PION-CARBON SCATTERING IN THE α PARTICLE MODEL

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A B S T R A C T

Elastic π - ^{12}C scattering in the region of the first pion-nucleon resonance is calculated within Glauber theory on the basis of the naïve α particle model. The input π - α scattering amplitudes are obtained from new measurements by an analysis dependent primarily on the nearby zeros of these amplitudes and the form factors of the nuclear model parametrized so as to give good behaviour at large momentum transfer. Agreement with experiment seems to get better with increasing energy, and the calculation makes clearer some of the limitations of using this scattering theory for very low energies.

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1. - INTRODUCTION

Soon after their first publication, the CERN medium energy pion carbon scattering measurements ¹⁾ were subject to analyses on the basis of optical potentials ²⁾ and Glauber theory ³⁾. Although both approaches have drawbacks, they do describe the general features which were observed, and they are understandable to the average experimentalist. Since this time the optical potential models have become a playground for the theoretician, where the formalism has become less transparent, but the results not significantly better ⁴⁾. There have not been as many analogous attempts to make the Glauber model more "realistic". There are two big question marks about using the present version for medium energy pion scattering. In the derivation of the approximation, Glauber assumes that the inequality $kR \gg 1$ be true for all characteristic distances R , including the nucleon radius ⁵⁾. Secondly, since it does not take place appreciably on a single nucleon, true pion absorption has been almost completely neglected. This latter problem is present also in the case of microscopic optical potentials, and it could be serious since absorption may account for up to half the inelastic π -¹²C cross-section ⁶⁾. These difficulties appear less troublesome if an α cluster model is taken for ¹²C, where an elementary π - α amplitude is used in the calculation instead of a pion-nucleon.

In the naïve α particle model, the nucleus is regarded as being made up of a collection of α particles in their ground states, and any dynamical consequence of the structure of the α particles, such as the exclusion principle between constituents of distinct clusters, studiously ignored except in so far as it gives rise to an α - α interaction. A particularly simple version of this, where the clusters are taken to lie at the vertices of an equilateral triangle, gives a good description of the electromagnetic form factor of ¹²C out to the first subsidiary maximum ⁷⁾. The same model has been used to calculate the cross-section for the scattering of 1 GeV protons from this nucleus in the framework of Glauber theory ⁸⁾. Although a qualitative success can be claimed for the model here, a detailed confrontation with experiment is difficult since the proton α amplitudes which must be used as input are largely unknown, especially in their spin structure. The situation is rather cleaner in the case of pion scattering. At zero energy the s wave π -¹²C scattering length can be expressed in terms of the s wave π - α scattering length using a low energy multiple scattering expansion ⁹⁾. In effect the scattering length is so small that the single scattering ansatz completely dominates, and there is almost no

residual sensitivity to the nuclear model once it is assumed that there are α particle subunits. The agreement with the scattering lengths extracted from the mesic atom data is rather good, even for the imaginary part of the amplitude. This is just one extra piece of evidence that pion absorption by light nuclei can be thought of as proceeding mainly via absorption on α particle subunits in the nucleus ^{6),10)}. Thus if we calculate the scattering of pions from ^{12}C using empirical α particle amplitudes as input, absorption has to a large extent been taken into account. The added bonus of this approach is that unlike the pion-nucleon system, many partial waves contribute to the π - α scattering even at relatively modest energies. This leads to an input amplitude which may be susceptible to an eikonal description, which that of the π -N $3-3$ resonance certainly is not. The Glauber derivation might therefore be valid for such amplitudes. Further advantages of this scheme are that binding corrections, Fermi motion corrections, multiple charge exchange and spin flip effects are to a large degree automatically included, and even the Pauli principle blocking ¹¹⁾ (if it is at all important) taken care of to some extent.

One of the motivations for the present investigation was the availability of accurate π - α elastic scattering data in the energy range 110-260 MeV ¹²⁾. In Section 3, we discuss the analysis of these results in order to obtain scattering amplitudes, particular attention being paid to the complex zeros of the amplitude in the momentum transfer plane. The trajectory of these zeros is another general feature of the scattering amplitude, which like the total cross-section or forward real part is smooth as a function of energy and possibly as a function of nucleon number as well ! The α structure of the nuclear model is treated in Section 2 and in Section 4 elastic π - ^{12}C scattering is calculated using Glauber theory with this model and α particle amplitudes. The agreement between theory and experiment, shown in Section 5, seems to get better as the energy increases, and the calculation sheds some light on the applicability of the Glauber scheme at low energies. This is discussed further in the conclusions of Section 6.

