CM-P00060476

Archives

Ref.TH.1889-CERN

MIXING OF OPERATORS IN WILSON EXPANSIONS

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A B S T R A C T

The tree-loop theorem of 't Hooft and Veltman is used in Fermi type gauges to show that gauge invariant operators in Wilson expansions mix in general with non-gauge invariant operators. Background field gauges are proposed in which this does not occur. Calculations of anomalous dimensions in these gauges are discussed.

1. INTRODUCTION

Recently field theories have been discovered which at large momenta behave as if they were free¹⁾. More precisely, in the language of the renormalization group, the effective coupling constant moves towards zero as the momentum increases. In applying these theories to deep inelastic phenomena it is necessary to consider currents J and in particular the short distance behaviour of the product of two currents. When the Wilson²⁾ or light-cone expansion

$$J(x) J(-x) \underset{x^2 \rightarrow 0}{\sim} \sum_j C^{(j)}(x^2, g) x^{\mu_1} \dots x^{\mu_j} R_{\mu_1 \dots \mu_j}^{(j)}(0) \quad (1.1)$$

is written, where for convenience we have suppressed all Lorentz and group indices of the J 's, we have to consider all operators $R_{\mu_1 \dots \mu_j}^{(j)}(0)$ which mix with each other under renormalization. The problem of mixing is present in all field theories. In ϕ^4 theory, for example, if we are interested in the Green functions

$$\langle 0 | T(\phi^4(x) \phi(x_1) \phi(x_2) \dots \phi(x_r) | 0 \rangle \quad (1.2)$$

which are generated by adding to the Lagrangian a source term $K(x)\phi^4(x)$, we must consider counter-terms linear in K but proportional to operators such as $(\partial_\mu \phi)^2$ which are different from ϕ^4 . This is basically all that the phenomenon of mixing is. Non-Abelian gauge theories are the only theories which can be asymptotically free. They are much more complicated than ϕ^4 theory because of gauge invariance.

In order to be specific we shall first consider the action³⁾ S

$$S[A] = -\frac{1}{4} F_{\mu\nu}^a F_{\mu\nu}^a + \bar{\psi} i \gamma^\mu (\partial_\mu + g \sigma^a A_\mu^a) \psi - \chi^\mu \chi^\lambda F_{\nu\mu}^a F_{\nu\lambda}^a \quad (1.3)$$

where $F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a - g f^{abc} A_\mu^b A_\nu^c$; A_μ^a is the gauge field; g is the bare gauge coupling constant; χ^μ, χ^λ are c-number sources such that $\chi^\mu \chi_\mu$ vanishes; ψ are quark fields transforming under a representation matrix σ of the (colour gauge) group; and f_{abc} are the structure constants of the gauge group. [Moreover, the supercondensed notation⁴⁾ is used for which indices are supposed implicitly to carry coordinate labels. In particular, summation over indices implies space-time integrations as well (in an obvious way).]

The reason for considering initially the operator $F_{\nu\mu}^a F_{\nu\lambda}^a$ is that it is the simplest operator of twist two which is invariant under the bare gauge transformation

$$A_a^\mu(x) = A_a^{\mu'}(x) + g f_{acb} A_\mu^c(x) \Lambda_a^b(x) - \partial_\mu \Lambda_a(x) \quad (1.4)$$

with Λ_a a c-number infinitesimal group transformation parameter. At short distances it is the lowest twist operators that dominate. There are of course other operators of twist two, e.g.

$$R^{(s)} = i^{s-2} F_{\mu_1 \nu}^a D_{\mu_2} \dots D_{\mu_{s-1}} F_{\nu \mu_s}^a \chi^{\mu_1} \chi^{\mu_2} \dots \chi^{\mu_s} \quad (1.5)$$

where D_μ is a covariant derivative. In general we know that operators of the same twist and Lorentz structure will mix. We will find some differences in the mixing of operators with s larger than two and the operators with s equal to two. The action given in (1.3) as it stands cannot be used without specifying a gauge fixing term $C_\alpha(A)$ and the associated Faddeev-Popov Lagrangian.

