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AMPLITUDES AND EXCHANGE MECHANISMS FOR $\pi N \rightarrow f_0 N$

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A B S T R A C T

Production amplitudes for the f_0 meson and for the lower spin background are obtained from an analysis of the angular distribution moments in 17 GeV/c $\pi^- p \rightarrow \pi^- \pi^+$ data. The energy dependence is studied by also analysing data at 4 GeV/c. The exchange mechanisms responsible are discussed and compared with those for $\pi N \rightarrow \rho N$.

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1. INTRODUCTION

Data on processes such as $\pi N \rightarrow \pi\pi N$ with a substantial π exchange contribution have most commonly been used to extract $\pi\pi$ scattering information by extrapolation to the π exchange pole. Another aspect of the data which is of considerable interest is the study of the production amplitudes themselves as functions of momentum transfer, helicity, $\pi\pi$ mass, etc. Of particular interest is the comparison of the production amplitudes for different $\pi\pi$ states -- for example, between f_0 meson and ρ meson production, or between the f_0 meson and the S-wave $\pi\pi$ background beneath it. Such a study of production amplitudes as the mass and spin of an external particle is varied provides an entirely new insight into the nature of the exchange mechanisms.

Rather than analyse the data directly in terms of exchange mechanisms, it has proved preferable to isolate the production amplitudes themselves in as model independent a way as possible. They then provide a solid base for future discussion of exchange contributions. For ρ meson production, the data yield directly ¹⁾ amplitude combinations ^{*)} S_0 , P_0 , P_{1-} and P_{1+} as well as the relative phases $\delta(S_0, P_0)$ and $\delta(P_0, P_{1-})$. These production amplitude combinations as functions of s and t can be compared with exchange models. As well as π and A_2 quantum number exchanges, the presence of a further contribution (commonly called the cut C) is also required by the data. This contribution does not vanish at $t'=0$ and is required theoretically in absorption models. Detailed discussion of the relative contributions of π , A_2 and C , and hence of models for their exchange contributions, has been given ^{2),3),4)}. We shall discuss the comparison of such analyses of ρ production with results for higher mass $\pi\pi$ production.

Previous attempts to analyse the production amplitudes in the f_0 region have either only produced bounds on quantities of interest ⁵⁾, or have over-simplified the amplitude structure ⁶⁾, or have fitted exchange mechanisms directly to the data ⁷⁾. We make here a systematic t -independent analysis of the amplitude structure of di-pion production in the f_0 mass region. Suitable data exist at 4 and 17.2 GeV ^{5),6),8)} and some comparisons of energy dependence can be made. Having obtained the production amplitudes, an analysis of the cut contributions is given and we discuss and compare exchange contributions as a function of $\pi\pi$ mass and angular momentum.

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*) The notation is $L_\lambda \pm$ where L is $\pi\pi$ angular momentum, λ is the helicity (in either s or t channel helicity frame as appropriate) and $\pm$ refers to natural/unnatural parity exchange. The amplitude combinations are summed over the nucleon flip and non-flip couplings.

2. AMPLITUDE ANALYSIS

For convenience we work with the combinations of $\pi^- p \rightarrow (\pi^- \pi^+) n$ helicity amplitudes which have definite asymptotic exchange naturality

$$L_{\mu\sigma}^{\lambda\pm} = \epsilon_\lambda \left[H_{\mu\sigma}^{L\lambda} \pm (-1)^{\lambda+\mu} H_{\mu\sigma}^{L-\lambda} \right] \quad (1)$$

where L , λ are the spin and helicity of the $\pi\pi$ system, μ , σ are the nucleon helicities, and where the normalization factor ϵ_λ is $\frac{1}{2}$ for $\lambda=0$ and $1/\sqrt{2}$ for $\lambda \neq 0$. Note that there is no $L^{\lambda+}$ amplitude for $\lambda=0$. The relationship between the moments [as defined in Ref. 8)] of the $\pi^- \pi^+$ angular distribution and the amplitudes $L^{\lambda\pm}$ is

