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REGGE COUPLINGS AND INTERCEPTS FROM THE
PLANAR DUAL BOOTSTRAP *)

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Planar unitarization of dual models is examined in terms of renormalized diagrams. The shift of the Reggeon intercept $1-\alpha(0)$ and the renormalized coupling $g^2_N/16\pi$ are both expected of order unity in a model with $SU(N)$ Chan-Paton factors. In four space-time dimensions Regge behaviour makes the shift of the dual photon mass reminiscent of Schwinger's mechanism.

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A new "topological expansion" for implementing unitarity in planar dual models has been recently proposed ¹⁾ and more precisely defined ²⁾ as a "large N" expansion.

It is of crucial interest to know properties of the first term of this expansion, defining a "planar unitary theory". It was already argued ¹⁾ that the renormalized Reggeon intercept $\alpha(0)$ had to be smaller than one in a planar unitary theory. It is the purpose of this note and of an accompanying paper ³⁾ to show how this shift of Regge intercept away from its bare value occurs. We are also able to strengthen the result and argue that the Reggeon intercept $\alpha(0)$ and the renormalized Reggeon coupling at $t=0$, g^2 , have to be such that both $1-\alpha(0)$ and $g^2N/16\pi$ are of order unity, in a model with $SU(N)$ Chan-Paton factors. As a consequence, one can show ²⁾ that the "large N" expansion is indeed a series in powers of $1/N^2 \approx 1/10$ so that good convergence can be expected.

We present here the essence of our argument, leaving details and comparison with the conventional approach for the longer article ³⁾. One important ingredient for reaching the results will be the presence, in dual models, of a fundamental scale parameter, $\alpha' \approx 1 \text{ GeV}^{-2}$, whose value relates the slope of trajectories to the Regge scale parameter and the momentum transfer cut-off. It is important to consider α' a given finite quantity whose value does not depend on the coupling constant (as it happens in ordinary field theory). Such scale parameter allows one to take and interpret s channel discontinuities of loop diagrams which would be purely real in ordinary field theory ($\alpha' \rightarrow 0$). These discontinuities have no ultra-violet divergence but are, instead, more infra-red singular than a corresponding Feynman graph with self-interacting Yang-Mills mesons. Regge behaviour reduces the effective number of momentum integrations from four to two, i.e., to a transverse momentum integral. One thus obtains infra-red divergences similar to those of Schwinger ⁴⁾ in his known model of two-dimensional massless QED, first example of spontaneous breaking of gauge invariance.

After having written the relevant equations of the theory and used the crucial assumption of Regge behaviour for the renormalized amplitudes, our argument proceeds in three steps.

1. We establish a proportionality between $1-\alpha(0)$ and $g^2N/16\pi$. This can be called a dual Schwinger mechanism ⁴⁾.
2. We argue that $g^2N/16\pi$ is of order unity, hence the same holds for $1-\alpha(0)$.
3. We extend our argument to cases where the previous reasoning gives no result.

We want to point out that although 1. appears to be a fairly rigorous result, the arguments in 2. and 3. leave room for improvements, but we certainly believe their validity in the dual context. The three sets of equations to be obeyed by the planar unitarized amplitudes are the following.

a) Unitarity sum rules for the leading particle

They have the form ^{1),3)}

$$\text{Disc}_s A_{abba'}(s,t) = \sum_c \text{Disc}_{M^2} A_{abcc'b'a'}(s,t,M^2,t_1,t_2), \quad (1)$$

$$s = (p_a + p_b)^2, M^2 = (p_a + p_b - p_c)^2, t = (p_a - p_{a'})^2, t_1 = (p_b - p_c)^2, t_2 = (p_c - p_{b'})^2.$$

This equation is depicted in Fig. 1.

b) Equations of the type discussed by Brower et al. ⁵⁾

Their exact form would be

$$\text{Disc}_s A_{abba'}(s,t) = \sum_{X_1, X_2} A_{abX_1X_2} A_{X_2X_1b'a'}^* \quad (2)$$

Equation (2) is sort of trivial until Regge behaviour is used, i.e., what we have done is simply to split the intermediate n particle state in the set X_1 of particles of rapidity $y > 0$ and the set X_2 of particles of rapidity $y < 0$, where a (b) is taken to have rapidity $-Y/2$ ($+Y/2$).

