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A STRONG CONSTRAINT ON CHIRAL SYMMETRY BREAKING
AT SHORT DISTANCES

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A B S T R A C T

It is shown that if chiral symmetry is broken by a piece of the Hamiltonian density which transforms as a member of a $(3, \bar{3}) + (\bar{3}, 3)$ multiplet of $SU(3) \times SU(3)$ and has scale dimension d , then it is necessary to have $d \geq 3$ to avoid inconsistency with low energy theorems which follow from spontaneous symmetry breaking. This is demonstrated using previously derived inequalities for $K_{\ell 3}$ decay, combined with a detailed analysis of the $SU(3) \times SU(3)$ structure of the short-distance expansion for products of current divergences. Asymptotically free gauge theories and some theories with an infrared stable hadronic fixed point can survive this constraint, and for the latter the cross-section for e^+e^- annihilation into hadrons should approach its asymptotically scale-invariant behaviour quite slowly. A softened quark-gluon model with $d = 3$ is excluded.

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1. INTRODUCTION

It was recently demonstrated by V. Baluni and the author ¹⁾ that the low-energy theorems of current algebra, combined with PCAC and $(3, \bar{3}) + (\bar{3}, 3)$ chiral symmetry breaking, strongly suggest the necessity for subtractions in the dispersion relations for the propagators of current divergences. Such subtractions imply a high-energy behaviour corresponding to a scale dimension ²⁾, d , of the chiral symmetry breaking Hamiltonian density, which is not less than 2.

In this paper we further analyze the relation between chiral symmetry breaking at short distances and low-energy current algebra, using Wilson's framework of broken scale invariance ²⁾. We find that the inequalities of Ref. 1) imply $d \geq 3$ if spontaneously broken $SU(3) \times SU(3)$ is an approximate symmetry of the strong interactions. We further demonstrate that these inequalities are inconsistent with the softened coloured quark-octet vector gluon model ^{3),4)}, which has $d = 3$, and from which Fritzsche and Leutwyler ⁵⁾ have abstracted leading and next-to-leading short-distance singularities of current and divergence operator products.

Our result is consistent with asymptotically free non-Abelian gauge theories of hadronic interactions which involve quarks ⁶⁾ and also with theories in which hadronic interactions at short distances are governed by an infra-red stable fixed point of the renormalization group ^{7),8)}, and in which d is (anomalously) greater than 3.

If it is a theory of the latter type which governs the asymptotic behaviour of hadronic processes initiated by a highly virtual photon, our lower bound for d is consistent with a slow approach of the cross-section for e^+e^- annihilation into hadrons (via one-photon exchange) to the asymptotic behaviour required by broken scale invariance ⁸⁾.

In Section 2 we prove that for $d < 3$ and $(3, \bar{3}) + (\bar{3}, 3)$ chiral symmetry breaking, there are two convergent sum rules for combinations of the propagators of current divergences. The first of these sum rules follows directly from Wilson's analysis of the Weinberg sum rules ²⁾. The proof of the second sum rule requires a detailed examination of the dependence on the Gell-Mann, Oakes and Renner ⁹⁾ parameter of the next-to-leading c-number short-distance singularity in the operator product expansion of two current divergences. In Section 3 it is shown that the saturation of the second sum rule by one- and two-pseudoscalar meson contributions violates the inequalities derived in Ref. 1). We conclude that either $d \geq 3$ or the strong interactions are far from the chirally symmetric limit. In Section 4 it is found that the sum rules are not true in the free quark

model (for which $d = 3$). They remain convergent, but contain anomalous constants. Nevertheless, we are able to show that the softened quark-gluon model is also inconsistent with the analysis of Ref. 1). We conclude in Section 5 with a discussion of two classes of theory that escape our strictures: asymptotically free gauge theories and theories in which d is anomalously greater than 3, due to the hadronic coupling constants sitting at an infra-red stable fixed point. Emphasis is given to the different predictions for e^+e^- annihilation.

2. DERIVATION OF SUM RULES

We are concerned with the propagators of current divergences:

$$\Delta_i^{(5)}(q^2) = i \int d^4x e^{iq \cdot x} \langle 0 | T(D_i^{(5)}(x) D_i^{(5)}(0)) | 0 \rangle, \quad (2.1)$$

where

$$D_i^{(5)}(x) = \partial_\mu J_i^{\mu(5)}(x) \quad (2.2)$$

and $J_i^{\mu(5)}(x)$, $i = 1, 8$, are the vector (axial) currents, whose charges,

$$Q_i^{(5)}(x_0) = \int d^3x J_i^{0(5)}(x), \quad (2.3)$$

generate chiral $SU(3) \times SU(3)$. We assume that the chiral symmetry breaking Hamiltonian density has scale dimension ²⁾ $d < 4$ and belongs to a $(3, \bar{3}) + (\bar{3}, 3)$ multiplet ⁹⁾ of $SU(3) \times SU(3)$. Thus one may write

$$\mathcal{H}_{csb}(x) = (\mu)^{4-d} (u_0(x) + c u_8(x)), \quad (2.4)$$

with

$$\begin{aligned} [Q_n(x_0), u_i(x)] &= i f_{nij} u_j(x) \\ [Q_n(x_0), v_i(x)] &= i f_{nij} v_j(x) \\ [Q_n^5(x_0), u_i(x)] &= i d_{nij} v_j(x) \\ [Q_n^5(x_0), v_i(x)] &= -i d_{nij} u_j(x) \end{aligned} \quad (2.5)$$

for $n = 1, 8$; $i = 0, 8$. We have introduced the mass parameter μ , which determines the magnitude of chiral symmetry breaking, whilst d determines

the strength of short-distance singularities. Note that no assumption has been made about the existence of an axial $SU(3)$ singlet current. Such a current exists in the quark model ⁴⁾, enabling the equal time commutation relations of Eqs. (2.5) to be extended to $n=0$. With such an extension, we could derive sum rules additional to those given below -- but that would be wrong. There are serious problems with a ninth pseudoscalar Goldstone boson ⁴⁾. We return to this point when discussing the operator product expansion.

