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A DIFFRACTIVE MODEL INCLUDING SPIN
FOR pp ELASTIC SCATTERING AT VERY HIGH ENERGY

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A B S T R A C T

In this paper we present a theoretical diffractive model for spin dependent proton proton scattering at very high energies using an extended version of the Chou-Yang model. The model describes a wide t range, but in this paper we use it for small t values. We obtain estimates of the ratio of spin dependent to spin independent amplitudes, β , consistent with a previous phenomenological analysis. Next we estimate the asymmetry, Σ , expected in a certain polarized inclusive process at high energy. We then compute the size of polarization effects in pp total cross-section measurements, using polarized beam and target at very high energy. Our estimates of the latter two quantities indicate that their measurement would not provide a sensitive test of Pomeron factorization.

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1. - INTRODUCTION

In this paper we present a theoretical model for proton-proton scattering at ultra high energies, which pays careful attention to spin dependent contributions. In particular we use the eikonal model together with a generalized Chou-Yang conjecture. We study in detail the spin dependent amplitudes in the forward direction.

The motivation for carrying out this analysis is as follows.

(i) Spin dependent amplitudes at high energies have previously been generally considered negligible. However, it was pointed out ¹⁾ on model independent grounds, by how much spin dependent amplitudes could affect, if present, the determination of σ_{tot} at ISR energies. A phenomenological analysis of spin dependence was therefore carried out over the range of energies p_{lab} between 9 and 500 GeV/c, in the very small t region where Coulomb interference experiments can give information on spin dependence ²⁾. It was found, although not conclusively proved, that the ratio of spin dependent amplitudes to spin independent amplitudes, β , [see Ref. 2] was approximately constant in energy, and of order of magnitude 10%. We have therefore wanted to follow this by making purely theoretical predictions for this parameter β , which we do in this paper.

(ii) Polarized beam experiments are already being carried out ³⁾ and hopefully will provide information for an amplitude analysis of nucleon nucleon scattering. The present available data at 6 GeV/c indicate that some spin dependent amplitudes are non-negligible compared with the spin independent part, and it seems urgent to set up a realistic model for these new physical quantities. As a first step we consider a model with purely imaginary amplitudes, valid at very high energy. We would point out that having a polarized proton beam at the ISR is a distinct possibility ⁴⁾.

(iii) In a recent paper ⁵⁾, it was pointed out that all previous tests of Pomeron factorization do not really prove that the Pomeron is a pole, even though factorization appears to hold in the sense of multiplication of residues ⁵⁾. We study a way of answering the question : does the Pomeron factorize ? Namely, one should look at amplitudes where, for kinematic

reasons (e.g., angular momentum constraints), there can be no pole contribution at all. This is precisely the case for high energy spin dependent pp amplitudes in the forward direction : a non-zero ρ indicates the extent of breakdown of true factorization. It is also the case for certain kinematic regions of polarized inclusive processes at high energies, and we will also make some theoretical predictions as to what may be expected experimentally. As we will see, according to this model, true factorization is very difficult to test experimentally.

(iv) Our model, which we use for spin dependence in pp scattering in the forward direction in this paper, could also be used at larger t values to investigate, for instance, breaks in the slope, filling in of the dip in pp scattering and any other structure in the t distribution which we believe due partly to spin dependent effects.

In Section 2 we describe in outline our model and its assumptions, leaving until Section 3 the detailed mathematical exposition. Thus the reader not interested in mathematical detail can skip Section 3. In Section 4 we consider a polarized inclusive process at high energy, and in Section 5 we discuss our results and how they tie in with points (i) to (iv) above.

2. - OUTLINE OF THE THEORETICAL MODEL

In this section we present in outline the physical features and assumptions of the model we use for spin dependent diffractive pp elastic scattering at very high energies.

