ON THE FACTORIZATION OF THE POMERANCHUK SINGULARITY
IN THE MUELLER-REGGE DIAGRAMS

CERN LIBRARIES, GENEVA

CM-P00060498

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A B S T R A C T

It is shown that the j plane singularity (Pomeron) which leads to a rising total cross-section like $(\ln s)^\epsilon$, $\epsilon > 0$, cannot be factorizable in the Mueller diagrams for inclusive production of particles with large rapidity differences. The relevance of this result for the theories of a renormalized Pomeron is discussed.

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The discovery of a rising cross-section in pp collisions at the ISR suggests that the scattering amplitude $A(s,t)$ has an asymptotic behaviour of the form $A(s,t) \sim C s^{\alpha_p(t)} (\ln s)^\epsilon$, $s \rightarrow \infty$, $\alpha_p(0) = 1$, $0 < \epsilon \leq 2$. The exact value of ϵ is model dependent and the coupling C is factorized on the basis of general arguments based on t channel unitarity¹⁾. This form corresponds to a rising cross-section like $\sigma_t \sim (\ln s)^\epsilon$ for $s \rightarrow \infty$. We expect that the rising of the total cross-section leads in general to a violation of scaling in the inclusive spectra due to the exchange of this leading singularity in the Mueller diagrams²⁾ of Fig. 1a.

In this note we want to show that Regge singularities leading to a rising cross-section σ_t cannot be factorizable in the Mueller diagrams since this is not allowed by positivity. In fact, consider the model in which the total cross-section rises asymptotically like $\sigma_t(s) = g_A g_B (\ln s)^\epsilon$, $0 < \epsilon \leq 2$. Then, assuming factorization in the Mueller diagrams, the μ particle inclusive distribution is written as follows :

$$\frac{d^\mu \sigma}{dY_1 dY_2 \dots dY_\mu} = g_A g_B \psi^\mu (Y_1 - Y_A)^\epsilon (Y_2 - Y_1)^\epsilon \dots (Y_\mu - Y_{\mu-1})^\epsilon \quad (1)$$

where $\psi \equiv \int \psi(P_\perp) d^2 P_\perp$ and Y_i the longitudinal rapidities of the particles as shown in Fig. 1a ($Y_A < Y_1 < Y_{i+1} < Y_B$ for $i = 1, 2, \dots, \mu$).

From the well-known sum rules :

$$\langle n(n-1) \dots (n-\mu+1) \rangle \sigma_t = \mu! \int_{Y_A}^{Y_B} dY_1 \int_{Y_1}^{Y_B} dY_2 \dots \int_{Y_{\mu-1}}^{Y_B} \frac{d^\mu \sigma}{dY_1 \dots dY_\mu} dY_\mu \quad (2)$$

with $\sigma_t = g_A g_B Y^\epsilon$ ($Y \rightarrow \infty$) and $Y \equiv Y_B - Y_A$, we obtain :

$$\langle n(n-1) \dots (n-\mu+1) \rangle = \mu! \psi^\mu Y^{(1+\epsilon)\mu} \frac{\Gamma^{\mu+1}(1+\epsilon)}{\Gamma(1+\epsilon+\mu+\epsilon\mu)} \quad (Y \rightarrow \infty) \quad (3)$$

In particular :

$$\langle n \rangle = \psi Y^{1+\epsilon} \frac{\Gamma^2(1+\epsilon)}{\Gamma(2+2\epsilon)}, \quad \langle n(n-1) \rangle = 2 \psi^2 Y^{2(1+\epsilon)} \frac{\Gamma^3(1+\epsilon)}{\Gamma(3+3\epsilon)} \quad (4)$$

Finally, we obtain :

$$\lim_{Y \rightarrow \infty} \frac{\langle n \rangle^2}{\langle n^2 \rangle} = \frac{\Gamma(1+\epsilon) \Gamma(3+3\epsilon)}{2 \Gamma^2(2+2\epsilon)} \quad (5)$$

From Eq. (5) and the condition $\langle n \rangle^2 / \langle n^2 \rangle \leq 1$, we obtain the following inequality :

$$\frac{\Gamma(1+\epsilon) \Gamma(3+3\epsilon)}{2 \Gamma^2(2+2\epsilon)} \leq 1 \quad (0 \leq \epsilon \leq 2) \quad (6)$$