Equations (2.2) and (2.3) imply that

$$D_i^{(5)}(x) = -i [Q_i^{(5)}(x_0), \mathcal{H}_{CSB}(x)], \quad (2.6)$$

which enables us to derive the Ward identity

$$\Delta_i^{(5)}(0) = -\langle 0 | [Q_i^{(5)}(x_0), [Q_i^{(5)}(x_0), \mathcal{H}_{CSB}(x)]] | 0 \rangle. \quad (2.7)$$

Using the commutation relations (2.5), one obtains

$$\Delta_i^{(5)}(0) = -\frac{1}{8} \mu^{4-d} \text{Tr} a [\Lambda_i^{(5)} \langle 0 | \lambda_0 u_0(0) + \lambda_8 u_8(0) | 0 \rangle], \quad (2.8)$$

with

$$\begin{aligned} \Lambda_i &= [\lambda_i, [\lambda_i, \lambda_m]], \\ \Lambda_i^5 &= \{ \lambda_i, \{ \lambda_i, \lambda_m \} \}, \\ \lambda_m &= \lambda_0 + c \lambda_8. \end{aligned} \quad (2.9)$$

Now there are only four linearly independent non-zero divergence propagators: $\Delta_4, \Delta_3^5, \Delta_4^5, \Delta_8^5$ and thus two independent linear combinations which vanish at zero momentum, by virtue of Eq. (2.8). We choose the combinations

$$\Delta_I \equiv \Delta_4 + \Delta_4^5 - \frac{1}{4} [\Delta_3^5 + 3 \Delta_8^5] \quad (2.10)$$

$$\Delta_{II} \equiv \Delta_4 + \left[\frac{3c}{2\sqrt{2}-c} \right] \Delta_4^5 - \left[\frac{3c}{2(\sqrt{2}+c)} \right] \Delta_3^5 \quad (2.11)$$

which satisfy

$$\Delta_{I,II}(0) = 0. \quad (2.12)$$

The first combination involves $D_8^5(x)$ but not c and vice versa for the second. Since the matrix elements of $D_8^5(x)$ and the value of c are not directly measurable, neither combination is ideal. It turns out, however, that Δ_{II} is directly related to the analysis of Ref. 1).

The asymptotic behaviour of $\Delta_{I,II}(q^2)$ depends upon the short-distance behaviour of $\langle 0 | \phi_\alpha(x) \phi_\beta(0) | 0 \rangle$, where ϕ_α stands for u_i or v_i , $i=0,8$. From Eqs. (2.1) and (2.4)-(2.6), we find

$$\Delta_{I,II}(q^2) = i(\mu^{4-d})^2 \int d^4x e^{iq \cdot x} \langle 0 | T(P_{I,II}^{\alpha\beta} \phi_\alpha(x) \phi_\beta(0)) | 0 \rangle, \quad (2.13)$$

with combinations

$$\begin{aligned} P_I^{\alpha\beta} \phi_\alpha \phi_\beta &= \frac{1}{6} [4v_u v_u - v_3 v_3 - 3v_8 v_8] \\ &+ \frac{\sqrt{2}c}{6} [3(v_8 v_8 - \sqrt{2}v_0 v_8) - v_3 v_3 - 2v_u v_u] \\ &+ \frac{c^2}{12} [9u_u u_u + v_u v_u - v_3 v_3 - 3(2v_0 v_0 + v_8 v_8 - 2\sqrt{2}v_0 v_8)], \end{aligned} \quad (2.14)$$

$$\begin{aligned} P_{II}^{\alpha\beta} \phi_\alpha \phi_\beta &= \frac{\sqrt{2}c}{2} [v_u v_u - v_3 v_3] \\ &+ \frac{c^2}{4} [3u_u u_u - 2v_3 v_3 - v_u v_u]. \end{aligned} \quad (2.15)$$

It remains to consider how the short-distance singularities of $\langle 0 | \phi_\alpha(x) \phi_\beta(0) | 0 \rangle$ depend upon the decomposition of the operator product into irreducible representations of $SU(3) \times SU(3)$. To answer this question we follow Wilson's treatment ²⁾ of the Weinberg sum rules. Consider the effect on the operator product expansion of a scale breaking Hamiltonian density

$$\mathcal{H}_{SB}(x) = (\mu)^{4-d} (u_0(x) + c u_8(x)) + (\lambda)^{4-\Delta} \delta(x) \quad (2.16)$$

where δ is an $SU(3) \times SU(3)$ singlet with scale dimension $\Delta < 4$. (In general, there is no reason to expect $\Delta = d$.) First we give the short-distance behaviour of the "skeleton theory" ($\mu = \lambda = 0$), which is both

scale and chirally invariant. The operator product expansion is

$$\frac{u_i(x)u_j(0)}{v_i(x)v_j(0)} = \delta_{ij} [C_1(x) + C_2(x)\delta(0)] + g_{ijk} C_3(x) u_k(0) + \dots \quad (2.17)$$

$$u_i(x)v_j(0) = g_{ijk} C_3(x) v_k(0) + \dots$$

where the terms omitted (e.g. $x^\mu J_\mu$) have vanishing vacuum expectation values. The coefficients of u and v in the expansion are fixed by $SU(3) \times SU(3)$ invariance. Scale invariance determines the singularities:

$$C_1(x) \sim 1/x^{2d}, \quad C_2(x) \sim 1/x^{2d-\Delta}, \quad C_3(x) \sim 1/x^d, \quad (2.18)$$

(where the symbol $\sim$ indicates the short-distance behaviour, up to possible logarithms). Chiral invariance determines the tensor g_{ijk} , up to an overall constant. Equations (2.5) and (2.17) require that g_{ijk} be totally symmetric and satisfy

$$f_{nim} g_{mjk} + f_{njm} g_{mki} + f_{nkm} g_{mij} = 0 \quad (2.19)$$

$$d_{nim} g_{mjk} + d_{njm} g_{mki} + d_{nkm} g_{mij} = R_{ijk} \delta_{no} \quad (2.20)$$

where all indices run from 0 to 8, and R_{ijk} is an arbitrary symmetric tensor [undetermined since Eqs. (2.5) are restricted to $n=1,8$]. Using standard identities¹⁰⁾ for the f 's and d 's, it is a matter of simple but tedious algebra to show that

$$g_{ijk} = \alpha \left[d_{ijk} - \sqrt{\frac{3}{2}} (\delta_{io} \delta_{jk} + \delta_{jo} \delta_{ki} + \delta_{ko} \delta_{ij} - 3 \delta_{io} \delta_{jo} \delta_{ko}) \right] \quad (2.21)$$

which satisfies Eqs. (2.19) and (2.20) with

$$R_{ijk} = 2\alpha \delta_{io} \delta_{jo} \delta_{ko}. \quad (2.22)$$

It is apparent that extending Eqs. (2.5) to $n=0$ requires $R_{ijk}=0$ and hence $g_{ijk}=0$. Thus, if $U(3) \times U(3)$ is broken only by a piece of the Hamiltonian density with scale dimension less than 4, $C_3(x)$ will be less singular than indicated in Eq. (2.18). We shall not make use of this fact since we are concerned with the scale dimension of $SU(3) \times SU(3)$ breaking.