Our model is a simple extension of the standard eikonal model to incorporate spin, which we remind the reader of here. In the spinless case, the scattering amplitude is given by [see, e.g., Ref. 6].

$$f(t) = \frac{k}{i} \int_0^\infty J_0(b\sqrt{-t}) (e^{i\chi(b)} - 1) b db$$

where k is the centre-of-mass incident momentum, t is the momentum transfer and where $\chi(b)$ is a phase shift which is dependent on the impact parameter b . The Born term f^B is identified as

$$f^B = k \int_0^\infty J_0(b\sqrt{-t}) \chi(b) b db$$

and hence if an assumption is made for the Born term f^B one can calculate the full diffractive amplitude ^{*)} f . The extension of this model to pp scattering with spin is described in detail in Section 3.

For the spinless case, the diffractive Born term f^B is often assumed to be pure imaginary and of the form $f^B \propto i k e^{At/2}$ while in the Chou-Yang model ⁸⁾, a form for $f^B \propto i k (F_1(E))^2$ is assumed, here F_1 is the electric (or magnetic) proton form factor. Of course, for small t , the two parametrizations for f^B are essentially identical ^{**)}. The Chou-Yang conjecture may be envisaged as implying that the same electromagnetic form factor is associated with the particle current whether the interaction is electromagnetic or strong. Hence a natural, but by no means unique, generalization of this model for particles with spin is to take the diffractive Born term proportional to $j_\mu j^\mu$ where j_μ is the electromagnetic current of the particle concerned. For protons,

$$j_\mu = \bar{u} \left(\gamma_\mu F_1 + i \sigma_{\mu\nu} q^\nu \frac{\mu_A}{2m} F_2 \right) u \quad (1)$$

where the u 's are proton spinors, $F_{1,2}$ are the usual form factors [normalized to $F_{1,2}(0) = 1$] μ_A is the anomalous magnetic moment of the proton, and q the momentum transfer between the protons of mass m . It has to be noticed that the value of μ_A determines the size of the spin-flip coupling; in particular if we want to describe np scattering as well as pp, we should consider the isoscalar current only since this ensures isospin zero exchange in the t channel. We will come back below to this important point for the numerical evaluation of the measurable quantities.

^{*)} A so-called λ enhancement factor can be introduced to attempt to account for inelastic intermediate states ⁷⁾. We do not include this factor in our calculations.

^{**)} If a suitable value for "A" is taken.

Our extension of the Chou-Yang conjecture is now clear : the Born term is proportional to $ij j^\mu$ which is then projected out onto the five helicity amplitudes ⁹⁾ φ_1^μ to φ_5^μ to obtain the input Born terms ^{*)} which correspond to f^B in the scalar case above. These helicity amplitudes ^{**)} φ_1^B to φ_5^B are then eikonalized using an extension of the model described above for the scalar case, to yield the final helicity amplitudes φ_1 to φ_5 .

In this paper we only calculate the amplitudes near the forward direction although all the machinery is set up for analysis at $t \neq 0$.

Finally in Section 4 we relate our results for $pp \rightarrow pp$ to polarized inclusive production at ultra high energies, $pp^\uparrow \rightarrow \pi X$ by a simple naïve assumption which has been used elsewhere ¹⁰⁾.

3. - FORMULATION OF THE MODEL

For a given value J of the total angular momentum we know that the two-nucleon system is described in terms of four eigenstates of parity π . One state is a singlet state with $J = \ell$ and $\pi = (-)^\ell$, another one is a triplet state with $J = \ell$ and $\pi = (-)^\ell$ and the last two states are triplet with $J = \ell \pm 1$ and $\pi = -(-)^\ell$. In the subspace of the first two states the S matrix is diagonal and can be expressed in terms of the corresponding conventional phase shifts δ_0^J and δ_1^J , so the usual partial wave helicity amplitudes ⁹⁾ read

$$\begin{aligned} \varphi_1^J - \varphi_2^J &= \frac{e^{i\delta_0^J} - 1}{i} \\ \varphi_3^J - \varphi_4^J &= \frac{e^{i\delta_1^J} - 1}{i} \end{aligned} \quad (2)$$

^{*)} In the spirit of the model, only the leading order term in energy is kept.