It is easy to realize that the condition (6) is satisfied only in the case of a simple pole or a soft singularity at $j = 1$, namely for $\epsilon \leq 0$ (by soft or hard singularities at $j = 1$, we mean these singularities which lead to $\sigma_t \rightarrow 0$ or $\sigma_t \rightarrow \infty$, respectively). This means that there is no factorized singularity in the Mueller diagrams leading to a rising total cross-section like a power of $\ln s$ since this would lead to a contradiction to positivity. We next show that not only the two first sum rules ($\mu = 1, 2$) are incompatible with positivity, but also for large μ equations (3) lead to the same contradiction. Therefore, factorization of a hard Pomeron in the Mueller diagrams can only be accidentally valid in a finite number of diagrams, without any physical significance.

In fact, Eqs. (3) lead to KNO scaling ³⁾ for the multiplicity distribution and the following moment equations are valid asymptotically ($Y \rightarrow \infty$).

$$\int_0^\infty z^\mu f(z) dz = \mu! \frac{\Gamma^\mu(2+2\epsilon)}{\Gamma(1+\epsilon+\mu+\epsilon\mu) \Gamma^{\mu-1}(1+\epsilon)} \quad (7)$$

where $Z \equiv n/\langle n \rangle$, $f(z) = \langle n \rangle \sigma_n / \sigma_t$. For large μ the integral in (7) is bounded as follows :

$$\int_0^\infty z^\mu f(z) dz \geq M \frac{b}{\mu+1} \quad (f(z) \geq 0) \quad (8)$$

where M and b are positive constants independent of μ . On the other hand, Eq. (7) gives for large μ :

$$\int_0^\infty z^\mu f(z) dz \simeq N \frac{\Gamma^\mu(2+2\epsilon)}{\Gamma^{\mu-1}(1+\epsilon)} \mu^{-\epsilon\mu} e^{\epsilon\mu} \mu^{-\epsilon} (1+\epsilon)^{-\mu(1+\epsilon)} \quad (9)$$

where N is a constant independent of μ . It can be easily seen that Eqs. (8) and (9) are not compatible for $\epsilon > 0$, since (9) goes to zero very fast, due to the factor $\mu^{-\epsilon\mu}$.

This argument completes the discussion on the factorizability of a hard Pomeron in the Mueller diagrams and it shows that the factorization of such singularities violates positivity.

The relevance of this result for the models of particle production at high energies can be illustrated with the following examples :

- 1) in the Gribov Reggeon field theory with a strong ϕ^3 coupling, the renormalized Pomeron is a hard singularity ⁴⁾ leading to an increasing total cross-section like, $\sigma_t \sim (\ln s)^\epsilon$ with $\epsilon \gtrsim 1/6$. Moreover, this singularity is factorized at the level of four point functions (elastic scattering) and of six point functions - Fig. 1b - (one-particle inclusive distribution) as it follows from the discussion of multiparticle production within the Gribov calculus ⁵⁾. Our result shows that in this theory of the Pomeron which satisfies t channel unitarity, one must have a drastic modification of the Mueller diagrams for $\mu \geq 2$, through the renormalization mechanism. Otherwise, the theory is not consistent with positivity ;
- 2) within the multiperipheral model, a similar renormalization of the bare Pomeron pole, leading to a rising total cross-section, can be achieved with the mechanism of rescattering in the final state ⁶⁾. In a model of

elastic rescattering of pairs in the final state, suggested by Ciafaloni and Marchesini ⁷⁾, the long range forces (absorption in the final state) which renormalize the Pomeron, disappear from the inclusive distribution and one has only short range inclusive correlations and a constant plateau for $\rho_1 = 1/\mathcal{J}_t(d\sigma/dy)$. In the Mueller approach this means that the diagrams of Fig. 1a, where the renormalized Pomeron is exchanged, fail to give the asymptotic inclusive distributions since a strong modification of the vertices due to rescattering cancels the long range effects of the diagrams of Fig. 1a and this mechanism makes the model consistent with positivity.

ACKNOWLEDGMENTS

One of us (N.G.A.) wishes to thank Professor M. Baker for helpful discussions and Professor G. Preparata for reading the manuscript.

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FIGURE CAPTIONS

Fig. 1a The Mueller diagram for the inclusive distribution of μ identical particles with large rapidity differences.

Fig. 1b The Mueller diagram for the one particle inclusive distribution in the central region.

(a)

(b)

Fig. 1