$$\langle Y_M^J \rangle = \frac{1}{\sqrt{4\pi}} \sum_{\substack{L L' \\ \lambda \lambda'}} \left[\frac{(2L+1)(2L'+1)}{2J+1} \right]^{1/2} \langle L L' 00 | J 0 \rangle (-1)^{\lambda'} \langle L L' \lambda - \lambda' | J M \rangle \cdot \text{Re } S_{\lambda\lambda'}^{L L'} \quad (2)$$

with

$$\text{Re } S_{\lambda\lambda'}^{L L'} = \frac{1}{8\epsilon_\lambda \epsilon_{\lambda'}} \sum_{\mu\sigma} \text{Re} \left(L_{\mu\sigma}^{\lambda+} L_{\mu\sigma}^{\lambda'+*} + L_{\mu\sigma}^{\lambda-} L_{\mu\sigma}^{\lambda'-*} \right) \quad (3)$$

We use the notation $L_{\lambda\pm}$ when spin coherence at the nucleon vertex is assumed

$$|L_{\lambda\pm}|^2 = |L_{++}^{\lambda\pm}|^2 + |L_{+-}^{\lambda\pm}|^2$$

We shall concentrate on the mass region $1.2 < M_{\pi\pi} < 1.34$ GeV which contains the dominant f meson contribution. In this region S , P and D wave $\pi\pi$ states are known to contribute. There are thus nine independent couplings to the $\pi\pi$ system, which with the choice of helicity flip or non-flip at the nucleon vertex yields 18 (complex) production amplitudes. The 15 measurable moments, $\langle Y_M^J \rangle$ with $J \leq 4$, $0 \leq M \leq J$, are clearly insufficient for a full amplitude analysis. Nucleon polarization data would be needed to continue in a model-independent manner. However, as in the ρ mass region, reasonable assumptions enable considerable progress to be made. In fact, to a very good approximation, we can reduce the complexity of the f region amplitude analysis essentially to that encountered in the ρ region. We describe how this may be done and then we present a more complete analysis.

The assumption of spin coherence at the nucleon vertex (in practice this implies that the flip coupling is dominant) reduces the problem to nine amplitudes. The $\pi\pi$ P wave in the f mass region is relatively small and has a phase which varies rather rapidly with $M_{\pi\pi}$ relative to the S and D waves. This makes it difficult to incorporate in an amplitude analysis, and a considerable simplification occurs if we neglect $|P|^2$ relative to $|S|^2$ and $|D|^2$, and just consider the even J moments which contain no SP or PD interference terms. This leaves six amplitude components (S_0 , D_0 , D_{1-} , D_{2-} ; D_{1+} , D_{2+}) to be determined from the nine even J moments. Further simplification occurs by working in the t channel frame where empirically the $M > 2$ moments are very small. This implies that $D_{2\pm} \approx 0$. D_0 , D_{1-} and D_{1+} can then be determined ⁶⁾ from $\langle Y_0^4 \rangle$, $\langle Y_1^4 \rangle$ and $\langle Y_2^4 \rangle$ if D_0 and D_{1-} are assumed to be relatively real. A more general approach is to determine S_0 , D_0 , D_{1-} and D_{1+} from the seven even J moments with $M \leq 2$. Using either of the $M=2$ moments to eliminate $|D_{1+}|$ the remaining moments can be solved analytically t by t for the magnitudes and relative phases of S_0 , D_0 and D_{1-} . As in the ρ region, two solutions are obtained in one of which D_0 and D_{1-} are phase coherent for $-t \lesssim 0.3 \text{ GeV}^2$. The two solutions are shown in Fig. 1, where the amplitudes represent the averages of the values obtained from 17 GeV/c data ⁸⁾ in the t bins with $-t < 0.1 \text{ GeV}^2$. In the ρ region, the physical solution is the one with P_0 and P_{1-} phase coherent, and so here we select the phase coherent solution for a more detailed study.

