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LARGE N EXPANSION IN DUAL MODELS *)

G. Veneziano
CERN - Geneva
and
Weizmann Institute of Science,
Rehovot, Israel

ABSTRACT

A "topological" expansion, recently proposed for unitarization of planar dual models, is reformulated as a $1/N$ expansion. The possible connection of this expansion to those used for multiparticle production at high energy is pointed out.

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In a recent paper ¹⁾ we have proposed a new "order of summation" for implementing unitarity in planar dual models.

The first stage of this programme consisted in summing all planar dual diagrams, the second in adding diagrams of cylindrical shape, the successive stages having been left somewhat unspecified.

In this short note we want to clarify and define in a more precise manner this new expansion, which we name the "topological expansion".

Take a planar dual model in which an internal symmetry group $SU(N)$ is introduced via Chan-Paton factors. The n point function is given, in the tree approximation, as a certain dimensionless function of the linear trajectories $\alpha(s_i) = \alpha_i(0) + \alpha' s_i$ times a factor g^{n-2} (where g is the dimensionless dual coupling constant), a certain power of α' that makes the amplitude exhibit the right dimensions, and, finally, a Chan-Paton trace factor.

The unitarized dual n point function A'_n is a function of g and of N and we can consider two types of expansions :

1. expansion in powers of g at N fixed. This is the conventional expansion ²⁾ in the loop number;
2. expansion in powers of g (or of $1/N$) at $g^2 N$ fixed. This is the topological expansion that we want to propose.

It is well known ²⁾ that any (orientable) dual loop diagram can be associated with a two-dimensional surface and, unlike an ordinary Feynman diagram, can be classified in terms of a few numbers. These are :

- a) the number b of boundaries on which the external legs are attached (with a specification of which legs are attached where, and in which cyclic order);
- b) the number w of windows, that is of boundaries with no legs attached. These correspond to planar loops;
- c) the number h of handles. This number is related to the degree of connectedness c of the corresponding surface by $c = 2h + 1$ ($c = 1$ for the sphere, $c = 3$ for the torus, etc.).

The g, N dependent factor associated with a loop contribution to A'_n exhibiting b boundaries, w windows and h handles is easily found to be

$$A'_n(b, w, h) \approx g^{n-2} (g^2)^{b-1+2h} (g^2 N)^w \quad (1)$$

where

$$1 \leq b \leq n \quad ; \quad h, w \geq 0 \quad (2)$$

The conventional expansion is in the loop number $\ell = b + w + 2h - 1$, the power of g^2 . On the other hand, an expansion in powers of g^2 at g^{2N} fixed amounts to sum at each order all diagrams with an arbitrary number of windows (planar loops) for a fixed number of boundaries and handles. Furthermore, for finite n , there is a finite number of inequivalent ways in which the external lines can be attached to the boundaries, each way giving a different structure of internal quantum numbers. Hence, collecting terms with the same structure of boundaries, the resulting $1/N$ expansion is in the number of handles h .

We now discuss the lowest order terms of the expansion :

- 1) $O(g^{n-2})$: $b=1, h=0$: sum of all planar diagrams;
- 2) $O(g^n)$: $b=2, h=0$: sum of cylindrical diagrams ("Pomeron");
- 3) $O(g^{n+2})$: two possibilities occur :
 - 3a) : $b=1, h=1$: planar diagrams with a Pomeron correction;
 - 3b) : $b=3, h=0$: triple Pomeron diagram;
- 4) $O(g^{n+4})$: two possibilities again :
 - 4a) : $b=2, h=1$: a Pomeron-Pomeron cut;
 - 4b) : $b=4, h=0$: a 4-Pomeron contribution.

We now add some comments on each of these terms.

1) The sum of all planar diagrams represents the Reggeon dominated amplitudes. This defines the planar unitary theory of Ref. 1), which satisfies planar duality with non-linear exchange degenerate trajectories dual to resonances of finite width (i.e., phenomenological duality). It is natural to assume that these amplitudes have smooth Regge behaviour on the real axis.

2) This second term can be obtained by sewing together two planes in a non-planar, but appropriate way [as explained in Ref. 1)]. In its vacuum channel this term contains a leading Regge pole, identified with the bare Pomeron pole, whose intercept is expected in the vicinity of $J=1$ according to the arguments of Ref. 3).

3a) This is the first contribution exhibiting a Reggeon-Pomeron cut, i.e., with absorption of the Reggeon exchange.