Next consider the effect upon the expansion (2.17) induced by the term $(\lambda)^{4-\Delta} \delta(x)$ in $\mathcal{K}_{SB}(x)$. Since δ is an $SU(3) \times SU(3)$ singlet, the only effect is to introduce terms in $C_{1,2,3}(x)$ which are less singular by some power of $(\lambda x)^{4-\Delta}$. The $SU(3) \times SU(3)$ structure stays the same. The effect of $\mathcal{K}_{CSB}(x)$ is radically different in that it induces extra terms in the operator product expansion, less singular by some power of $(\mu x)^{4-d}$ and with an $SU(3) \times SU(3)$ structure which depends upon that power. For our purpose, it is sufficient to consider the leading effect of $SU(3) \times SU(3)$ breaking in the expansion of Eq. (2.17).

The most general expression for $\langle 0 | \phi_\alpha(x) \phi_\beta(x) | 0 \rangle$ is therefore

$$\begin{aligned} \langle 0 | \frac{u_i(x) u_j(0)}{v_i(x) v_j(0)} | 0 \rangle &= \delta_{ij} A(x) \mp \\ &[g_{ij0}(B(x) + \langle u_0 \rangle C(x)) + g_{ij8}(C B(x) + \langle u_8 \rangle C(x))] \\ &+ T_{ij}^\pm(x), \end{aligned} \quad (2.23)$$

where the representation content of $T_{ij}^\pm$ is unrestricted and

$$A(x) \sim 1/x^{2d}, \quad B(x) \sim \mu^{4-d}/x^{3d-4}, \quad C(x) \sim 1/x^d, \quad (2.24)$$

$$T_{ij}^\pm(x) \sim (\mu^{4-d})^2 / x^{4d-8}, \quad \text{for } d \geq 2, \quad (2.25)$$

$$\sim \mu^{4-d} \langle u_{0,8} \rangle / x^{2d-4}, \quad \text{for } d \leq 2. \quad (2.26)$$

We see that a sufficient condition for $\langle 0 | P^{\alpha\beta} \phi_\alpha(x) \phi_\beta(x) | 0 \rangle$ to behave according to Eq. (2.25) or (2.26) is that it contains no $(1,1)$ nor $(3,\bar{3}) + (\bar{3},3)$ piece in its $SU(3) \times SU(3)$ decomposition. That this condition is not necessary will become apparent.

It is clear that $\Delta_{I,II}$ contain no $(1,1)$ piece, since by inspection of Eqs. (2.14) and (2.15)

$$P_{I,II}^{\alpha\beta} \delta_{\alpha\beta} = 0. \quad (2.27)$$

Using Eqs. (2.21) and (2.23), it is straightforward to show that neither $B(x)$ nor $C(x)$ contribute to $P_I^{\alpha\beta} \phi_\alpha(x) \phi_\beta(0)$ -- each of the three terms in Eq. (2.14) contains neither a $(1,1)$ nor a $(3,\bar{3}) + (\bar{3},3)$ piece. This is

not true for the combination of Eq. (2.15). We find that

$$\langle 0 | P_{II}^{\alpha\beta} \phi_\alpha(x) \phi_\beta(0) | 0 \rangle = \frac{\sqrt{6}\alpha c}{4} [c(B(x) + \langle u_0 \rangle C(x)) - (cB(x) + \langle u_8 \rangle C(x))] + O(1/x^{4d-8}, 1/x^{2d-4}). \quad (2.28)$$

Thus although Δ_{II} contains a $(3, \bar{3}) + (\bar{3}, 3)$ piece, it receives no contribution from $B(x)$. We were only able to show this using the explicit forms of Eqs. (2.21) and (2.23). At the price of introducing the parameter c in combinations of operator products, more singularities may be cancelled than is apparent from purely group theoretic arguments.

We obtain our results

$$\langle 0 | P_I^{\alpha\beta} \phi_\alpha(x) \phi_\beta(0) | 0 \rangle \sim 1/x^{4d-8} \text{ or } 1/x^{2d-4}, \quad (2.29)$$

$$\langle 0 | P_{II}^{\alpha\beta} \phi_\alpha(x) \phi_\beta(0) | 0 \rangle \sim 1/x^{4d-8} \text{ or } 1/x^d, \quad (2.30)$$

using Eqs. (2.24) to (2.26).

The Fourier transform of Eq. (2.13) now gives the asymptotic behaviour of $\Delta_{I,II}(q^2)$. Defining absorptive parts by

$$\rho_i^{(5)}(s) \equiv \lim_{\epsilon \rightarrow 0^+} \frac{1}{\pi} \text{Im} \Delta_i^{(5)}(s+i\epsilon), \quad (2.31)$$

we obtain

$$\begin{aligned} \Delta_I(s) &\sim s^{2(d-3)}, \text{ for } d \geq 2, \\ &\sim s^{d-4}, \text{ for } d \leq 2; \end{aligned} \quad (2.32)$$

$$\begin{aligned} \Delta_{II}(s) &\sim s^{2(d-3)}, \text{ for } d \geq 8/3, \\ &\sim s^{\frac{1}{2}(d-4)}, \text{ for } d \leq 8/3; \end{aligned} \quad (2.33)$$

and from the Ward identity (2.12) we conclude

$$d < 7/2 \Rightarrow \Delta_{I,II}(s) = s \int \frac{ds' \rho_{I,II}(s')}{s'(s'-s)}, \quad (2.34)$$

$$d < 3 \Rightarrow 0 = \int \frac{ds}{s} \rho_{I,II}(s), \quad (2.35)$$

$$d < 5/2 \Rightarrow 0 = \int ds \rho_I(s), \quad (2.36)$$

$$d < 2 \Rightarrow 0 = \int ds \rho_{II}(s). \quad (2.37)$$

Our two sum rules are given by Eqs. (2.10), (2.11) and (2.35). For $d < 3$

$$\int \frac{ds}{s} [\rho_4(s) + \rho_4^s(s) - \frac{1}{4}(\rho_3^s(s) + 3\rho_8^s(s))] = 0, \quad (2.38)$$