The results of an analysis including the $\lambda = 2$ amplitudes and the $M > 2$ moments are shown by the points in Fig. 2. Here the six amplitude moduli and two relative phases (S_0 to D_0 to D_{1-}) are determined t by t from the nine even J moments. The assumption of phase and spin coherence between D_{2-} and D_0 and between D_{2+} and D_{1+} is not very strong, since $D_{2\pm}$ are very small in the t channel helicity frame. As a check on the size of the P wave contributions and on the stability of the analysis to the inclusion of the P wave amplitudes, we performed a t dependent fit to the full 15 odd and even J moments for $-t < 0.2 \text{ GeV}^2$. We used a simple parametrization for the π exchange and non-evasive contributions with $P_{1+} = P_{1-}$ and with phase coherence between amplitudes with the same L. The result is shown by the curves in Fig. 2 and agrees well with the amplitudes obtained in the t by t analysis and shows the relative size of the P wave components.

Data at 4 GeV/c also exist for $\pi^+n \rightarrow \pi^+\pi^-p$ for the same $\pi\pi$ mass region ⁵⁾. Since the statistics are less, we fixed the relative phase of D_0 and S_0 at 38° , and D_0 and D_{1-} to be relatively real. Such

phases are compatible with the 17 GeV/c data (see Fig. 2) at smaller t values. We present the results in the form of the amplitude fractions to the total cross-section as a function of t in Fig. 3. For comparison, the curves in Fig. 3 are interpolations of the same quantities determined from the 17 GeV/c data. Studying the energy dependence in this way avoids the problems of the normalization of $d\sigma/dt$ which arise in comparing bubble chamber and counter data from deuterium and hydrogen targets. The main features of interest are the relative decrease of $\sigma_U^D(\lambda=1,2)$, in particular D_{1-} , with energy for $-t \sim 0.3 \text{ GeV}^2$ and the relative increase of σ_N^D with energy for $-t \gtrsim 0.4 \text{ GeV}^2$.

3. EXCHANGE MECHANISMS AND COMPARISONS

It is possible to relate theoretically the exchange mechanisms for different $\pi\pi$ mass intervals -- the quantum numbers exchanged will be the same, only the external mass and spin change. For Regge pole exchange in particular, one can define Regge-particle scattering amplitudes and apply duality techniques to relate resonance production contributions. Using either Regge + $\pi \rightarrow \pi + \pi$ finite energy sum rules ^{9),10)} or the explicit dual model structure ^{3),11)} for the Regge recurrence vertex, one obtains relationships between the Regge pole exchange (π or A_2) contributions to $\pi N \rightarrow \rho N$, $\pi N \rightarrow f_0 N$ etc. A comparison between the D wave di-pion resonance (f_0) and the S wave background in the same mass region, is less constrained by a duality approach.

To apply such considerations, the most suitable amplitude is $\lambda_S = 0$ which is expected to be pure π exchange. A comparison of the t dependences of P_0 for the ρ and D_0 for the f_0 is made in Fig. 4 in the s channel helicity frame. Here D_0 has slightly steeper slope than P_0 . Using

$$L_0 = g_L \sqrt{-t} e^{a(t-\mu^2)} / (\mu^2 - t) \quad (4)$$

we find $a(\rho) = 4.4 \text{ GeV}^{-2}$, $a(f_0) = 5.2 \text{ GeV}^{-2}$; while the expectation ¹¹⁾ of the explicit dual factor is $a(f) - a(\rho) = 0.9 \text{ GeV}^{-2}$. Neither P_0 nor D_0 show any clear evidence for a dip at larger t , although both become relatively small for $|t| \gtrsim 0.4 \text{ GeV}^2$. In the t channel frame, as shown by the curves in Fig. 2, the t dependence of S_0 and of D_0 are found to be identical for small t .

Another expectation of the Regge-particle duality approach is that natural parity exchange contributions arising from A_2 exchange have a dependence on $\pi\pi$ mass relative to π exchange contributions as

$$\langle M_{\pi\pi}^2 \rangle^{\alpha_{\pi} - \alpha_{A_2}} \sim M_{\pi\pi}^{-1}$$

This relative suppression of natural parity exchange in going from ρ to f_0 production is illustrated in Fig. 4. The energy dependence of the natural parity exchange fraction shown in Fig. 3 is consistent with what is found for ρ production²⁾. Thus natural parity exchange, in s dependence at fixed $M_{\pi\pi}^2$ and also in $M_{\pi\pi}^2$ dependence at fixed s , suggests dominance of a relatively high-lying trajectory (A_2 exchange) for $|t| > 0.4$ while for smaller t the trajectory is close to those present in unnatural parity exchange.