3b) This is not very relevant for the four-point function but is relevant instead for the six-point function $a+b+c \rightarrow a+b+c$. If we pair the legs on the three boundaries as $a\bar{a}$, $b\bar{c}$, $\bar{b}c$, we have the Mueller diagram for diffraction dissociation of b into c .

4a) This is the two-Pomeron cut diagram and is needed, of course, for consistency, if the bare Pomeron has intercept $\alpha_p(0) \geq 1$.

4b) Similar to 3b), is relevant for a double diffraction dissociation $a+b \rightarrow c+d+X$ analyzed à la Mueller from the eight-point function $a+b+c+d \rightarrow a+b+c+d$.

In order to see whether our expansion can be a good one, consider the four-point function. We have :

$$A'_4 = g^2 \left(A_R + g^4 A_{R \times P^+ \dots} \right) + g^4 \left(A_P + g^4 A_{P \times P^+ \dots} \right) \quad (3)$$

where we have collected terms with the same structure of boundaries. The crucial question then becomes : is the value of g sufficiently small ?

In order to answer this question, one has to analyze in detail the planar unitary theory. Such a theory, although non-unitary, satisfies a set of non-linear discontinuity equations ¹⁾ (or, equivalently, of unitarity sum rules) that we named planar unitarity. The detailed study of these equations ⁴⁾ has led to the conclusion that, if we insist on Regge behaviour on the real axis, one must have the renormalized Reggeon intercept $\alpha_r(0)$ and the renormalized coupling g_r satisfy the equations

$$1 - \alpha_r(0) \approx 0(1) \quad (4)$$

$$\frac{g_r^2 N}{16\pi} \approx 0(1) \quad (5)$$

A way to see the origin of Eq. (5) is to write the unitarity sum rule [see Ref. 1) for the exact definition]

$$\text{Disc}_s A_4^{ab \rightarrow ab} = \sum_c \text{Disc}_{M^2} A_6^{abc \rightarrow abc} \quad (6)$$

Notice that $A_4 \simeq g^2 f(g^2 N)$, $A_6 \approx g^4 \bar{f}(g^2 N)$ with

$$\begin{aligned} f(g^2 N) &= a_1 + a_2 g^2 N + \dots \\ \bar{f}(g^2 N) &= \bar{a}_1 + \bar{a}_2 g^2 N + \dots \end{aligned} \quad (7)$$

The $\sum_1$ in Eq. (6) gives a factor $N/16\pi$ and we get

$$g^2 f(g^2 N) = g^2 \left(\frac{g^2 N}{16\pi} \right) \bar{f}(g^2 N) \quad (8)$$

which has a solution (if any) for $g^2 N/16\pi \approx 1$.

Equation (5) is equivalent to the statement that a planar loop is of the same order of the tree itself. This seems to rule out the possibility of summing the planar loops perturbatively. Notice, however, that our expansion adds non-planar loops and therefore the first correction is of order $(g^2/16\pi)^2 \approx 1/N^2$. Using $N=3$, we get a nice factor $\sim 1/10$ for the first correction.

We thus expect our expansion to be a good one, after a non-perturbative way has been found to calculate A_R , the planar leading term. Actually, our scheme could provide a theoretical justification for various types of Regge expansions presently used in multiparticle production at high energy (e.g., the fireball expansion).

The usual assumption that the input production amplitudes contain no diffraction and no absorption is for us a consequence of planarity. The shadow (overlap function) produced by these production amplitudes generates a bare Pomeron pole with short-range correlations, pionization, etc. [if $\alpha_p(0) > 1$ a rise of σ_{total} too !]. Absorption and diffraction dissociation come next together to order $1/N^2$ and give together a negative and relatively small two-Pomeron cut ⁵⁾.

A final remark concerns the relation of our expansion to the one of 't Hooft ⁶⁾. The two, although formally similar, are physically quite different. 't Hooft starts with an ordinary field theory with a non-Abelian gauge symmetry related to an $SU(M)$ colour group. His $1/M$ expansion selects as leading only planar diagrams with one quark loop on the boundary (to be identified, hopefully, with the dual Born term). Planar and non-planar dual loops are still mixed together at higher order in $1/M$.

We are instead expanding in $1/N$ where $SU(N)$ is the internal symmetry group. If the dual coupling constant g^2 is of order $1/M$, (as 't Hooft argues) or of order $1/N$ (as we say), the planar loop expansion is still bad, because the expansion parameter is $g^2 N \approx O(N/M) \approx O(1)$. Conversely, the "topological expansion" is in $1/N^2$ and is probably a decent one.

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R E F E R E N C E S

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