c) Chew-Pignotti-type equations ⁶⁾

These have been used in Ref. 1). They involve calculation of the average multiplicity $\langle n \rangle$ or, more generally, of the so-called pressure $P(z)$ in terms of Regge couplings ⁷⁾. By constructing $P(1) - P(0) = 1 - \alpha(0)$ one relates intercepts to coupling constants. The formula for $\langle n \rangle \text{Disc}_s A_{abba'}$ is similar to Eq. (2), except that X_1 and X_2 are now simply ordered in rapidity, i.e., particles in X_1 have rapidity larger than in X_2 . This clearly counts $(n-1)$ times (X_1, X_2 are not empty) each n particle contribution.

If we now make the (crucial) assumption of Regge pole dominance, one can use a triple-Regge expansion for the six-point function and write Eq. (1) in the form :

$$\frac{\pi g_{aa'}(t) g_{bb'}(t)}{\Gamma(1+\alpha(t))} = c \sum_{R, R'} \int d^2 p_{\perp} g_{ac}^R(t_1) g_{ca'}^{R'}(t_2) g_{bb'}(t) \cdot$$

$$\cdot \frac{\pi G^{RR'}(t, t_1, t_2) \Gamma(-\alpha_R(t_1)) \Gamma(-\alpha_{R'}(t_2)) \cos \pi(\alpha_R(t_1) - \alpha_{R'}(t_2))}{\Gamma(2+\alpha(t) - \alpha_R(t_1) - \alpha_{R'}(t_2)) (1+\alpha(t) - \alpha_R(t_1) - \alpha_{R'}(t_2))} \quad (3)$$

Here $g^R(t)$ is the coupling of the Reggeon α_R to two particles (the label R is omitted for the leading trajectory). The four-point function is parametrized as $A_{abb'a'} \approx g_{aa'}(t) g_{bb'}(t) \Gamma(-\alpha(t)) (-s/s_0)^{\alpha(t)}$ and the triple Regge coupling is defined as $\pi G^{RR'}(t, t_1, t_2) \Gamma^{-1}(2+\alpha(t) - \alpha_R(t_1) - \alpha_{R'}(t_2))$. The $\cos \pi(\alpha_R - \alpha_{R'})$ factor comes from the Regge phase $e^{-i\pi\alpha}$ of a planar theory, and the factor $(1-\alpha(t) - \alpha_R(t_1) - \alpha_{R'}(t_2))^{-1}$ comes from the longitudinal integration. This, together with other phase space factors, makes the constant c take the value $c = \alpha' N / 16\pi^2$.

Using also Regge pole dominance for Eq. (2), we can insert a sum of Regge poles between aX_2 and bX_1 (see Fig. 2) and, in the same normalizations used in Eq. (3), we obtain ^{*}

$$\frac{\pi g_{aa'}(t) g_{bb'}(t)}{\Gamma(1+\alpha(t))} = c \sum_{R, R'} \int d^2 k_{\perp} g_{aa'}(t) g_{bb'}(t) \cdot$$

$$\cdot \frac{\pi [G^{RR'}(t, t_1, t_2)]^2 \Gamma(-\alpha_R(t_1)) \Gamma(-\alpha_{R'}(t_2)) \cos \pi(\alpha_R(t_1) - \alpha_{R'}(t_2))}{\Gamma^2(2+\alpha(t) - \alpha_R(t_1) - \alpha_{R'}(t_2)) (1+\alpha(t) - \alpha_R(t_1) - \alpha_{R'}(t_2))^2} \quad (4)$$

Now $k = p_a - p_{X_2} = p_b - p_{X_1}$ and the two factors $(1+\alpha(t) - \alpha_R(t_1) - \alpha_{R'}(t_2))^{-1}$ come from the two integrations over $k_{\pm} = (k_0 \pm k_3)$.