An analysis of D_{1-} and D_{2-} (as of P_{1-} in the ρ region) is conventionally made in the s channel helicity frame. In Fig. 5, the behaviour of the s channel helicity amplitudes is shown for $|t| < 0.2$. The π exchange contribution is dominantly in $\lambda_t = 0$ and vanishes as $\sqrt{-t}$ due to the γ_5 coupling at the nucleon vertex. Crossing of the π exchange to the s channel then gives contributions to D_{1-} vanishing as $-t'$ and to D_{2-} vanishing as $(-t')^{\frac{3}{2}}$. Figure 5 shows clearly, however, the presence of contributions to D_{1-} and D_{2-} behaving as $(-t')^0$ and $(-t')^{\frac{1}{2}}$, respectively, as allowed by angular momentum conservation. These additional contributions, conventionally called cuts C_1 and C_2 , can be studied by separating them from the π exchange component. Near $t=0$, this is unambiguous, since π exchange vanishes as t relative to the cut contributions (neglecting corrections due to $t_{\min}$ for simplicity)

$$D_{1-} \sim \frac{\sqrt{3}}{M_{\pi\pi}} g_D \left[\frac{-2t}{\mu^2 - t} - C_1 \right] \quad (5)$$

$$D_{2-} \sim \frac{\sqrt{3}}{M_{\pi\pi}^2} g_D \sqrt{-t} \left[\frac{-2t}{\mu^2 - t} - C_2 \right] \quad (6)$$

The values $C_1 = C_2 = 1$ are then equivalent to replacing $-t/(\mu^2 - t)$ by $-\mu^2/(\mu^2 - t)$ in the original π pole expressions for H_{+-}^1 and H_{+-}^2 and this is known as "Poor Man's Absorption" or "the Williams model" and corresponds to completely suppressing the low partial wave (central) spike in the impact parameter representation of π exchange in the limit of no form factor or Regge slope ($a=0$).

Whereas in the ρ region $C_1(P) \sim 1.0$ is found empirically, the values $C_1(D) \sim 0.57 \pm 0.02$ and $C_2(D) \sim 0.9 \pm 0.2$ are found for f_0 production (Figs. 2 or 5) and correspond to a relative suppression of the cut

effect. No clear understanding of this suppression exists. The cut $C_1(P)$ in the f_0 region is very poorly determined. The value 0.5 ± 0.2 is consistent with the value of $C_1(D)$.

Unfortunately, away from $t=0$, the determination of C_1 and C_2 depends on the detailed evaluation of the π exchange contribution. For example, in Eqs. (5) and (6) for D_{1-} and D_{2-} [and also Eq. (4) for D_0] the crossing factors have been approximated by setting $\cos \chi = 1$, $\sin \chi = 2\sqrt{-t/M_{\pi\pi}}$ [as is done in the Williams' model ¹²⁾]. These π exchange contributions to D_{1-} and D_{2-} are shown by the dotted lines in Fig. 5, whereas if the correct crossing factors are included the dashed lines are obtained. In the ρ region, the approximation of the crossing factors is suggested by the data (viz. the constancy of S_0/P_0 in the s channel) and coincides with the π contributions obtained in a dual model calculation which contains $\lambda_t = 1$, as well as the dominant $\lambda_t = 0$, π couplings. In the f region the inclusion of the dual model $\lambda_t = 0, 1, 2$ couplings ^{3), 11)} leads to the π contributions shown by the dot-dashed curves. Between these three different prescriptions for π exchange helicity structure, the resulting differences in the t dependence of the cuts can be seen clearly in Fig. 5. Such cuts also contribute to D_{1+} and D_{2+} along with A_2 exchange which provides further constraints on exchange mechanism decompositions.

A further feature of note is the strong energy dependence of the relative fraction of D_{1-} plus D_{2-} shown in Fig. 3 between 4 and 17.2 GeV/c. Such a decrease of D_{1-} with energy implies a lower-lying trajectory -- and since the π contribution should have the same energy dependence in the D_{1-} as in D_0 , this must be due to the cut contribution C_1 . Since C_1 subtracts from π in D_{1-} , this implies that C_1 must have a somewhat higher effective trajectory than the π contribution. This is indeed what is expected in absorption-type models.