^{**)} In fact since, for present purposes we are only interested in the small t region, we approximate $[\mathbb{F}(t)]^2 \sim e^{At/2}$, see Section 3.

In the subspace of the two triplet states with $J = \ell \pm 1$ the S matrix is no longer diagonal because transitions between a $J = \ell + 1$ state and a $J = \ell - 1$ state are allowed. If we consider the states of definite ℓ value it is possible to find the eigenstates of the S matrix which provide a basis where S has a diagonal form in terms of the two eigenphase shifts δ_I^J and δ_{II}^J ¹¹⁾. The S matrix is then obtained in any other basis by means of a rotation which involves the so-called "mixing parameter" ϵ_J ¹¹⁾. Therefore we can express the partial wave helicity amplitudes ⁹⁾ in terms of δ_I^J , δ_{II}^J and ϵ_J and we have in the limit $J \gg 1$

$$\begin{aligned} \phi_1^J + \phi_2^J &= \frac{1}{2i} \left[e^{i\delta_I^J} + e^{i\delta_{II}^J} - 2 - (e^{i\delta_I^J} - e^{i\delta_{II}^J}) \sin 2\epsilon_J \right] \\ \phi_3^J + \phi_4^J &= \frac{1}{2i} \left[e^{i\delta_I^J} + e^{i\delta_{II}^J} - 2 + (e^{i\delta_I^J} - e^{i\delta_{II}^J}) \sin 2\epsilon_J \right] \\ \phi_5^J &= \frac{1}{4i} (e^{i\delta_I^J} - e^{i\delta_{II}^J}) \cos 2\epsilon_J \end{aligned} \quad (3)$$

From Eqs. (2) and (3) we obtain

$$\begin{aligned} \phi_1^J &= \frac{1}{2i} \left[e^{i\chi_+^J} \cos \chi_-^J - 1 - i e^{i\chi_+^J} \sin \chi_-^J \sin 2\epsilon_J + e^{i\delta_0^J} - 1 \right] \\ \phi_2^J &= \frac{1}{2i} \left[e^{i\chi_+^J} \cos \chi_-^J - 1 - i e^{i\chi_+^J} \sin \chi_-^J \sin 2\epsilon_J - e^{i\delta_0^J} \right] \\ \phi_3^J &= \frac{1}{2i} \left[e^{i\chi_+^J} \cos \chi_-^J - 1 + i e^{i\chi_+^J} \sin \chi_-^J \sin 2\epsilon_J + e^{i\delta_1^J} - 1 \right] \\ \phi_4^J &= \frac{1}{2i} \left[e^{i\chi_+^J} \cos \chi_-^J - 1 + i e^{i\chi_+^J} \sin \chi_-^J \sin 2\epsilon_J - e^{i\delta_1^J} \right] \\ \phi_5^J &= \frac{1}{2} e^{i\chi_+^J} \sin \chi_-^J \cos 2\epsilon_J \end{aligned} \quad (4)$$

where

$$\chi_+^J \equiv \frac{\delta_I^J + \delta_{II}^J}{2} \quad \text{and} \quad \chi_-^J \equiv \frac{\delta_I^J - \delta_{II}^J}{2}$$