Although Eqs. (3) and (4) may look different, they are closely related. This is because planar dual models satisfy superconvergence equations and FESR without fixed pole contributions. The absence of the fixed pole implies

^{*}) The possible use of this equation in this context and some of its properties have been pointed out to us by M. Bishari.

$$\sum_c g_{ac}^R(t_1) g_{ca'}^{R'}(t_2) \underset{1+\alpha-\alpha_R-\alpha_{R'} \approx 0}{\approx} \frac{g_{aa'}(t) G(t, t_1, t_2)^{RR'}}{1+\alpha(t)-\alpha_R(t_1)-\alpha_{R'}(t_2)} \equiv$$

$$\equiv g_{aa'}(t) \cdot \tilde{G}^{RR'}(t, t_1, t_2) \quad (5)$$

and this makes Eqs. (3) and (4) very similar. In particular, we find that G has to exhibit the usual triple-Regge zero for $\alpha(0) \rightarrow 1$, $t \rightarrow 0$. Equation (5) is satisfied in the Born approximation, where, for the leading trajectory, one has, in our normalizations ^{*)}

$$g(t) = \tilde{G}(t, t_1, t_2) \equiv \frac{G(t, t_1, t_2)}{1+\alpha(t)-\alpha(t_1)-\alpha(t_2)} = g \quad (6)$$

We now sketch how to prove our results. More details on this and on the above equations will be found in Ref. 3). As we said, we proceed in three steps.

1. Take Eq. (4) at $t=0$. Using positivity, one can easily show ⁵⁾, under weaker assumptions than those used to derive Eqs. (3) and (4), that

$$\frac{1}{\Gamma(1+\alpha(0))} > c \cdot \pi \cdot \int dt \frac{\tilde{G}^2(0, t, t) \Gamma^2(-\alpha(t))}{\Gamma^2(2+\alpha(0)-2\alpha(t))} \quad (7)$$

where $\tilde{G}$ is defined in Eq. (5) and approaches $g^2(0)$ for $t \rightarrow 0$, $\alpha(0) \rightarrow 1$. In that limit, Eq. (7) gives

$$1 \gtrsim \frac{g^2(0)N}{16\pi} \int \alpha' dt (1-\alpha(t))^{-2} \approx \frac{g^2 N}{16\pi} \frac{1}{1-\alpha(0)} \quad (8)$$

*) We consider for simplicity the ordinary beta function model, which unfortunately has a tachyon. In models without the tachyon, some shifts of one unit will occur in the arguments of the various Γ functions without affecting, we believe, our conclusions.

where we have taken $\alpha(t) = \alpha(0) + \alpha' t + \dots$, i.e., we have assumed a finite slope at $t=0$. The same conclusion is obtained by use of Eq. (3). We thus conclude that there is a mass shift whenever $\int dt (1-\alpha(t))^{-2} \xrightarrow{\alpha(0) \rightarrow 1} \infty$. Comparison with ordinary field theory is postponed to the end of the paper.

2. We now want to argue that $g^2(0) \approx 16\pi/N$ and therefore that $1-\alpha(0) \approx 0(1)$.

In order to do that we take Eq. (4) at $t \neq 0$. We then bound the absolute values and get

$$c^{-1} < \sum_{R, R'} \int d^2 k_1 \left| \tilde{G}^{RR'}(t, t_1, t_2) \right|^2 \quad (9)$$

where we have omitted other factors since, at $t \neq 0$, they are not singular (they just produce a transverse momentum cut-off at $\sim 1/\alpha'$). Hence $\tilde{G}_{RR'}^2(t, t_1, t_2) \gtrsim 0(1)$ for some R, R' and in some finite region of $t, t_1, t_2 \neq 0$ ^{*}). Using Eq. (5), we then find that $g_R^2(t) \approx 0(1)$, and, finally, using Eq. (3) at $t=0$ and again Eq. (5), we get :

$$g(0) > \sum_{R, R'} \int dt g_R(t) g_{R'}(t) \tilde{G}^{RR'}(t) \approx \sum_{R, R'} \int dt \frac{g_R^2(t) g_{R'}^2(t)}{g(0)} \quad (10)$$

which implies $g^2(0) \approx 0(1)$, the result we wished to prove. We want to point out again that, although the above proof is not too rigorous, we nevertheless think that it should be possible to make our argument more rigorous. Certainly, if some counter-examples can be found, they have to have features drastically different from those of the dual Born term ^{**}).