An aspect of the data which is rather striking in the t channel frame, is the absence of significant $M > 2$ moments and even the small size of the $M = 2$ moments for $|t| < 0.2$. This leads to substantial $M \geq 2$ moments in the s channel frame and to a relatively complicated picture of cuts in $\lambda_s = 1$ and $\lambda_s = 2$ as well as the π and A_2 contributions. The similarity of the t dependence of S_0 and D_0 is yet another reminder of the relative simplicity of the data in the t channel frame. No theoretical explanation exists for this suppression of $\lambda = 2$ in the t channel as well as for the approximate equality of D_{1+} and D_{1-} and the simple behaviour of S_0 relative to D_0 .

4. CONCLUSIONS

We have presented f_0 region amplitudes which show the following features:

- i) the π exchange contributions in $\lambda_s = 0$ and a natural parity exchange contribution which are related to those for ρ production by the Regge-particle duality approach;
- ii) cut contributions which are relatively weaker for $\lambda_s = 1$ than in the ρ region, while an additional $n=1$ non-evasive contribution is needed. This latter contribution is indeed expected in an absorption approach where central partial waves are suppressed;
- iii) the apparent simplicity of the data in the t channel helicity frame is quite remarkable and is somewhat of a coincidence from an s channel absorption viewpoint.

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Figure captions

- Fig. 1 : The two solutions for the magnitudes and relative phases of the unnatural parity exchange t channel amplitudes in the f_0 region for the interval $-t < 0.1 \text{ GeV}^2$.
- Fig. 2 : Magnitudes and relative phases of the t channel f_0 region amplitudes $L_{\lambda\pm}$ at $17.2 \text{ GeV}/c$. The points with error bars are from a t independent fit to the even J moments while the curves are from a t dependent fit ($-t < 0.2 \text{ GeV}^2$) to all moments. The signs of the amplitude magnitudes have been chosen to facilitate display.
- Fig. 3 : Fractions of contributions of different $L_{\lambda\pm}$ to the cross-section at $17 \text{ GeV}/c$ (curves) and $4 \text{ GeV}/c$ (points with error bars). The fractions are represented by the differences between the curves (points) and add up to 1. The error bars represent the errors on σ_U^S , $\sigma_U^D(\lambda=0)$ and σ_N^D , respectively, reading from the top of the figure. σ_U^S and σ_N^D are independent of choice of helicity frame, while $\sigma_U^D(\lambda=0)$ and $\sigma_U^D(\lambda=1,2)$ are divided according to their s channel helicity.
- Fig. 4 : A comparison between ρ and f_0 region amplitudes at $17 \text{ GeV}/c$ where D_0 and P_0 (s channel) have been normalized to each other at small t to have equal π exchange contributions. The resulting natural parity exchange contributions P_+ and $D_N = \{|D_{1+}|^2 + |D_{2+}|^2\}^{\frac{1}{2}}$ are then compared.
- Fig. 5 : Magnitudes of the s channel f_0 mass region amplitudes $L_{\lambda\pm}$ at $17.2 \text{ GeV}/c$. The continuous curves come from a fit to the t range $0.01 < -t < 0.1 \text{ GeV}^2$ including S , P and D wave $\pi\pi$ contributions. Phase coherence between amplitudes with the same L was used and relative phases found were 38° for S_0 to D_0 , and 42° with a larger error for P_0 to D_0 . Taking D_0 to be pure π exchange, the contributions of π exchange to D_{1-} and D_{2-} are shown for various assumptions:
- i) crossing exactly from a pure $\lambda_t = 0$ coupling gives the dashed lines;

- ii) approximating the crossing factors by $\cos \chi = 1$ and $\sin \chi = 2\sqrt{-t/M_{\pi\pi}}$ gives the dotted lines; and
- iii) the dual model vertex coupling gives the dashed-dotted lines. The differences between these π contributions and the resultant D_1- or D_2- amplitudes gives the cut contributions.

FIG.1

T CHANNEL $\pi^- \pi^+$ PRODUCTION AMPS IN THE f REGION

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FIG. 2

FIG. 3

FIG. 4

FIG.5