The most straightforward way of seeing which operators mix with $F_{\nu\mu}^a F_{\nu\lambda}^a \chi^\mu \chi^\lambda$ is just to insert it into various Green functions and obtain counter-terms⁵⁾ of the form

$$\begin{aligned} & U \partial_\alpha A_a^\alpha A_b^\beta A_c^\gamma f^{abc} \chi_\beta \chi_\gamma \\ & + V \chi^\beta \chi^\gamma A_a^\alpha A_b^\beta A_c^\gamma A_d^\alpha \delta_{ab} \delta_{cd} \\ & + W \chi^\beta \chi^\gamma A_a^\alpha A_b^\beta A_c^\gamma A_d^\alpha f^{abe} f^{cde} \\ & + X \partial_\rho c_c^\dagger f^{aes} f^{skd} f^{cde} \chi^\alpha \chi^\rho A_a^\alpha c_k \\ & + Z \chi^\alpha \partial_\alpha A_a^\mu \chi^\beta \partial_\beta A_a^\mu \\ & + Y c_c^\dagger f^{ekd} f^{cde} \chi^\beta \chi^\rho \partial_\beta \partial_\rho c_k \end{aligned} \quad (1.6)$$

+ possible permutations of indices (where c_a denotes the Faddeev-Popov field, and U, V, W, X, Y , and Z are divergent constants).

In this note we will show how invariance arguments^{6,7)} can tell us the connection between the above constants, at least at the one-loop level. Taylor⁸⁾ has recently adopted a similar philosophy but we are unable to follow his methods. The formalism and techniques that we adopt are those of Ref. 6. We find in Fermi-type gauges for $s=2$ that the Lagrangian containing one-loop counter-terms is invariant under a χ -dependent gauge transformation, and in particular the Faddeev-Popov mixing is nothing other than the consequent χ -dependence of the Faddeev-Popov

term arising from this gauge transformation. For higher s we find that the Lagrangian in general is invariant under a transformation of A which is non-linear. The gauge invariance of the Lagrangian then becomes more difficult to disentangle. Since the one-loop Lagrangian is invariant under some transformation and the bare Lagrangian is invariant under a different transformation (namely the bare gauge transformation), the difference (i.e. the set of one-loop counter-terms) is not invariant under either of these transformations. Consequently there is much mixing between gauge-invariant and non-gauge-invariant operators. We will discuss, however, a class of gauges which do not suffer from such problems; these are the background field gauges⁹⁾. Gauge-invariant operators mix only with gauge-invariant operators in such gauges.

In Section 2 we shall give the invariance arguments in Fermi-type gauges. In Section 3 we will introduce the background field gauge and discuss its consequences.

2. FERM-TYPE GAUGES

We will consider a bare Lagrangian $\mathcal{L}_0$ invariant under the gauge transformation

$$A_a^\mu = A_a'^\mu + g \hat{s}^\mu_{ab}(A') \Lambda_b + \hat{t}^\mu_{ab} \Lambda_b \quad (2.1)$$

where Λ is an infinitesimal parameter, $\hat{s}$ is field-dependent but $\hat{t}$ is not. We must also choose a gauge-fixing function $C_a(A)$ for $\mathcal{L}_0$. Under (2.1) we have

$$C_a(A) = C_a(A') + g \hat{l}_{ab}(A') \Lambda_b + \hat{m}_{ab} \Lambda_b \quad (2.2)$$

When $C_a(A)$ is $(1/\sqrt{\xi}) \partial_\mu A_\mu^a$, ξ being the gauge-fixing parameter,

$$\hat{s}^\mu_{ab} = f_{acb} \delta_{\mu\nu} A_\nu^c$$

and

$$\hat{t}^\mu_{ab} = -\delta_{ab} \partial_\mu$$

Taking (1.3) for $\mathcal{L}_0$, regularized Ward identities can be written down in n -dimensions. Diagrammatically they are represented as

$$= \sum_{j=1}^q \left\{ \begin{array}{l} \text{Diagram 1} \\ \text{Diagram 2} \\ \text{Diagram 3} \end{array} \right\} \quad (2.3)$$

Bold lines denote vector mesons and dotted lines ghosts. These diagrams often seem to cause confusion and so we shall explain their meaning using simple examples. We would first like to note that (2.3) is valid for the total amplitudes on the left- and the right-hand side. In particular it is true at the tree diagram level and at the one-loop level. Since we shall only be interested in the one-loop counter-terms of the theory, this is all that we shall need. Below we shall omit fermion lines because they are not essential to the argument.