These equations agree with that of a previous work on the same subject ¹²⁾, but from now on we will not make the same approximations so that in our case we will calculate all helicity amplitudes. The generalization of the eikonal model to the case of spinning particles is achieved by turning the partial wave expansion of the scattering amplitude into an integral over the impact parameter $b = J/k$ where the reduced rotation matrix $d_{\lambda\mu}^J(\theta)$ is replaced by the appropriate ^{*}) Bessel function $J_{|\lambda-\mu|}(k\theta b)$. We therefore have

$$\begin{aligned}
 \varphi_1 &= \frac{k}{2i} \int_0^\infty J_0(b\sqrt{-t}) \left[e^{i\chi_+ \cos \chi_-} - 1 - i e^{i\chi_+ \sin \chi_-} \sin 2\varepsilon + e^{i\delta_0} - 1 \right] b db \\
 \varphi_2 &= \frac{k}{2i} \int_0^\infty J_0(b\sqrt{-t}) \left[e^{i\chi_+ \cos \chi_-} - i e^{i\chi_+ \sin \chi_-} \sin 2\varepsilon - e^{i\delta_0} \right] b db \\
 \varphi_3 &= \frac{k}{2i} \int_0^\infty J_0(b\sqrt{-t}) \left[e^{i\chi_+ \cos \chi_-} - 1 + e^{i\chi_+ \sin \chi_-} \sin 2\varepsilon + e^{i\delta_1} - 1 \right] b db \\
 \varphi_4 &= \frac{k}{2i} \int_0^\infty J_2(b\sqrt{-t}) \left[e^{i\chi_+ \cos \chi_-} + i e^{i\chi_+ \sin \chi_-} \sin 2\varepsilon - e^{i\delta_1} \right] b db \\
 \varphi_r &= \frac{k}{2} \int_0^1 J_1(b\sqrt{-t}) e^{i\chi_+ \sin \chi_-} \cos 2\varepsilon b db
 \end{aligned} \tag{5}$$

We get the Born terms by expanding to leading order in the phase shifts, so

$$\begin{aligned}
 \varphi_1^B(t) &= \frac{k}{2} \int_0^\infty J_0(b\sqrt{-t}) \left[\chi_+ - \chi_- \sin 2\varepsilon + \delta_0 \right] b db \\
 \varphi_2^B(t) &= \frac{k}{2} \int_0^\infty J_0(b\sqrt{-t}) \left[\chi_+ - \chi_- \sin 2\varepsilon - \delta_0 \right] b db \\
 \varphi_3^B(t) &= \frac{k}{2} \int_0^\infty J_0(b\sqrt{-t}) \left[\chi_+ + \chi_- \sin 2\varepsilon + \delta_1 \right] b db \\
 \varphi_4^B(t) &= \frac{k}{2} \int_0^\infty J_2(b\sqrt{-t}) \left[\chi_+ + \chi_- \sin 2\varepsilon - \delta_1 \right] b db \\
 \varphi_r^B(t) &= \frac{k}{2} \int_0^\infty J_1(b\sqrt{-t}) \chi_- \cos 2\varepsilon b db
 \end{aligned} \tag{6}$$

^{*}) The generalization of the eikonal model including spin presented in Ref. 13) is not generally correct because it does not involve the appropriate Bessel functions.

By inverting these equations we get

$$\begin{aligned}
 \chi_+ - \chi_- \sin 2\varepsilon + \delta_0 &= \frac{2}{k} \int_0^\infty J_0(b\sqrt{-t}) \varphi_1^B(t) \sqrt{-t} d(\sqrt{-t}) \\
 \chi_+ - \chi_- \sin 2\varepsilon - \delta_0 &= \frac{2}{k} \int_0^\infty J_0(b\sqrt{-t}) \varphi_2^B(t) \sqrt{-t} d(\sqrt{-t}) \\
 \chi_+ + \chi_- \sin 2\varepsilon + \delta_1 &= \frac{2}{k} \int_0^\infty J_0(b\sqrt{-t}) \varphi_3^B(t) \sqrt{-t} d(\sqrt{-t}) \\
 \chi_+ + \chi_- \sin 2\varepsilon - \delta_1 &= \frac{2}{k} \int_0^\infty J_2(b\sqrt{-t}) \varphi_4^B(t) \sqrt{-t} d(\sqrt{-t}) \\
 \chi_- \cos 2\varepsilon &= \frac{2}{k} \int_0^\infty J_1(b\sqrt{-t}) \varphi_5^B(t) \sqrt{-t} d(\sqrt{-t})
 \end{aligned} \tag{7}$$