2. - THE α PARTICLE STRUCTURE OF ^{12}C

The internal wave function of ^{12}C in the α particle model depends upon two Jacobi variables $\vec{v}$ and $\vec{w}$, which can be defined in terms of the α co-ordinates $\vec{r}_i$ by

$$\begin{aligned}\vec{v} &= \vec{r}_1 - \vec{r}_2 \\ \vec{w} &= (\vec{r}_1 + \vec{r}_2 - 2\vec{r}_3) / \sqrt{3}\end{aligned}\quad (2.1)$$

and the centre-of-mass co-ordinate of the nucleus has been put at the origin. Now because of the Bose symmetry, the wave function must be symmetric under the interchange of the $\vec{r}_i$, and hence under either

$$\vec{v} \rightarrow -\vec{v} \quad (2.2a)$$

or

$$\begin{aligned}\vec{v} &\rightarrow \frac{1}{2} (\vec{v} - \sqrt{3} \vec{w}) \\ \vec{w} &\rightarrow -\frac{1}{2} (\vec{w} + \sqrt{3} \vec{v})\end{aligned}\quad (2.2b)$$

Inopin⁷⁾ made the ansatz that the nuclear density was just a simple product of Jastrow factors in the inter-alpha separation.

$$\rho(\vec{v}, \vec{w}) \sim f(|\vec{r}_1 - \vec{r}_2|) f(|\vec{r}_2 - \vec{r}_3|) f(|\vec{r}_3 - \vec{r}_1|) \quad (2.3)$$

which is manifestly symmetric. Once this form is chosen, then in principle the values of f are completely determined by the carbon charge form factor $F_c(q)$. Thus

$$F_c(q) = F_\alpha(q) S^1(q) \quad (2.4)$$

where F_α is the α particle form factor and S^1 the Fourier transform of the distribution of α 's.

$$S^1(q) = \int \rho(\vec{v}, \vec{w}) e^{i\vec{q} \cdot \vec{w} / \sqrt{3}} d^3 v d^3 w \quad (2.5)$$

where without loss of generality the charge has been put on particle 3. The only practical way to invert this relation is to choose a functional form for the $f(r)$. Inopin⁷⁾ assumed that the distance between the α 's was a constant d , so that they lie at the vertices of an equilateral triangle of side d ,

$$f(r) = \delta(r-d) \quad (2.6)$$

leading to two-body density

$$\rho_I(\vec{v}, \vec{w}) \approx \delta(v-d) \delta(w-d) \delta(\vec{v} \cdot \vec{w}) \quad (2.7)$$

which has an additional symmetry under the interchange of v and w . This yields a form factor

$$S_I^1(q) = j_0(qd/\sqrt{3}) \quad (2.8)$$

which gives a remarkably good representation out to and including the height of the subsidiary maximum. However, this form factor decreases only like $1/q$ for large q , and it has an infinite number of oscillations. This is connected with the choice of a fixed distance between the α 's. If this is blurred a little, then $S^1(q)$ can be made well behaved for large q , without destroying the agreement over the observed region. To this end we have considered the trial function

$$f(r) = e^{-2\alpha^2 r^2/3} (1 - C e^{-\gamma^2 \alpha^2 r^2}) \quad (2.9)$$

for which the form factor is expressible as the sum of four terms Gaussian in the momentum transfer, well behaved at infinity and having only a finite number of oscillations. A numerical fit to the charge form factor¹³⁾, at least as good as that of Inopin yields parameters

$$\begin{aligned} \alpha^2 &= 0.448 \text{ fm}^{-2} \\ \gamma &= 0.165 \\ C &= 1.14 \end{aligned} \quad (2.10)$$

where F_α was taken to be a Gaussian with a charge radius of 1.63 fm. It has the obvious disadvantage that for a very small range of r near the origin the density becomes negative ! This could be cured by the introduction of more terms in f , but it brings us to the most serious problem of the naïve α particle picture. It is clear from Eq. (2.4) that the carbon form factor should have a zero whenever the helium one does. Now the true helium form factor ¹⁴⁾ has such a zero for $q^2 \sim 10 \text{ fm}^{-2}$, whereas there is no sign of such out to the limit of the carbon measurements ($q^2 \sim 11 \text{ fm}^{-2}$). Inopin ⁷⁾ has suggested that perhaps the size of the α particle is changed slightly inside a nucleus, and that the zero is displaced to just outside the measured region. In other words he considers including small admixtures of excited states of the α particle in his carbon wave function to explain the very high momentum transfer phenomena, and the simplicity of the picture is lost. In our opinion, we cannot trust the model for $q^2 \geq 10 \text{ fm}^{-2}$, which leaves rather in the dark about the small r behaviour. The pion scattering that we shall discuss later occurs at smaller momentum transfers than this, but in the multiple scatterings large q 's may intervene, and we must be careful not allow an artificially large S in this region.