We will first consider $q=1$ and the tree approximation; (2.3) then becomes just

$$= 0 \quad (2.4)$$

where the central vertex is

$$- 2 \delta_{ab} (p \cdot q \chi^\alpha \chi^\beta - \chi \cdot q \chi^\alpha p^\beta - \chi \cdot p q^\alpha \chi^\beta + \chi \cdot p \chi \cdot q \delta^{\alpha\beta}) \quad (2.5)$$

the left vertex is

$$- i q_\mu \delta_{\beta\mu} \quad (2.6)$$

and the right vertex is

$$\delta_{a,b} \delta_{\beta\mu} \quad (2.7)$$

(Our conventions are such that all momenta are taken pointing into the vertex; p and q denote momenta; α, β Lorentz indices; and a, b group indices.) In order to understand the conventions for the right-hand side of (2.3) we will consider the one-loop version of (2.3)

$$\begin{array}{c}
 + \quad \text{[Diagram: A vertex with a loop and a dashed line]} \quad + \quad \text{[Diagram: A triangle loop]} \quad + \quad \text{[Diagram: A vertex with a loop and a dashed line]} \\
 = 0
 \end{array}$$

In (2.8) the new vertices that appear (apart from the standard vertices of $\mathcal{L}_0$) are

$$\begin{array}{c}
 \text{[Diagram: A vertex with three lines labeled } a, \alpha, p; b, \beta, q; c, \gamma, k \text{]} \\
 2ig f^{abc} (\chi^\alpha \chi^\gamma (-p^\beta + k^\beta) + \chi^\alpha \chi^\beta (p^\gamma - q^\gamma) + \chi^\beta \chi^\gamma (q^\alpha - k^\alpha) \\
 + \chi^\gamma \chi \cdot (p - q) \delta^{\alpha\beta} + \chi^\beta \delta^{\alpha\gamma} \chi \cdot (k - p) \\
 + \chi^\alpha \delta^{\beta\gamma} \chi \cdot (q - k))
 \end{array} \quad (2.9)$$

and the non-Lagrangian vertices

$$\begin{array}{c}
 \text{[Diagram: A vertex with a dashed line labeled } b \text{ and a solid line labeled } c \text{ meeting at } (a, \alpha) \text{]} \\
 g f^{abc}
 \end{array} \quad (2.10)$$

$$\begin{array}{c}
 \text{[Diagram: A dashed line labeled } b, p \text{]} \\
 i \delta_{ab} p_\alpha
 \end{array} \quad (2.11)$$

One method of neatly deriving the Ward identities is by using the path integral representation of the generating functional and regarding a gauge transformation simply as a change of variables of the path integral^{7,10}). The identities obtained in this way are a general expression of the gauge invariance of (1.3); however, our problem is to learn something not about the Lagrangian in (1.3) but about the Lagrangian obtained from it by adding the counter-terms (1.6), $\mathcal{L}_1$ say. The counter-term Lagrangian is defined by saying that it is the sum of terms required to cancel the poles at $n=4$ of one-loop diagrams of (1.3). Here n is the continued dimension of space-time required in dimensional regularization.

So far we have -- knowing the invariance properties of a Lagrangian -- derived identities between Feynman diagrams. We would like to invert this procedure and ask whether we can deduce the invariance properties of a Lagrangian if we know something about identities between its diagrams. Such an inversion is

provided by the tree-loop theorem of 't Hooft and Veltman⁶⁾. This is not to be confused with another theorem of the same name due to Feynman.