What remains to be done now is to parametrize the Born terms ; this will be achieved in the manner outlined in the previous section. In Ref. 2), we have calculated exactly all the amplitudes coming from $j_\mu j^\mu$ (see Section 2 for j_μ), so, to leading order in s , in the small t approximation, and projecting out the helicity amplitudes we get the following contributions from $j_\mu j^\mu$:

$$\begin{aligned}
 \varphi_1(t) &= \varphi_3(t) \sim \alpha \sqrt{s} (F_1)^2 \\
 \varphi_2(t) &= -\varphi_4(t) \sim \alpha t \sqrt{s} \left(\frac{M_A}{2m} \right)^2 (F_2)^2 \\
 \varphi_5(t) &\sim \alpha \sqrt{-t} \sqrt{s} \frac{M_A}{2m} F_1 F_2
 \end{aligned} \tag{8}$$

We approximate to first order in t the form factors F_1 and F_2 , for calculational convenience only, by the same exponential $e^{At/2}$ since for small t , $F_1 \sim F_2$. The diffractive Born terms are assumed to be purely imaginary so we multiply Eqs. (8) by a factor i and an over-all constant to obtain, for the Born terms :

$$\begin{aligned}
 \varphi_1^B(t) &= \varphi_3^B(t) = ic^2 e^{At} \\
 \varphi_2^B(t) &= -\varphi_4^B(t) = it d^2 e^{At} \\
 \varphi_5^B(t) &= i\sqrt{-t} c d e^{At}
 \end{aligned} \tag{9}$$

where c and d are two over-all constants with the normalization condition $d/c = (\mu_A)/2m$ obtained by comparing with Eqs. (8). By solving Eqs. (7) using Eqs. (9), we finally get

$$\begin{aligned}\chi_+ &= \frac{ic^2}{2kA} \left[1 - \frac{d^2}{c^2} \frac{1}{(2A)^2} (2A - b^2) \right] e^{-b^2/4A} \\ \delta_1 &= \frac{ic^2}{2kA} \left[1 - \frac{d^2}{c^2} \frac{b^2}{(2A)^2} \right] e^{-b^2/4A} \\ \delta_0 &= \frac{ic^2}{2kA} \left[1 + \frac{d^2}{c^2} \frac{1}{(2A)^2} (4A - b^2) \right] e^{-b^2/4A} \\ \chi_- &= -\frac{icd}{2kA} \sqrt{b^2 + \frac{d^2}{4c^2}} e^{-b^2/4A} \\ \tan \epsilon &= \frac{d}{2cb}\end{aligned}\tag{10}$$

If we take the value of A as determined by the form factors [see Ref. 2] one finds $A = 5.64 \text{ GeV}^{-2}$. Now, if one takes μ_A equal to the proton anomalous magnetic moment, $d^2/c^2 = 0.913 \text{ GeV}^{-2}$, so we see that χ_+ , δ_1 and δ_0 are comparable since they get their main contributions from $ic^2/2kA$; however, χ_- is much less than the other three phase shifts.

At this stage, since we are interested in evaluating the parameter β [see Ref. 2], we compute the relevant amplitudes. As an illustration we shall give detailed calculations for the spin independent part only. We have, by using Eqs. (5)

$$\begin{aligned}a(t) &= \text{Im} \left(\frac{g_1 + g_3}{2} \right) = \text{Im} \left\{ \frac{1}{2i} \int_0^\infty J_0(b\sqrt{-t}) \left[e^{i\chi_+ \cos \chi_-} - 1 + \frac{e^{i\delta_1} + e^{i\delta_0} - 2}{2} \right] b db \right\} \\ &= \text{Im} \left\{ \frac{1}{2i} \int_0^\infty J_0(b\sqrt{-t}) \left[e^{i\chi_+ \cos \chi_-} - 1 + e^{i\Delta_+ \cos \Delta_-} - 1 \right] b db \right\}\end{aligned}$$

