- 5) G.H. Thomas, Phys.Rev. D8, 3042 (1973).
- 6) F. Czikor, I. Montvay and F. Niedermayer, Phys.Letters 49B, 47 (1974).
- 7) P. Grassberger and H.I. Miettinen, CERN preprint TH.1870 (1974).
- 8) J.E. Paton and H.M. Chan, Nuclear Phys. B10, 516 (1969).
- 9) D. Swanson et al., Phys.Letters 48B, 479 (1974).
- 10) H. Blumenfeld et al., Phys.Letters 45B, 525 (1973).
- 11) F.T. Dao et al., Phys.Rev.Letters 30, 1151 (1973).
- 12) See, e.g., A. Biaxas, IVth International Symposium on Multiparticle
Hadrodynamics, Pavia 1973 (F. Duimio et al., eds.).
- 13) M. Le Bellac, Acta Phys.Polon. B4, 901 (1973).
- 14) L. Caneschi and A. Schwimmer, Phys.Rev. D3, 1588 (1971).
- 15) D.K. Campbell and S.-I. Chang, Phys.Rev. D8, 2996 (1973).
- 16) P.W. Coulter and D.R. Snider, Phys.Rev. D8, 4055 (1973).

- 17) U.M. Wambach, Bielefeld preprint Bi-74/05 (1974).
- 18) E.-M. Ilgenfritz and J. Kripfganz, Nuclear Phys. B62, 141 (1973).
- 19) F. Cerulus, Nuovo Cimento 19, 528 (1960).
- 20) F.T. Dao et al., Phys.Rev.Letters 29, 1627 (1972).

FIGURE CAPTIONS :

Fig. 1 : The second Mueller moment f_2^{0-} , as computed from the associated neutral multiplicities of Refs. 9)-11). The three curves are our prediction Eq. (2) and the predictions of the ϵ and ω models. For them, experimental values of f_2^{--} were used. Corrections due to fragmentation regions are expected to shift all three curves down by 0.15 - 0.3 units.

Fig. 2 : Average number $\langle n_{0^+ n_-} \rangle$ of neutral pions associated with n_- negative pions at 303 GeV/c. $\bullet$: Data from F.T. Dao et al., Ref. 11) ; $\circ$: Our prediction using prong cross-sections of Ref. 20), with corrections due to fragmentation regions.

Fig. 3 : Difference between $(\gamma, \text{charged})$ and $(\text{charged}, \text{charged})$ normalized correlations in the central region. The shaded band represents data of Refs. 2) and 3).

Fig. 4 : Generalized optical theorem and isospin decomposition of the inclusive cross-section.

FIG.1

FIG. 2

FIG.3

$$\rho_{c_1 \dots c_n}(\vec{p}_1, \dots, \vec{p}_n) = \frac{1}{\sigma_{\text{inel}}}$$

$$= \frac{1}{\sigma_{\text{inel}}} \sum_{I_1, \dots, I_{n+1}}$$

FIG.4