$$\int \frac{ds}{s} [\rho_4(s) + (\chi-1)\rho_4^s(s) + (1/\chi-1)\rho_3^s(s)] = 0, \quad (2.39)$$

where

$$\chi \equiv \frac{\sqrt{2+c}}{\sqrt{2-c/2}}. \quad (2.40)$$

Finally, in this Section, we discuss related work on the short-distance expansion. As far as we are aware, our results for Δ_{II} are new (and unexpected on the basis of group theory alone). The result of Eq. (2.32) for Δ_I follows, however, quite simply from Wilson's original analysis of current propagators²⁾. Wilson observed that the combination

$$\begin{aligned} & 4 [J_4^r(x) J_4^v(0) + J_4^{rs}(x) J_4^{vs}(0)] \\ & - [J_3^r(x) J_3^v(0) + J_3^{rs}(x) J_3^{vs}(0)] \\ & - 3 [J_8^r(x) J_8^v(0) + J_8^{rs}(x) J_8^{vs}(0)] \end{aligned}$$

transforms as a member of a $(1,27) + (27,1)$ multiplet and thus at short distances behaves as

$$[(\mu x)^{4-d}]^4 / x^6, \text{ for } d \geq 2,$$

or

$$\langle u_{0,8} \rangle [(\mu x)^{4-d}]^3 / x^{6-d}, \text{ for } d \leq 2,$$

since at least four $(3, \bar{3}) + (\bar{3}, 3)$ multiplets are required to make a member of a $(1, 27) + (27, 1)$ multiplet. Differentiating twice with respect to x (to obtain the combination of divergence propagators Δ_I) and Fourier transforming, we obtain Eq. (2.32) directly. We have been unable to find an analogous argument for Eq. (2.33). More recently, Auvil and Deshpande ¹¹⁾ have derived various sum rules under the assumption that $d < 5/2$. Equation (2.36) is a linear combination of three of their sum rules. These authors did not, however, give the explicit d dependence of the $1/x^{3d-4}$ term in the short-distance expansion, and thus from their analysis all one can say is that $d < 8/3$ is a sufficient condition for Eq. (2.39). We have shown that $d < 3$ is sufficient, which is an essential improvement, of considerable relevance to the question of whether the short-distance expansion is that of an asymptotically free quark coloured vector gluon theory (for which $d=3$).

3. CONSTRAINT ON d

In Ref. 1), a rigorous lower bound was obtained for the contribution of s -wave, $I = \frac{1}{2}$, $K\pi$ states to the sum rule (2.39). Using the elastic $K\pi$ phase shift and current algebra predictions for the scalar $K_{\rho 3}$ form factor, and assuming $(3, \bar{3}) + (\bar{3}, 3)$ chiral symmetry breaking with $d < 2$, Baluni and the author were able to show that this lower bound is badly inconsistent with approximate pion and kaon pole dominance of axial divergence propagators. The tentative conclusion was that $d \geq 2$. The analysis of the previous Section enables us to extend this result to $d \geq 3$, making the further assumption of the validity of Wilson's operator product expansion.

Let

$$A = \int_{(m_K + m_\pi)^2}^{\infty} \frac{ds}{s} [\rho_4(s)]_{K\pi}, \quad (3.1)$$

then A is given in terms of the scalar $K_{\rho 3}$ form factor, since

$$[\rho_4(s)]_{K\pi} = \frac{3}{64\pi^2} \left[1 - \frac{(m_K + m_\pi)^2}{s} \right]^{\frac{1}{2}} \left[1 - \frac{(m_K - m_\pi)^2}{s} \right]^{\frac{1}{2}} |F(s)|^2, \quad (3.2)$$

where

$$F(s) = (m_K^2 - m_\pi^2) f_+(s) + s f_-(s) \quad (3.3)$$

and $f_\pm(s)$ are the standard K form factors, normalized such that $f_+(0) = 1$ in the limit of $SU(3)$ symmetry. Note that A is finite, provided the scalar form factor is asymptotically vanishing; a condition almost certainly fulfilled whatever the value of d and necessarily for $d < 2$.

Let us first consider the situation when $d < 2$. Then all propagators obey unsubtracted dispersion relations and

$$\Delta_4(0) > A > 0. \quad (3.4)$$

Also, the sum rule (2.39) reduces to the Ward identity

$$\Delta_4(0) = (1-\chi) \Delta_4^S(0) + (1-1/\chi) \Delta_3^S(0), \quad (3.5)$$

which implies

$$\Delta_4(0) \leq \left[[\Delta_4^S(0)]^{\frac{1}{2}} - [\Delta_3^S(0)]^{\frac{1}{2}} \right]^2, \text{ for } \chi \geq 0, \quad (3.6a)$$

or

$$\Delta_4(0) \geq \left[[\Delta_4^S(0)]^{\frac{1}{2}} + [\Delta_3^S(0)]^{\frac{1}{2}} \right]^2, \text{ for } \chi \leq 0. \quad (3.6b)$$

We reject the possibility that χ is negative by appealing to continuity with the $SU(3)$ limit ($\Delta_4(0) = 0$) where inequality (3.6b) is plainly false. [Alternatively, one may rely upon the Gell-Mann, Oakes and Renner determination⁹⁾ $\chi \approx (m_\pi/m_K)^2$ or the "weak PCAC" scheme of Brandt and Preparata⁹⁾ with $\chi \approx 0.7$.] We next assume approximate pion and kaon PCAC in the form

$$\begin{aligned} \Delta_3^S(0) &\approx (m_\pi f_\pi)^2, \\ \Delta_4^S(0) &\approx (m_K f_K)^2. \end{aligned} \quad (3.7)$$