where from Eq. (10)

$$\Delta_+ \equiv \frac{\delta_1 + \delta_0}{2} = \frac{ic^2}{2kA} \left[1 + \frac{d^2}{c^2} \frac{1}{(2A)^2} (2A - b^2) \right] e^{-b^2/4A}$$

and

$$\Delta_- \equiv \frac{\delta_1 - \delta_0}{2} = - \frac{id^2}{4\hbar A^2} e^{-b^2/4A}$$

If we expand to first order in the small quantities χ_- and Δ_- we get

$$a(t) = \text{Im} \left\{ \frac{\hbar}{i} \int_0^\infty J_0(b\sqrt{-t}) [e^{iD_+} \cos D_- - 1] b db \right\} \quad (11)$$

where

$$D_+ \equiv \frac{\chi_+ + \Delta_+}{2} = \frac{ic^2}{2\hbar A} e^{-b^2/4A}$$

and

$$D_- \equiv \frac{\chi_+ - \Delta_+}{2} = - \frac{id^2}{\hbar(2A)^2} (2A - b^2) e^{-b^2/4A}$$

By expanding e^{iD_+} and since D_- is small, $\cos D_- \sim 1$, we obtain from Eq. (11)

$$a(t) = -\hbar \sum_{m=1}^{\infty} \frac{1}{m!} \int_0^\infty J_0(b\sqrt{-t}) \left(-\frac{c^2}{2\hbar A}\right)^m e^{-b^2/4A} b db$$

which yields after a straightforward integration

$$a(t) = -\frac{c^2}{u} \sum_{m=1}^{\infty} \frac{(-u)^m}{m m!} e^{At/m} \quad (12)$$

where

$$u \equiv \frac{c^2}{2\hbar A}$$

In a similar manner, we find, to leading order in t

$$f(t) = \frac{\text{Im}(\varphi_2 - \varphi_1)}{2} = \left(\frac{d^2}{c^2}\right) \frac{d^2}{4A^2} u \sum_{m=1}^{\infty} \frac{(-u)^m m e^{At/m+2}}{(m+2)^2 m!} \quad (13)$$

and

$$e(t) \equiv \text{Im} \left(\frac{G_2}{2} \right) = -\frac{d^2}{2A} \sum_{m=1}^{\infty} \frac{(-u)^m e^{At/m+1}}{(m+1)^2 m!} \quad (14)$$

We also find the interesting relation

$$G_r(t) = \frac{d}{c} \sqrt{-t} \left(\frac{G_1(t) + G_2(t)}{2} \right) \quad (15)$$

In the limit $t=0$ the above sums can be carried out analytically and we get

$$\begin{aligned} a(0) &= -\frac{c^2}{u} \left[\text{Ei}(-u) - c - \ln u \right] \\ f(0) &= \left(\frac{d^2}{c^2} \right) \frac{d^2}{4A^2} \left[3 - 3e^{-u} - ue^{-u} + 2\text{Ei}(-u) - 2c - 2\ln u \right] \\ e(0) &= -\frac{d^2}{2A} \frac{1}{u} \left[\text{Ei}(-u) - c - \ln u - e^{-u} + 1 \right] \end{aligned} \quad (16)$$

where $c = 0.577$ is the Euler constant and $\text{Ei}(-u)$ is the exponential integral function.