Equation (2.3) when written in terms of tree diagrams is given by

$$\begin{aligned}
 & \text{Diagram: A central vertex with a double line leg labeled } C, \text{ a single line leg labeled } R, \text{ and } q \text{ single line legs labeled } A_{\mu_1}^{a_1}, A_{\mu_2}^{a_2}, \dots, A_{\mu_q}^{a_q}. \\
 & = - \sum_{\text{blobs}} \sum_{j=1}^q \left(\begin{array}{c} \text{Diagram 1: A blob with } j \text{ legs, each connected to a circle with a vertical line. The blob has a dashed line leg labeled } \hat{s}^{(j)}. \\ \text{Diagram 2: A blob with } j \text{ legs, each connected to a circle with a vertical line. The blob has a dashed line leg labeled } \hat{t}^{(j)}. \end{array} \right) \quad (2.12)
 \end{aligned}$$

This is a simple consequence of the fact that the ghost interaction vertex is given by $c_a^\dagger \ell_{ab}(a) c_b$. On the right-hand side of (2.12) we have intentionally not explicitly marked the R and A legs. These are distributed in all possible ways in the blobs.

We shall now state the tree-loop theorem:

Given

- i) the existence of functions C'_a as well as $\hat{s}'$, $\hat{t}'$, $\hat{\ell}'$, and $\hat{m}'$ such that

$$C'_a(A_b + g \hat{s}'_{bc}(A) \mathcal{L}_c + \hat{t}'_{bc} \mathcal{L}_c) = C'_a(A) + g \hat{\ell}'_{ab}(A) \mathcal{L}_b + \hat{m}'_{ab} \mathcal{L}_b + O(\mathcal{L}^2)$$

and that

- ii) the equation (2.12) is satisfied with C replaced by C' , $\hat{\ell}$ replaced by $\hat{\ell}'$, and the ghost propagator by $-\hat{m}^{-1}$,

and, provided all the hierarchy of identities expressed by (2.12) are satisfied, then the Lagrangian $\mathcal{L}_1$ under consideration can be written as

$$\mathcal{L}_1 = \mathcal{L}_{\text{invariant}} - \frac{1}{2} (C'_a)^2$$

where $\mathcal{L}_{\text{invariant}}$ is invariant under the transformation

$$A_b = A'_b + \hat{s}'_{bc}(A') \mathcal{L}_c + \hat{t}'_{bc} \mathcal{L}_c$$

To $\mathcal{L}_1$ a Faddeev-Popov part is added which is given by

$$- c_a^\dagger (g \hat{\ell}'_{ab}(A) + \hat{m}'_{ab}) c_b$$

Now that we have stated the theorem we will see how we can obtain tree identities for $\mathcal{L}_1$. From (2.8) we can consider the pole parts at 'n=4' and immediately deduce

$$\begin{aligned}
 & \text{Diagram 1} + \text{Diagram 2} + \text{Diagram 3} \\
 & + \text{Diagram 4} + \text{Diagram 5} + \text{Diagram 6} \\
 & + \text{Diagram 7} = 0
 \end{aligned}
 \tag{2.13}$$

Crosses replace loops in (2.8). In particular a cross above a big black dot denotes replacing a loop involving a source vertex. The factor associated with a cross is just the pole part (at $n=4$) of the bubble being replaced. To illustrate this we will obtain the factor associated with

This is the pole part (P.P.) of

(where p is the momentum flowing in the ghost line). All calculations involved can be performed using

$$\int d^n l \frac{l_\mu l_\nu}{(l^2 - m^2 - i\epsilon)^\alpha} = \frac{1}{2} \frac{i\pi^{\frac{n}{2}}}{(-m^2)^{\alpha - \frac{n}{2} - 1}} \frac{\delta_{\mu\nu}}{\Gamma(\alpha)} \Gamma(\alpha - 1 - \frac{n}{2}) \tag{2.14}$$

and

$$\int d^n l \frac{l_\mu l_\nu l_\lambda l_\delta}{(l^2 - m^2 - i\epsilon)^\alpha} = \frac{1}{4} \frac{i\pi^{\frac{n}{2}}}{(-m^2)^{\alpha - \frac{n}{2} - 2}} \frac{\Gamma(\alpha - 2 - \frac{n}{2})}{\Gamma(\alpha)} \{ \delta_{\mu\nu} \delta_{\lambda\delta} + \delta_{\mu\lambda} \delta_{\nu\delta} + \delta_{\nu\lambda} \delta_{\mu\delta} \} \tag{2.15}$$

We find in the Feynman gauge that

$$\frac{2g^2}{4-n} f^{ebd} f^{ade} \pi^2 (\chi \cdot p) \chi^\alpha \tag{2.16}$$

Here (a, α) is the quantum number associated with $\times$ and b with the ghost line.