3. Finally, we present another argument still leading to the same result on $1-\alpha(0)$. In fact, this argument holds even if the trajectory has infinite slope at $t=0$ or in higher dimensionality of space time. The argument is based on calculating $\langle n \rangle \text{Disc}_s A_{\text{abb}'a'}$, as explained earlier.

Using again Regge behaviour for $A_{abX_1X_2}$ of Eq. (2), and taking into account the new limits of integration, one gets at $t=0$ ^{***})

*) By $0(1)$ we mean actually $0(c^{-1}/\pi \langle t \rangle) \approx (16\pi/N)$.

**) In ordinary field theory, for example, $\alpha(t)$ differs from an integer by an amount proportional to g^2 . Since $\text{Im} A / \text{Re} A \approx \tan \pi \alpha$ this makes $\text{Im} A$ and $|A|^2$ of the same order in g and a solution with $g \rightarrow 0$ is possible.

***) This is actually a lower bound on $\langle n \rangle$ if there is clustering.

$$\langle n \rangle \text{Disc}_s A_{abba'} = \log s \cdot c \cdot \sum_{R, R'} \int d^2 k_\perp I_{RR'}(0, t, t) (1 + \alpha(0) - \alpha_R(t) - \alpha_{R'}(t)) \quad (11)$$

where $I_{RR'}(t, t_1, t_2)$ is the integrand on the right-hand side of Eq. (4). Comparing with Eq. (4), one gets

$$\begin{aligned} \frac{1}{\log s} \langle n \rangle &= \langle 1 + \alpha(0) - \alpha_R(t) - \alpha_{R'}(t) \rangle = \\ &= 1 - \alpha(0) + \langle \alpha(0) - \alpha_R(t) \rangle + \langle \alpha(0) - \alpha_{R'}(t) \rangle = \\ &= 1 - \alpha(0) + \langle \alpha(0) - \alpha_R(0) \rangle + \langle \alpha(0) - \alpha_{R'}(0) \rangle + \langle \alpha_R(0) - \alpha_R(t) \rangle + \langle \alpha_{R'}(0) - \alpha_{R'}(t) \rangle, \end{aligned} \quad (12)$$

where the $\langle \rangle$ sign on the right-hand side is obviously defined. On the other hand, $\langle n \rangle / \log s = (d/dz)P(z)|_{z=1}$. Assuming $\langle \alpha(0) - \alpha_R(0) \rangle \geq 0$, we find

$$P'(1) \gtrsim P(1) - P(0) + 2 \langle \alpha_R(0) - \alpha_R(t) \rangle \quad (13)$$

In those cases where $\int dk_\perp (1 - \alpha(t))^{-2}$ was convergent, we also expect $\langle \alpha' t \rangle \approx \langle \alpha_R(0) - \alpha_R(t) \rangle \approx 0(1)$ and therefore $P'(1) \gtrsim 0(1)$. Since $P(z)$ is an increasing function of z a finite slope at $z=1$ implies $P(1) - P(0) \equiv 1 - \alpha(0) \approx 0(1)$ unless higher derivatives will show up infinitesimally close to $z=1$. Explicit calculations have shown that this does not happen³⁾, but a more careful analysis of the situation at $1-z \ll 1$ should be done, with particular attention to the possible role of Regge cut contributions at $z \neq 1$ (these are absent at $z=1$ because of planarity). In more phenomenological terms, Eq. (12) is quite appealing. It indicates first of all that one cannot have a pure Poisson distribution ($P(z) = a + bz$) unless one has just one flat trajectory. Experimentally $P'(1)$ should be identified with about half of the observed multiplicity⁸⁾, i.e., $P'(1) \approx 1.5$. Of this $P(1) - P(0)$ accounts for ~ 0.5 and $2\langle \alpha' t \rangle$ should give about another 0.5 if we use the $e^{-4\alpha' P_1^2}$ cut-off. The rest can be due to lower exchanges (especially π exchange) and clustering.