Let us recall the definition of β

$$\beta = \sqrt{\frac{2 e(0)^2 + f(0)^2}{a(0)^2}} \quad (17)$$

that we can now calculate in terms of $a(0)$, $f(0)$ and $e(0)$ given by Eqs. (16). At this stage we will allow some freedom in the model and we will not take the value of A determined by the form factor but we will keep the predicted ratio d/c . From Eqs. (15), we see that β is only a function of u and A which remains to be determined. Our two experimental data input will be firstly σ_{tot} by means of the optical theorem which says that $a(0) = (k/4\pi)\sigma_{\text{tot}}$, i.e.,

$$2A \left(\frac{c^2}{u} \right) \left[\text{Ei}(-u) - c - \ln u \right] = \frac{\sigma_{\text{tot}}}{4\pi} \quad (18)$$

and secondly the slope $2B$ of the elastic cross-section at $t=0$, which is taken to be that of the spin independent amplitude $a(t)$. So we have from Eqs. (12)

$$2B = 2A \frac{\sum_{n=1}^{\infty} \frac{(-u)^n}{n! n!}}{Ei(-u) - \gamma - \ln u} \quad (19)$$

By solving numerically Eqs. (18) and (19) in terms of $2B$ and σ_{tot} we obtain for $2B = 10 \text{ GeV}^{-2}$ and $\sigma_{\text{tot}} = 40 \text{ mb}$, $u = 1.23$ and $A = 4.37 \text{ GeV}^{-2}$. These values correspond ^{*)} to $e(0)/a(0) = 2.5\%$, $f(0)/a(0) = 0.15\%$ and $\beta = 3.5\%$ with $d/c = 0.956$.

If, on the other hand, one takes isoscalar magnetic and electric form factors, one gets $d/c = -0.06$, and the numbers just quoted are then all quite negligibly small. We have applied the model to πN elastic scattering ¹⁴⁾, and compared it with recent amplitude analyses ¹⁵⁾ of the data. We find an effective d/c (for diffractive contributions) given by $d/c = -0.15$. This value is not unreasonable in view of the uncertainties involved in introducing spin and isospin in a Chou-Yang model. Using this value $d/c = -0.15$, we find $e(0)/a(0) = 0.06\%$, $\beta(0) = 0.1\%$ (f/a is negligible).

Finally, we conclude this section by noting that the eikonal model presented here has strong justification theoretically because it can be easily checked that the 4×4 T matrix used in the above analysis is

$$T = e^{i(\chi_D + \chi_{ND})} - 1$$

^{*)} With these parameters we find $D_+(b=0) = i(1, 2, 3)$ and indeed the remaining phase shift parameters are smaller than 1 ; in fact $\chi(b=0) = \Delta_-(b=0) = D_-(b=0) = (-id^2)/(4kA^2) = -i(0.13)$, so we consider our approximations justified.

where $\chi_{D^{(*)}}$ a multiple of the unit matrix, and χ_{ND} is a general matrix with elements $\ll 1$. Hence on expansion, one finds

$$T = e^{i\chi_D} - 1 + i\chi_{ND} e^{i\chi_D}$$

and if the Born term is taken as

$$T_B = i\chi_D + i\chi_{ND}$$

the χ 's may be eikonalized ^{**))} in a unique way ^{***))}.

4. - POLARIZED INCLUSIVE PROCESS AT HIGH ENERGY

In this section, we consider the process $pp^\dagger \rightarrow \pi X$, where X is anything, at a very high energy and using a polarized proton target. We are only interested in the kinematic region where the pion is produced in the beam fragmentation region. In the Mueller picture, the polarization asymmetry Σ , is given by

$$\Sigma = \frac{\text{Disc } \langle + | M | - \rangle}{\text{Disc } \langle + | M | + \rangle}$$

where $\langle \lambda | M | \mu \rangle$ is the forward amplitude for $pp\pi \rightarrow pp\pi$ and the protons have helicity λ, μ in the initial and final states, respectively. The model we take for $\langle \lambda | M | \mu \rangle$ at high energy is shown in the Figure.

Since Σ is a ratio, one may obtain a crude estimate of its size in the same way as in Ref. 10). There, it was argued that the ratio could reliably be obtained from taking the p trajectory to its pole, and cancelling the $pp\pi$ vertices appearing in the Figure. In our case, this gives

$$\Sigma = \frac{g_2}{g_1 + g_3}$$

*) See previous footnote.