If we add (2.4) to (2.13) we indeed have a tree identity for $\mathcal{L}_1$ of the type mentioned in the tree-loop theorem. However, we need to consider the hierarchy of tree Ward identities of $\mathcal{L}_1$. So far we have extracted information out of (2.3) for $q=1$. In fact for $q=2$ we can follow through exactly the same procedure to obtain Fig. 1. This is a rather long expression but we think it is instructive to see it in full. The interesting feature of these identities is that of the non-Lagrangian vertices. In addition to (2.16) we have now

which arises as the pole part of

$$\begin{aligned}
 & \begin{array}{c} (a, \alpha) \\ \diagup \\ \text{triangle} \\ \diagdown \\ (c, \rho) \end{array} + \begin{array}{c} (a, \alpha) \\ \diagup \\ \text{triangle} \\ \diagdown \\ (c, \rho) \end{array} \\
 & + \begin{array}{c} \text{triangle} \\ \diagup \\ (a, \alpha) \end{array} + \begin{array}{c} \text{triangle} \\ \diagdown \\ (a, \alpha) \end{array} \\
 & \text{with momentum } (k) \text{ on the left dashed line}
 \end{aligned} \tag{2.17}$$

The quantities in brackets are associated with the appropriate line or vertex. A straightforward calculation reveals that

$$\begin{array}{c} \diagup \\ \text{triangle} \\ \diagdown \end{array} = \frac{4 g^3 \chi^\alpha \chi^\rho \pi^2 f^{a b c} f^{b c d} f^{c d e}}{4-n} \tag{2.18}$$

Moreover the pole part of (2.17) is not in general zero for 'higher s ' operators. We can write down the Ward identities for $q=3$, $q=4$, etc., but the purpose of showing $q=2$ identity in detail is to obtain some intuition about the nature of these tree identities. It is clear that apart from vertices (having crosses) with a ghost line entering but not leaving, all the other vertices can be readily identified with counter-terms calculated in the usual approach. The question is whether we get any new non-Lagrangian vertices as we go to $q=3$ and higher Ward identities. The only type of graphs that can give rise to such 'cross' vertices are, for example,

and other topological arrangements of the vector prongs. From power counting we know that this is finite and consequently there is no corresponding cross vertex. The higher 'q' Ward identities are clearly of the topological form required for the tree loop theorem. We have now to check that the condition (i) of the tree-loop theorem is satisfied.

Looking at the structure of (2.13) and Fig. 1 as well as the 'higher' tree identities we see that

$$C'_a(A) = C_a(A) = \frac{1}{\sqrt{\xi}} \partial \cdot A_a \quad (2.19)$$

Moreover, from the identities we note that

$$\begin{aligned} & (g \hat{\Gamma}'_{ab}(A) + \hat{m}'_{ab}) \Lambda_b \\ &= (g \hat{\Gamma}_{ab}(A) + \hat{m}_{ab}) \Lambda_b \\ &+ \text{P.P.} \left(\begin{array}{c} \text{Diagram 1} + \text{Diagram 2} \\ + \text{Diagram 3} + \text{Diagram 4} \\ + \text{Diagram 5} + \text{Diagram 6} \\ + \text{Diagram 7} + \text{Diagram 8} \end{array} \right) \end{aligned} \quad (2.20)$$

The infinities in the non-Lagrangian vertices that we have already discussed cannot be absorbed by terms in $\mathcal{L}_1$. They have to be removed by renormalization of $\hat{s}$ and $\hat{t}$, i.e. by a renormalization of the gauge transformation.