Inequalities (3.4) and (3.6a) then yield

$$(m_K f_K - m_\pi f_\pi)^2 > A. \quad (3.8)$$

Now consider the situation when $2 \leq d < 3$. The dispersion relations for $\Delta_1^{(5)}(0)$ need one subtraction, but the sum rule (2.29) is still true. The dominant low-energy contributions are the $K\pi$ states in $\rho_4(s)$ and the K and π poles in $\rho_4^5(s)$ and $\rho_3^5(s)$ respectively. The pole contributions are of first order in chiral symmetry breaking [assuming that $SU(3) \times SU(3)$ is spontaneously broken by an octet of pseudoscalar Goldstone bosons]. At high energies the integrand vanishes faster than $s^{-5/3}$ if $2 \leq d < 8/3$. For $8/3 \leq d < 3$ it behaves as s^{2d-7} , but is of fourth order in chiral symmetry breaking. In either case it should be a good approximation to neglect the three meson and higher contributions to $\rho_{3,4}^5(s)$ and the four meson and higher contributions to $\rho_4(s)$. We therefore obtain

$$A \approx (1-\chi)(m_K f_K)^2 + (1-1/\chi)(m_\pi f_\pi)^2, \quad (3.9)$$

in lieu of Eq. (3.5). Again assuming χ is positive, we recover inequality (3.8), for $d < 3$. Note that without the detailed analysis of Section 2, would have had to assume $d < 8/3$ in deducing (3.9). The explicit c dependence of Eq. (2.23) is crucial in cancelling $B(x)$ in Eq. (2.28) and ensuring that there is no $s^{(3d/2)-5}$ piece in the integrand.

The upper bound (3.8) is to be compared with the lower bounds for A given in Ref. 1). The archetype of such lower bounds was given by Okubo¹²⁾ and by Li and Pagels¹³⁾, and follows merely from the analyticity of $F(s)$ and the normalization condition $F(0) = (m_K^2 - m_\pi^2)f_+(0)$. It is

$$A \geq \frac{3(m_K + m_\pi)}{256\pi} \left[f_+(0)(m_K - m_\pi)(m_K^{\frac{1}{2}} + m_\pi^{\frac{1}{2}}) \right]^2 \quad (3.10)$$

$$\approx 0.17 (m_K f_K - m_\pi f_\pi)^2$$

with $f_\pi = 0.66 m_\pi$, $f_K/f_\pi f_+(0) = 1.27 \pm 0.03$. In Eq. (3.10) we have used the Ademollo-Gatto theorem¹⁴⁾, which says $f_+(0) \approx 1$, the renormalization being of second order in $SU(3)$ breaking. The sensitivity to the value of $f_+(0)$ is in any case very slight. Inequalities (3.8) and (3.10) are clearly consistent. The lower bound (3.10) is improved by a factor of 3-4 using the elastic $K\pi$ phase shift and either the Callan-Treiman relation¹⁵⁾ or the slope of $F(s)$ as measured by the high statistics SLAC-Santa Cruz experiment¹⁶⁾. This is just on the borderline of possibility if we substitute $\chi \approx (m_\pi/m_K)^2$ in Eq. (3.9), and is still consistent with inequality

(3.8), which is true for arbitrary positive χ and $d < 3$. However, it was shown in Ref. 1) that the lower bound for A exceeds the upper bound (3.8) by at least a factor of 2, if it is also demanded that the low-energy theorem of Dashen et al. ¹⁷⁾

$$F'(m_\pi^2 + m_\pi^2) \approx \frac{1}{2} \left[\frac{f_\pi}{f_\pi} - \frac{f_\pi}{f_\pi} \right] \quad (3.11)$$

be satisfied to within 40%. The leading corrections to Eq. (3.11) are calculable given $(3, \bar{3}) + (\bar{3}, 3)$ symmetry breaking and are found to be only about 10% ¹⁷⁾. We therefore conclude that the assumptions of spontaneously broken $SU(3) \times SU(3)$ and of $(3, \bar{3}) + (\bar{3}, 3)$ chiral symmetry breaking require $d \geq 3$. To maintain $d < 3$ it would be necessary either to abandon $(3, \bar{3}) + (\bar{3}, 3)$ chiral symmetry breaking, or seriously to question the validity of the expansion in powers of chiral symmetry breaking which leads to the low-energy theorems of Callan and Treiman ¹⁵⁾ and of Dashen et al. ¹⁷⁾ and the saturation of convergent sum rules by low-energy pseudoscalar meson contributions.

4. SUM RULES IN THE QUARK MODEL

We now consider whether the sum rules (2.38) and (2.39) are true in the simplest model with $(3, \bar{3}) + (\bar{3}, 3)$ symmetry breaking and $d = 3$: the free quark model. The Lagrangian is

$$\mathcal{L}(x) = \bar{\psi}(x) [i \not{\partial} - m] \psi(x),$$

where m is the quark mass matrix,

$$m = \begin{bmatrix} m_u & & \\ & m_d & \\ & & m_s \end{bmatrix},$$

and $SU(2)$ invariance is ensured by requiring $m_u = m_d$. $SU(3) \times SU(3)$ and scale invariance are broken by

$$\mathcal{H}_{S3}(x) = \mathcal{H}_{CS3}(x) = \bar{\psi}(x) m \psi(x).$$

We define currents by

$$J_i^{\mu(5)}(x) = \bar{\psi}(x) \frac{\lambda_i}{2} \gamma^\mu (\gamma^5) \psi(x)$$

and scalar and pseudoscalar densities by

$$\begin{aligned} u_i(x) &= \bar{\psi}(x) \lambda_i \psi(x), \\ v_i(x) &= \bar{\psi}(x) \lambda_i (i \gamma^5) \psi(x). \end{aligned}$$

The commutation relations (2.5) are satisfied (including $n=0$). On comparison with Eqs. (2.4) and (2.40), we find

$$d = 3, \quad \mu = (m_s + 2m_u)/\sqrt{6},$$

$$C = -\sqrt{2} \left(\frac{m_s - m_u}{m_s + 2m_u} \right), \quad \chi = \left(\frac{2m_u}{m_s + m_u} \right).$$