**) We thank L. Castillejo for discussion of this point.

***) Except perhaps for the one very small amplitude $\varphi_1 - \varphi_3$ which we believe is given to leading order by Eq. (13).

This reduces to $\Sigma \approx \varphi_2/2\varphi_1 \approx \beta/\sqrt{2}$ since φ_3 is approximately equal to φ_1 and "f" is small compared with "e" (see Section 4).

As pointed out in Ref. 10), Σ must be proportional to $p_{\perp}$ for kinematic reasons. This is not found because we have normalized to the nucleon poles. Hence we introduce this factor with the same justification as in Ref. 10), in the following way. Using $t=(q_p-q_{\pi})^2$ one has the kinematic relation at high energy :

$$p_{\perp} = \sqrt{|x| [m^2(1-|x|) - t]}$$

where x is the usual Feynman scaling variable, $p_{\perp}$ is the transverse momentum of the outgoing π , and m is the nucleon mass. As in Ref. 10) we normalize the amplitude to the particle pole (here the nucleon pole), while introducing the $p_{\perp}$ factor. Thus we have to multiply the above expression for Σ by

$$\frac{p_{\perp}}{\sqrt{|x| [m^2(1-x) - m^2]}} \sim \frac{p_{\perp}}{m}$$

for x not too far from one. Hence the final answer for Σ is

$$\Sigma \approx \beta \frac{p_{\perp}}{\sqrt{x}}$$

We emphasize that this is a crude first estimate of the size of this parameter Σ , and more refined calculations are left to a later stage.

5. - DISCUSSION

Using the values quoted in Section 3 for σ_{tot} , the forward slope $2B$, and $d/c = -0.15$, we find $\beta(t=0) = 0.1\%$ which appears not consistent with the phenomenological analysis²⁾ referred to in part (i) of the Introduction. However, one has to remember that some amplitudes increase very rapidly away from t exactly zero. We have calculated β for t ($\sim 10^{-2} \text{ GeV}^2$) in the Coulomb interference region, where the analysis was

done, and find $\beta \sim 3\%$ [using Eqs. (13), (14), (15)]. Hence within the experimental errors, the theoretical prediction for β is not inconsistent with the phenomenological analysis.

In part (iii) of the Introduction, questions were raised as to how one might sensitively test factorization of the Pomeron. If factorization holds $\beta = 0$ strictly but we find only minor deviations from zero due to non-factorizing contributions. An experimental check would be to measure total cross-sections with polarized beam and target. If these are transverse to the beam direction, we predict at high energies

$$\frac{\sigma_{tot}^{\uparrow\uparrow} - \sigma_{tot}^{\uparrow\downarrow}}{\sigma_{tot}^{\uparrow\uparrow} + \sigma_{tot}^{\uparrow\downarrow}} = e(0)/a(0) = 0.06\%$$

while factorization gives zero for this quantity. If the experimental quantity turns out to be larger, this would indicate a larger value for d/c .

Another test of factorization is in the polarized inclusive experiment $pp^{\uparrow} \rightarrow \pi X$, where we have predicted in Section 4

$$\Sigma \approx \frac{\beta(0) p_{\perp}}{\sqrt{s}}$$

at very high energies. Hence we predict for $p_{\perp} \sim 2$ GeV/c $\Sigma \sim 0.1\%$, while a factorizing Pomeron predicts $\Sigma = 0$. Hence it appears that asymmetry, Σ , does not provide a sensitive test of factorization.

In fact it appears ¹⁴⁾ that the most sensitive tests of Pomeron factorization are obtained by measuring C, K, D parameters in polarized beam and target experiments away from $t=0$ at very high energies. Such experiments have already yielded results at 6 GeV/c, and at this energy we expect our diffractive model to give essential features of the data, while real parts must, of course, be included at these energies.

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