Hence

$$\begin{aligned}
 & (g \hat{s}'_{ab}(A) + \hat{t}'_{ab}) \mathcal{L}_b \\
 &= (g \hat{s}_{ab}(A) + \hat{t}_{ab}) \mathcal{L}_b \\
 &+ \text{P.P.} \left(\begin{array}{c}
 \text{Diagram 1: A horizontal dashed line with an arrow pointing right. A curved line (loop) starts from the middle of the dashed line, goes up and then down, ending at the same point. A solid line with a dot at its end is attached to the top of the loop.} \\
 + \text{Diagram 2: A horizontal dashed line with an arrow pointing right. A curved line (loop) starts from the middle of the dashed line, goes up and then down, ending at the same point. A solid line with a dot at its end is attached to the bottom of the loop.} \\
 + \text{Diagram 3: A horizontal dashed line with an arrow pointing right. A triangle is formed by two solid lines meeting at a vertex above the dashed line. A solid line with a dot at its end is attached to the top vertex.} \\
 + \text{Diagram 4: A horizontal dashed line with an arrow pointing right. A triangle is formed by two solid lines meeting at a vertex above the dashed line. A solid line with a dot at its end is attached to the bottom vertex.} \\
 + \text{Diagram 5: A horizontal dashed line with an arrow pointing right. A triangle is formed by two solid lines meeting at a vertex above the dashed line. A solid line with a dot at its end is attached to the left vertex.} \\
 + \text{Diagram 6: A horizontal dashed line with an arrow pointing right. A triangle is formed by two solid lines meeting at a vertex above the dashed line. A solid line with a dot at its end is attached to the right vertex.} \\
 + \text{Diagram 7: A horizontal dashed line with an arrow pointing right. A triangle is formed by two solid lines meeting at a vertex above the dashed line. A solid line with a dot at its end is attached to the top vertex.} \\
 + \text{Diagram 8: A horizontal dashed line with an arrow pointing right. A triangle is formed by two solid lines meeting at a vertex above the dashed line. A solid line with a dot at its end is attached to the bottom vertex.} \\
 + \text{Diagram 9: A horizontal dashed line with an arrow pointing right. A triangle is formed by two solid lines meeting at a vertex above the dashed line. A solid line with a dot at its end is attached to the left vertex.} \\
 + \text{Diagram 10: A horizontal dashed line with an arrow pointing right. A triangle is formed by two solid lines meeting at a vertex above the dashed line. A solid line with a dot at its end is attached to the right vertex.}
 \end{array} \right) \quad (2.21)
 \end{aligned}$$

In order to have condition (i) holding, we need to show schematically that

$$\xi^{-\frac{1}{2}} \partial_\mu [\text{Expression of (2.21)}] = [\text{Expression of (2.20)}] .$$

Fortunately it is necessary only to do combinatorics to verify this. We can see that the identity is valid from noting that

$$\partial_\mu \left(\text{Diagram: A horizontal dashed line with an arrow pointing right. A solid line with a dot at its end is attached to the middle of the dashed line, pointing upwards and to the right.} \right) = \text{Diagram: A horizontal dashed line with an arrow pointing right. A vertical solid line with a dot at its end is attached to the middle of the dashed line, pointing upwards.}$$

Then all the integrals become identical and so condition (i) is satisfied. We now have shown that the tree-loop theorem can be applied to $\mathcal{L}_1$. Consequently the whole Lagrangian at the one-loop level is

$$\mathcal{L}_1 + (\text{gauge part}) .$$

Here the gauge part is

$$-\frac{1}{2} \xi^{-1} (\partial \cdot A)^2 \quad + \text{Faddeev-Popov part corresponding to this gauge fixing.}$$

Explicitly the Faddeev-Popov part is

$$c_a^\dagger (g \hat{l}'_{ab}(A) + \hat{m}'_{ab}) c_b$$

Now the χ -dependent parts of $\hat{l}'$ and $\hat{m}'$ are calculated from the χ -dependent parts of $\hat{s}'$ and $\hat{t}'$:

$$\hat{s}'_{ck} = \frac{4}{n-4} g^2 \pi^2 f^{aes} f^{skd} f^{cde} \chi^\alpha \chi^\rho A_\alpha^\rho + \chi - \text{independent terms} \quad (2.22)$$

and

$$\hat{t}'_{ck} = \frac{2}{n-4} g^2 \pi^2 f^{ekd} f^{cde} \chi^\beta \chi^\rho \partial_\beta + \chi - \text{independent terms} \quad (2.23)$$