Let $\sigma_i^{(5)}(s)$ denote the free quark result for $\rho_i^{(5)}(s)$. It is straightforward to show that the quark-antiquark contributions at high energy give ⁵⁾

$$\sigma_i^{(5)}(s) = \frac{\mu}{32\pi^2} \text{Trace} [\Lambda_i^{(5)} m(s - 2m^2)] + O\left(\frac{1}{s}\right)$$

where $\Lambda_i^{(5)}$ is given by Eq. (2.9). Comparing this expression with Eq. (2.8), we conclude that the same cancellations which gave the Ward identities (2.12) also imply

$$\lim_{s \rightarrow \infty} s \delta_{I,II}(s) < \infty \quad (4.1)$$

This result is perhaps rather unexpected. It appears that for any combination which vanishes at zero momentum by virtue of the Ward identity, the convergence of the integrals of (2.35) is guaranteed, whereas our short-distance analysis allowed the possibility of logarithmic divergence for $d=3$. The really interesting question is whether the sum rules (2.35) remain true. It is by no means obvious that they do, since we assumed $d < 3$ in deriving them. To answer this question, it is most appropriate to study the asymptotic behaviour of $\Delta_{I,II}(s)$. The Feynman diagrams contributing to each $\Delta_i^{(5)}(s)$ are quadratically divergent, but for the combinations $\Delta_{I,II}(s)$ both the quadratic and logarithmic divergences cancel, as indicated by Eq. (4.1). We find

$$\Delta_{I,II}(s) = -B_{I,II} D_{I,II}(s) \quad (4.2)$$

where

$$B_I = (m_s^2 - m_u^2)^2 / 4\pi^2 \quad (4.3)$$

$$B_{II} = m_u(m_s + m_u)(m_s - m_u)^2 / 4\pi^2 \quad (4.4)$$

$$\begin{aligned} D_I(s) &= D(s) + m_s^2 E(s), \\ D_{II}(s) &= D(s), \end{aligned} \quad (4.5)$$

$$D(q^2) = \frac{i}{\pi^2} \int d^4p \frac{p \cdot q}{[(p+q)^2 - m_u^2][p^2 - m_u^2][p^2 - m_s^2]}, \quad (4.6)$$

$$E(q^2) = \frac{i}{\pi^2} \int d^4p \frac{p \cdot q}{[(p+q)^2 - m_u^2][(p+q)^2 - m_s^2][p^2 - m_u^2][p^2 - m_s^2]}. \quad (4.7)$$

From Eqs. (4.2) to (4.7), it is immediately clear that $\Delta_{I,II}(s)$ satisfy the Ward identities (2.12) and the once subtracted dispersion relations (2.34) (proved for $d < 7/2$). It is simple to show that

$$D(s) \rightarrow 1 + \frac{3m_u^2 + m_s^2}{2s} \log(-s) + O\left(\frac{1}{s}\right), \quad (4.8)$$

$$E(s) \rightarrow \frac{1}{s} \log(-s) + O\left(\frac{1}{s}\right), \quad (4.9)$$

as $s \rightarrow \infty$, consistent with the asymptotic behaviour (4.1). Thus we have

$$\Delta_{I,II}(\infty) = -B_{I,II} \quad (4.10)$$

and

$$B_{I,II} = \int \frac{ds}{s} \sigma_{I,II}(s). \quad (4.11)$$

As a cross-check on this calculation, we have evaluated the integrals of Eq. (4.11) using the explicit forms of $\sigma_{I,II}(s)$. After considerable labour we found agreement with Eqs. (4.3) and (4.4). It is important to realize, however, that it is the asymptotic results of Eq. (4.10) together with the Ward identities,

$$\Delta_{I,II}(0) = 0, \quad (4.12)$$

which determine Eqs. (4.11).

From this analysis we have learnt two things. First, in the free quark model, the sum rules (2.38) and (2.39) are no longer true. The integrals remain convergent, but the right-hand sides are no longer zero. In coordinate space this corresponds to a leading singularity proportional to $\delta^4(x)$ in the combinations of time-ordered products. Such singularities are normally excluded by fiat in the definition of T products¹⁸⁾. In Section 2 we were rather cavalier in not defining the T products of Eq.(2.1) before giving the momentum space Ward identities of Eq. (2.7), thereby ignoring Wilson's strictures¹⁸⁾. This is easily remedied for $d < 3$ (as assumed in Section 2), since in that case

$$\begin{aligned}\Delta_{I,II}(q^2) &= q^2 \int \frac{ds}{s(s-q^2)} \rho_{I,II}(s) \\ &= i \int d^4x e^{iq \cdot x} \partial_\mu [T_{I,II}^\mu(x)]\end{aligned}\quad (4.13)$$

and the $T_{I,II}^\mu(x)$ are less singular than $1/x^3$ as $x \rightarrow 0$. Thus the Fourier transforms of Eq. (4.13) are finite, covariant and independent of the order of integration¹⁸⁾, and

$$\Delta_{I,II}(0) = \Delta_{I,II}(\infty) = 0$$

which immediately gives the sum rules of Eq. (2.35). However, for the free quark model

$$T_{I,II}^\mu(x) = - \frac{\beta_{I,II}}{2\pi^2} \left[\frac{x^\mu}{(x^2 - i\epsilon)^2} \right] + \text{less singular.}$$

Using

$$\partial_\mu \left[\frac{x^\mu}{(x^2 - i\epsilon)^2} \right] = -2\pi^2 i \delta^4(x),$$

we obtain Eq. (4.10), provided the point $x=0$ is not excluded from the integration of Eq. (4.13). Alternatively, one can adopt Wilson's prescription for T products¹⁸⁾:

$$\begin{aligned}\Delta_{I,II}^W(q^2) &= \lim_{\eta \rightarrow 0^+} i \int d^4x e^{iq \cdot x} \Theta(|x_0| - \eta) \partial_\mu [T_{I,II}^\mu(x)] \\ &= \Delta_{I,II}(q^2) + \beta_{I,II}.\end{aligned}$$

In which case $\Delta_{I,II}^W(q^2)$ satisfy unsubtracted dispersion relations, but not the momentum space Ward identities of Eq. (4.12). The sum rules of

Eq. (4.11) then correspond to

$$\Delta_{I,II}^w(0) = B_{I,II}. \quad (4.14)$$

It seems reasonable to call the result of Eq. (4.14) a "canonical anomaly" -- canonical because it follows from free field theory, anomaly because the Green's functions $\Delta_{I,II}^w(q^2)$ defined to satisfy unsubtracted dispersion relations do not satisfy the expected momentum space Ward identities:

$$\Delta_{I,II}^w(0) = 0.$$

Chanowitz and Ellis¹⁸⁾, who coined this phrase, have argued that canonical anomalies do not occur in the Ward identities for two-point functions. In our case, the argument would say that if the $T_{I,II}^\mu(x)$ of Eq. (4.13) are less singular than $1/x^3$ there can be no anomaly, and if they are as singular as, or more singular than, $1/x^3$ some regulation prescription is required, in defining the Fourier transform, which corresponds to choosing a subtraction constant. The subtraction constant can then be chosen so as to satisfy the Ward identity. This argument breaks down in the quark model where the singularity is $x^\mu/(x^2 - i\epsilon)^2$, but no regulation is required.