Our second estimate, $g^2(0) \approx 16\pi/N$, also seems to be of the right order. It would predict

$$\sigma_{\text{Resonance}} = \sigma_{\pi^+\pi^-} - \sigma_{\pi^+\pi^+} \approx \frac{32\pi}{N} \alpha' (\alpha' s)^{\alpha(0)-1} \approx 15 \text{ mb} \cdot (s/16s_0)^{-1/2} \quad (14)$$

A similar estimate was obtained some time ago in a multiperipheral model context⁹⁾.

Finally, we want to compare our results with those of ordinary massless Yang-Mills field theory. In that case the first correction to the bare propagator $1/q^2$ will be $(q^2)^{-2}\pi(q^2)$ with :

$$\Pi_{\mu\nu}(q^2) = g_{\mu\nu} \pi(q^2) + \dots = e^2 \int \frac{d^4 k}{k^2 (q-k)^2} k_\mu k_\nu \quad (15)$$

where e is the dimensionless Yang-Mills coupling constant. After ultra-violet divergences have been removed in a gauge invariant way, one finds

$$\Pi_{\mu\nu}(q^2) = e^2 g_{\mu\nu} q^2 + (q_\mu q_\nu \text{ terms}) \quad (16)$$

indicating no-mass renormalization. Although the possibility of a mass renormalization from higher order loops or from a non-perturbative mechanism has not been ruled out ¹⁰⁾ we see that naïve power counting indicates no shift.

In the two-dimensional Schwinger model ⁴⁾ one has instead :

$$\Pi_{\mu\nu}(q^2) = e^2 \int \frac{d^2 k}{k^2 (q-k)^2} \text{Tr} [\gamma_\mu \not{k} \gamma_\nu (q-k)] \approx e^2 g_{\mu\nu} + \dots \quad (17)$$

and a mass is produced of order e (which is now dimensional). In the (Reggeized) dual theory the loop of Fig. 2 can be written at large s in light cone variables as

$$\int \frac{d^2 k_\perp d k_+ d k_-}{k^2 (k-q)^2} G^2(k) (\alpha' k_+ p_-^a)^{\beta} (\alpha' k_- p_+^b)^{\beta} (\alpha' p_-^a p_+^b)^{\alpha(k^2) + \alpha((k-q)^2)} \quad (18)$$

where $\beta = \alpha(q^2) - \alpha(k^2) - \alpha(k-q)^2$ and $G(k)$ contains a zero at $k=0$ corresponding to the derivative coupling in Yang-Mills. The integrations over k_+ and k_- can be easily done to give

$$(\alpha' p_-^a p_+^b)^{\alpha(q^2)} \frac{1}{\alpha'} \int \frac{d^2 k_\perp}{k^2 (k-q)^2} \frac{G^2}{(1+\beta)^2} \quad (19)$$

We see that the longitudinal integrations instead of helping for the infra-red divergence make it worse, by producing the additional factor $(1+\beta)^{-2}$. Now $\tilde{G} \equiv G(1+\beta)^{-1}$ is regular at $k \rightarrow 0$, $\alpha(0) \rightarrow 1$ and we are left with a divergence as bad as $\tilde{G}^2/\alpha' q^2$. This is what makes the renormalization of $\alpha(0)$ unavoidable. It looks as if the dual theory has traded, through the existence of a non-zero α' , ultra-violet for infra-red divergences so as to necessitate an infra-red cut-off (a mass) of the same order of magnitude as the ultra-violet one ($\sim 1/\sqrt{\alpha'}$).

In conclusion, we have found that planar unitarization of dual models should yield a reasonable Regge intercept and coupling constant. The value of g would be such that perturbation theory is ruled out for the planar theory, the relevant expansion parameter being $g^2 N/16\pi \approx 0(1)$. Since non-planar corrections miss the factor of N , the hope ²⁾ is that a better expansion can be made in the "degree of non-planarity" of the graph. This turns out ²⁾ to be an expansion in $(g^2/16\pi)^2 \approx 0(1/N^2)$ and it has an appealing similarity with present Regge analysis of multiparticle production data ¹¹⁾.

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FIGURE CAPTIONS

Figure 1 : Unitarity sum rule for the planar theory [Eqs. (1) and (3)].

Figure 2 : Graphical form of Eqs. (2) and (4).

- Fig. 1 -

$$= \sum X_1 X_2$$

- Fig. 2 -