Thus the χ -dependent Faddeev-Popov contribution is

$$\begin{aligned} & \partial_\rho c_c^\dagger \left(\frac{4 g^2 \pi^2}{n-4} f^{aes} f^{skd} f^{cde} \chi^\alpha \chi^\rho A_\alpha^\rho \right) c_k \\ & - c_c^\dagger \left(\frac{2 g^2 \pi^2}{n-4} f^{ekd} f^{cde} \right) \chi^\beta \chi^\rho \partial_\beta \partial_\rho c_k \end{aligned} \quad (2.24)$$

The χ -independent terms in $\hat{s}'$ and $\hat{t}'$ reproduce the bare Faddeev-Popov interactions together with the one-loop renormalizations of the ghost propagator and vertex. There is a question whether the invariance of $\mathcal{L}_1$ proved using the tree-loop theorem is a group invariance. A proof can be given using the tree identities, but we shall not reproduce it here.

There are other twist-two operators such as those given in (1.5). The procedure followed here would go through exactly for them. However, in general we would expect in the Ward identities new varieties of infinities. Apart from the expected ones such as

for high enough s , we also have the new non-Lagrangian infinities, which we recall are responsible for the renormalized invariances of the theory. These are of the type given below:

Such infinities would give rise to χ -dependent invariance transformations which are non-linear in A , and so the Lagrangian would no longer be invariant under a simple gauge transformation. We thus see, just from invariance arguments, that Fermi-type gauges do not allow us to consider just only the mixings between gauge-invariant operators. Because of such difficulties we feel that the problem of mixing should be looked at in a different gauge. In the next section we shall discuss such gauges.

3. BACKGROUND FIELD QUANTIZATION

There exist renormalizable gauges in which the gauge renormalization found in Section 2 does not occur. These are the background field gauges⁹⁾ which are based on splitting the vector field into a classical part $A_{\mu}^{cl\ a}(x)$ and a quantum part $Q_{\mu}^a(x)$. It is the quantum parts of the fields which flow in loops, and the classical parts of the field flow in any tree structures attached to loops.

The invariance (1.4) can be regarded as being made up of

$$A_{\mu}^{cl\ a}(x) = A_{\mu}^{cl'\ a}(x) + g f^{acb} A_{\mu}^{cl'\ c}(x) \Lambda^b(x) - \partial_{\mu} \Lambda^a(x) \quad (3.1)$$

$$Q_{\mu}^a(x) = Q_{\mu}^{'a}(x) + g f^{acb} Q_{\mu}^{'c}(x) \Lambda^b(x)$$

Alternatively the invariance (1.4) can be thought of as

$$\begin{aligned} A_{\mu}^{cl\ a}(x) &= A_{\mu}^{cl'\ a}(x) \\ Q_{\mu}^a(x) &= Q_{\mu}^{'a}(x) + g f^{acb} Q_{\mu}^{'c}(x) \Lambda^b(x) - (\bar{D}_{\mu} \Lambda(x))^a = Q_{\mu}^{'a}[\Lambda] \end{aligned} \quad (3.2)$$

where

$$\bar{D}_{\mu}^{ac} = \partial_{\mu} \delta^{ac} + g f^{acb} A_{\mu}^{cl\ b}$$

The gauge-fixing condition is taken to be

$$(\bar{D}_\mu Q^\mu_a) = C_a(x)$$

The attractive feature of such a term is that it is invariant under (3.1) but not under (3.2); so although we are breaking a gauge symmetry we are still maintaining a very helpful degeneracy. This forces the one-loop counter-terms (which are functions of just A_μ^{cl}) to be invariant under (3.1) and consequently gauge invariant. At this stage we must take note of the fact that the classical field concept is defined with respect to sources; so, for example, to (1.3) we must add $J_\mu^a(x)$. We want J_μ^a to be field-independent. However, the equations of motion give

$$\begin{aligned} & - D_{\mu'}^{a'a} F_{\mu'\nu'}^a - \chi^{\mu'} \chi^\lambda D_{\mu'}^{a'a} F_{\nu'\lambda}^a + \chi^{\nu'} \chi^\lambda D_{\mu'}^{a'a} F_{\mu'\lambda}^a - J_{\nu'}^{a'} \\ & = 0 \end{aligned} \quad (3.3)$$