The second interesting feature of the free quark model result is that the constants $B_{I,II}$ are positive for arbitrary quark masses. Indeed for the second sum rule we find that

$$\sigma_{II}(s) \geq 0$$

for arbitrary quark masses. This is intriguing, since the sign is what is required to remove the contradiction between upper and lower bounds demonstrated for $d < 3$ in the previous Section. It is tempting then to suggest that $d=3$ and that the quark masses (and hence the magnitude of chiral symmetry breaking) must be large enough to respect the lower bounds of Ref. 1). But to use the field theory of free quarks to relate quark masses to the couplings of supposedly composite Goldstone bosons, seems to us to be stretching it far past breaking point. Even if quark fields are relevant to understanding the couplings of hadrons to currents, the underlying dynamics must be generated by the interactions (and containment?) of these fields and the consequences of these interactions for the short-distance expansion must surely be considered.

It is, however, possible to use the results of this Section to deduce a contradiction with the recent proposal of Fritzsche and Leutwyler ⁵⁾ for the leading singularities of current products.

They start with the picture of Fritzsche, Gell-Mann and Leutwyler ⁴⁾ in which the dynamics of hadrons and their interactions with currents are supposed in some sense to be described by a Lagrangian which couples three triplets of coloured massive quarks to an octet of coloured massless vector gluons, with $[SU(3)]_{\text{colour}}$ as a local gauge group of the strong interaction. Fritzsche and Leutwyler further ask that there be some mechanism which softens ³⁾ the theory at high energies and avoids the logarithms required by renormalization group analysis, so that Bjorken scaling ¹⁹⁾ for deep inelastic lepto-production is exact. They claim that this is tantamount to treating the gluons as classical external fields. In lepto-production, next-to-leading singularities then depend upon both quark masses and gluon field strengths ²⁰⁾, but for the colour-singlet parts of current or divergence products the gluons are supposed to give corrections of the form

$$\frac{G_{\mu\nu}^A G_{\lambda\sigma}^A}{x^2} \quad + \text{less singular},$$

where $G_{\mu\nu}^A$ is the gluon field strength, and $A = 1, 8$ is a colour index. Fritzsche and Leutwyler have pursued some consequences of this radical assumption. Their prediction for e^+e^- annihilation is embarrassingly specific:

$$\sigma(e^+e^- \rightarrow \text{hadrons}) = \frac{4\pi\alpha^2}{3s} R_{\infty} \left[1 + O\left(\frac{1}{s^2}\right) \right] \quad (4.15)$$

(the $1/s^2$ corrections depend upon both the quark masses and gluon field strengths -- there is no $1/s$ term for free massive quarks). For three coloured triplets $R_{\infty} = 2$ and Eq. (4.15) is violently at odds with the data ²¹⁾. They argue, however, that it may be possible to escape this problem by allowing for a fourth charmed quark ²²⁾ or colour thaw with Han-Nambu quarks ²³⁾, without amending their predictions for the $1/x^6$ and $1/x^4$ singularities of $SU(3) \times SU(3)$ current or divergence products. We now show that they still encounter difficulties with the results of Ref. 1).

Their scheme implies that

$$\lim_{s \rightarrow \infty} \left(\Delta_i^{(5)}(s) - [\Delta_i^{(5)}(s)]_{\text{free}} \right) = 0 \quad (4.16)$$

and hence that

$$3B_{I,II} = \int \frac{ds}{s} \rho_{I,II}(s), \quad (4.17)$$

where the factor of 3 comes from the sum over colour. Low-energy saturation of these convergent sum rules gives

$$3B_I \approx A + (m_K f_K)^2 - \frac{1}{4} (m_\pi f_\pi)^2 - \frac{3}{4} [(m_\eta f_\eta)^2 + (m_{\eta'} f_{\eta'})^2], \quad (4.18)$$

$$3B_{II} \approx A - \left(\frac{m_s - m_u}{m_s + m_u} \right) (m_K f_K)^2 + \left(\frac{m_s - m_u}{2m_u} \right)^2 (m_\pi f_\pi)^2. \quad (4.19)$$

These relations must be compared with our lower bound ¹⁾

$$A \geq N (m_K f_K - m_\pi f_\pi)^2 \quad (4.20)$$

where N is greater than 2 given the $K\pi$ phase shift, the Callan-Treiman relation, and a very conservative statement of the Dashen-Weinstein relation [see Ref 1) for details]. This is only consistent with the second sum rule of Eq. (4.19) if

$$\frac{3}{4\pi^2} m_u (m_s + m_u) (m_s - m_u)^2 \gtrsim (N-1) (m_K f_K - m_\pi f_\pi)^2. \quad (4.21)$$

Leutwyler ²⁴⁾ has suggested the values $m_u = 5.4$ MeV, $m_s = 125-150$ MeV from an analysis of meson null-plane quark wave functions. With these quark masses, inequality (4.21) fails by more than three orders of magnitude if $N \geq 2$. Even if one allows for larger quark masses, we can combine Eqs. (4.18) and (4.19) to obtain

$$A \lesssim (m_K f_K)^2 - \left(\frac{2m_s^2 - m_u^2}{4m_s m_u} \right) (m_\pi f_\pi)^2, \quad (4.22)$$

eliminating the unknown contributions of η and η' to the first sum rule. The only way of reconciling inequalities (4.20) and (4.22) with $N \geq 2$ is to have $m_u \gg m_s$, which is violently at odds with one's expectation of the quark model. We conclude that the scheme of Fritzsche and Leutwyler is inconsistent with the lower bounds of Ref. 1). In fact, they themselves suggested that

$$A \lesssim (m_K f_K)^2$$

but only from an analysis which involved cutting off divergent integrals. Inequality (4.22) follows directly from low-energy saturation of a convergent sum rule.

5. DISCUSSION

We have shown that for a theory in which $SU(3) \times SU(3)$ is spontaneously broken, the chiral symmetry breaking Hamiltonian density should have scale dimension $d \geq 3$, provided it belongs to a $(3, \bar{3}) + (\bar{3}, 3)$ multiplet of $SU(3) \times SU(3)$. Our result follows from the $SU(3) \times SU(3)$ structure of Wilson's operator product expansion ²⁾, which enables one to prove a convergent sum rule for $d < 3$. Saturation of this sum rule by one and two pseudoscalar meson contributions should be a good approximation in the chiral symmetry breaking expansion, but is inconsistent with the inequalities of Ref. 1), given the low-energy theorems for the scalar $K_{\rho 3}$ form factor. Although the sum rule is modified in the free quark model ($d=3$), the inequalities of Ref. 1) are still inconsistent with the softened quark-vector gluon model, from which Fritzsche and Leutwyler ⁵⁾ have extracted leading and next-to-leading short-distance singularities of current operator products.