This has the immediate consequence that

$$(D_{\nu'} J_{\nu'})^{a'} = 0$$

and so J is field-dependent, contrary to our starting assumption. Instead of considering the equation (3.3) we can take as our classical field a solution of

$$\begin{aligned} & - D_{\mu'}^{a'a} F_{\mu'\nu'}^a - \chi^{\mu'} \chi^\lambda D_{\mu'}^{a'a} F_{\nu'\lambda}^a + \chi^{\nu'} \chi^\lambda D_{\mu'}^{a'a} F_{\mu'\lambda}^a \\ & - J_{\nu'}^{a'} + (D_{\nu'} \bar{G} D_\sigma J_\sigma)^{a'} = 0 \end{aligned} \quad (3.4)$$

where $\bar{G}$ is defined by

$$(D_\tau D^\tau)^{ab} \bar{G}_{bc}(x, x'; A^{cl}) = \delta_{ac} \delta(x, x')$$

Following standard arguments⁹⁾ we are led finally to the generating functional Z

$$\begin{aligned} & Z(J_\mu, \chi) \\ & = \exp i \left(S(A^{cl}) + \int d^n x J_\mu^a(x) A_\mu^{cl a}(x) \right) \\ & \int \prod_x \prod_\mu \prod_a dQ_\mu^a(x) \delta(\bar{D}_\mu Q^\mu - C) \bar{\Delta}(Q_\mu, A_\mu^{cl}) \\ & \exp -i (\bar{D}_\sigma J^\sigma)_a \bar{G}_{ab} (\bar{D}_\mu Q^\mu)_b \exp \frac{1}{2} i S_{,\mu\nu}^{ab} Q_\mu^a Q_\nu^b (1 + \dots) \end{aligned} \quad (3.5)$$

where S is the action of (1.3), $S_{,\mu\nu}^{ab}$ is $\delta^2 S / \delta A_\mu^a \delta A_\nu^b$, and

$$\Delta(Q^\mu, A_\mu^{cl}) \int d\Lambda \delta(\bar{D}_\mu Q_a^\mu [\Lambda] - C_a) = 1$$

this being the Faddeev-Popov prescription. If we now introduce the weight factor $\exp(-i/2\xi C_a^2)$ and integrate over C (owing to the C -independence of Z) we still retain a term involving JQ . In the background field method we want the external lines to be classical and so this could, in general, be unsatisfactory, at least for off-shell matrix elements. The simplest way of avoiding a JQ term is not to integrate over C but rather to take a particular value $C=0$. This has the effect of replacing $S_{\mu\nu}^{ab} Q_a^\mu Q_b^\nu$ by $Q_a^\mu S_{\mu\nu}^{ab} Q_b^\nu - (\bar{D}_\mu Q_a^\mu)^2$. Unlike in the Fermi-type gauges, the background field gauge condition helps not only to define the propagator, but also gives rise to interaction terms, namely

$$g f^{abc} A_\nu^b Q_\nu^c \partial_\mu Q_a^\mu$$

and

$$g f^{abc} A_\mu^b Q_\mu^c D_\nu Q_a^\nu \quad (3.6)$$

The second of these terms does not contribute owing to the δ -function constraint in (3.5), but the first term certainly leads to additional diagrams for the mixing of the operators $R^{(s)}$. These are

N.V. denotes the new vertex arising from the first interaction given in (3.6).

(We have for clarity not indicated in the diagrams the momentum or quantum numbers of the external legs.)

Owing to the specially attractive property of the background field gauge we know that the sum of these diagrams plus the diagrams of the Fermi gauges

[where no vertex of (3.6) occurs] gives the true gauge-invariant mixing. (For simplicity we have left out the fermion contribution.) Owing to the intricacies of the calculation we shall report the numerical value of the anomalous dimensions of the operators $R^{(s)}$ elsewhere. However, any equality of our number with those obtained from calculations in Fermi-type gauges would seem to be fortuitous.

I would like especially to thank G. 't Hooft for many discussions. C.H. Llewellyn Smith and R.J. Crewther have also made helpful comments.

After this work was completed, H. Kluberg-Stern and J.B. Züger showed me their work on the mixing of the twist-four operator $F_{\mu\nu}^a F_{\mu\nu}^a$ at the Marseilles Conference, June 1974.

Figure 1

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