Our analysis does not appear to be inconsistent with either an asymptotically free ⁶⁾ quark-vector gluon theory ($d=3$) or any theory in which hadronic interactions at short distances are governed by an infra-red stable fixed point of the renormalization group ^{7), 8)} and d is anomalously greater than 3. In the former case, inconsistency with Ref. 1) can presumably be avoided by the logarithmic divergence of our sum rule, Bjorken scaling is modified by logarithms in deep inelastic electroproduction ²⁵⁾ and approached logarithmically in e^+e^- annihilation ²⁶⁾. In the latter case, the divergence of our sum rule is worse than logarithmic, anomalous dimensions cause a violation by powers of q^2 of Bjorken scaling in electroproduction ²⁷⁾, and the e^+e^- annihilation cross-section tends to an unknown scale invariant limit with non-leading corrections which are necessarily of order $s^{-\frac{1}{2}}$ or greater ⁸⁾.

We are unable to distinguish between these two possibilities, but in view of the recent data from CEA and SPEAR ²¹⁾ it is appropriate to spell out the consequences of each for e^+e^- annihilation.

If we write the cross-section for producing hadrons by one-photon exchange as

$$\sigma(e^+e^- \rightarrow \text{hadrons}) = \frac{4\pi\alpha^2}{3s} R(s) \quad (5.1)$$

then the prediction of an asymptotically free non-Abelian gauge theory of the strong interactions ^{6), 26)} is that

$$R(s) \rightarrow R_\infty \left[1 + \frac{c}{\log(s/\mu^2)} + O\left(\frac{\log \log(s/\mu^2)}{[\log(s/\mu^2)]^2}\right) \right] \quad (5.2)$$

where μ is an unknown mass scale, but R_∞ and c are simply calculable from the form of the Lagrangian. For the coloured quark-octet vector gluon Lagrangian^{4),26)} $R_\infty = 2$, $c = 4/9$. If it is accepted that the hadronic events at SPEAR and CEA are produced by one-photon exchange, it appears²¹⁾ that $R(s)$ increases from 2-3 at $s \approx 9 \text{ (GeV)}^2$ to 4-7 at $s \approx 25 \text{ (GeV)}^2$, which means that asymptotia has certainly not been reached and throws serious doubt on the gauge theory prediction for R_∞ .

On the other hand, Nachtmann⁸⁾ has pointed out that such a behaviour is not inconsistent with Wilson's suggestion of an infra-red stable hadronic fixed point⁷⁾, provided the dimension of scale breaking is large enough. The general prediction of broken scale invariance is

$$R(s) \rightarrow R_\infty \left[1 + O\left(s^{\left[\frac{d_\theta}{2}-4\right]}\right) \right], \quad (5.3)$$

where

$$d_\theta = \text{Max}[d, \Delta] \geq d \quad (5.4)$$

is the scale dimension of θ , the trace of the energy momentum tensor. [We assume that $4 > d_\theta > 2$.] Now R_∞ and d_θ are determined by the hadronic coupling constant(s) at the non-trivial fixed point and are not calculable from the underlying Lagrangian, given current technology. In particular, there is no reason to expect $R_\infty = 2$ (or $d_\theta = 3$) even if three triplets of fractionally charged quarks describe the internal degrees of freedom of composite hadrons. In fact it is not even possible, as yet, to say whether d_θ is larger or smaller than its canonical value (which is, for example, 2 for ϕ^4 theory and 3 in QED). For example, Baker and Johnson²⁸⁾ found in QED that

$$d_\theta = 3 \left[1 - \left(\frac{\alpha_0}{2\pi}\right) - \frac{1}{4} \left(\frac{\alpha_0}{2\pi}\right)^2 + O\left(\left(\frac{\alpha_0}{2\pi}\right)^3\right) \right], \quad (5.5)$$

where $\alpha_0 = e_0^2/4\pi$ and e_0 is supposed to be given by the lowest non-trivial fixed point of the renormalization group. On the other hand, the sign of the anomalous dimension of $\bar{\psi}\psi$ in the Thirring model depends upon the sign of coupling constant²⁹⁾. Thus our result that $d_\theta \geq 3$ enables one to understand why, in such a theory, $R(s)$ approaches its asymptotic value slowly. Moreover, Nachtmann⁸⁾ has argued that the approach should be from below:

$$R(s) = R_\infty \left[1 - \left(\frac{\gamma_{\theta 0}^2}{s}\right)^{\left[\frac{4-d_\theta}{2}\right]} + \text{higher order} \right]. \quad (5.6)$$

If we suppose that $m_0 \approx 1 \text{ GeV}$ (a typical hadronic mass scale), $d_0 \geq 3$ implies that $R(s)$ is still at least 20% below its asymptotic value at the highest SPEAR and CEA energies.

It will be of great interest to know how the annihilation cross-section behaves at higher energies and to learn whether Bjorken scaling in electro-production is violated by logarithms or by powers of q^2 . If it eventually emerges that a theory with an infra-red stable hadronic fixed point gives a better understanding of processes initiated by highly virtual photons, it seems likely that a fairly large value of d_0 will be required to explain the slow approach of the e^+e^- annihilation cross-section to the asymptotic form required by broken scale invariance. It is therefore rather interesting that the low-energy theorems of current algebra and the $SU(3) \times SU(3)$ structure of the operator product expansion seem to require that $d_0 \geq 3$.

Finally, we remark that it would be interesting to know the detailed prediction of asymptotically free quark-gluon models for the asymptotic behaviour of the spectral functions of our sum rules (2.38) and (2.39). Our guess would be that they behave like some (inverse?) power of a logarithm. If, however, they vanish like $1/s$ (modulo logarithms), as in the free quark model, one would like to know whether there are relations like Eqs. (4.10) which could embarrass asymptotically free theories as they do the softened quark-gluon model. Such a situation is not inconceivable if the gauge fields carry no isospin or hypercharge, but only colour, since in that case one might expect $SU(3) \times SU(3)$ combinations of Green's functions to behave in the same fashion asymptotically as they do in the free quark model (modulo logarithms).

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