In the Glauber calculation of the later sections, we shall need also the two-body form factor defined as

$$S^2(\vec{q}', \vec{q}) \equiv \int \rho(\vec{v}, \vec{w}) e^{i\vec{q}' \cdot \vec{v}} e^{i\vec{q} \cdot \vec{w}/2\sqrt{3}} d^3v d^3w \quad (2.11)$$

which has the property that

$$S^1(q) = S^2(0, 2\vec{q}) = S^2(\frac{1}{2}\vec{q}, \vec{q}) \quad (2.12)$$

the latter arising from the Bose symmetry of Eq. (2.2). In the Inopin model the S^2 can be expanded in Legendre polynomials in the angle δ between q' and q ,

$$S_I^2(\vec{q}', \vec{q}) = \sum_{\ell=0}^{\infty} \frac{(4\ell+1)(2\ell)!}{2^{2\ell}(\ell!)^2} (-1)^\ell j_{2\ell}(q'd) j_{2\ell}(qd/2\sqrt{3}) P_{2\ell}(\cos \delta) \quad (2.13)$$

For $\delta = 0$ the series can be summed to give

$$S_I^2(\vec{q}', \vec{q}) = j_0(d\sqrt{q'^2 + q^2/12}) \quad (2.14)$$

with its rather poor asymptotic behaviour.

The two-body form factor associated with the Gaussian trial density of Eq. (2.9) is the sum of eight Gaussian terms, easily calculable in closed form.

The Inopin and Gaussian models are just slightly different ansätze to the same $f(r)$ and as a result have qualitatively similar two-body correlation functions. To show the sensitivity to the nuclear model we want to adjust the correlation function without changing the nuclear form factor, and one very simple way of doing this is to assume that there is no dependence of S^2 upon the angle δ . Then for any one-body form factor S^1 the two-body one is uniquely determined as

$$S_{nc}^2(\vec{q}', \vec{q}) = S^1(\sqrt{3q'^2 + q^2/4}) \quad (2.15)$$

which is of the form noted in Eq. (2.14). It allows the three α 's to line up in a straight line as well as forming a triangle.

3. - THE PION ALPHA SCATTERING AMPLITUDE

After many years of careful work, the IISN-IPN group have now finished the analysis of their π - α scattering and total cross-section data in the energy range 110-260 MeV ¹⁴⁾. Previously lower energy measurements ¹⁵⁾ had been successfully analyzed in terms of phase shifts, the solutions being relatively well defined apart from certain discrete ambiguities ¹⁶⁾. Now it is not very profitable to try this on the new data, since at 260 MeV some six or seven complex phase shifts are required, and the data cannot really fix them all, leading to large coupling of the errors.

Phase shifts are interesting quantities because the wave equation is diagonal in angular momentum, so that unitarity can be applied in each partial wave. However, we have as yet no reliable pion nucleus optical potential to insert into the wave equation, and moreover unitarity is a very weak constraint when there are so many open inelastic channels. It seems therefore more useful to seek an alternative formalism where the parameters have a more physical interpretation.

If a partial wave analysis is terminated after $L+1$ terms, the resulting scattering amplitude is a polynomial of order L which can be factorized into L terms ¹⁷⁾

$$F(q) = F(0) \prod_{i=1}^L (1 - q^2/q_i^2) \quad (3.1)$$

Since, apart from small effects on the Coulomb nuclear interference, the differential cross-section does not depend upon the sign of the imaginary part of q_i^2 (the zeros of the amplitude), this demonstrates the well-known 2^L fold discrete ambiguity of phase shift analyses. Some of the combinations of the signs of the $\text{Im}(q_i^2)$ will be ruled out by unitarity, but in general only at low energies. At 260 MeV the π - α differential cross-section shows two well-defined minima, so that the real parts of the first two zeros of Eq. (3.1) are well determined from their positions, and the imaginary parts from their depths (up to a sign). The errors on the remaining q_i^2 are very strongly coupled. We therefore propose to analyze the helium data with the functional form

$$F(q) = F(0) (1 - q^2/q_1^2) (1 - q^2/q_2^2) e^{-\frac{1}{2}\beta^2 q^2} \quad (3.2a)$$

$$F(0) = \frac{i k \sigma}{4\pi} (1 - i\rho) \quad (3.2b)$$

To a first approximation the seven parameters are determined by different parts of the data, and so are well constrained independent of each other. The phase shifts can be easily deduced from this representation, but, of course, they are very model dependent. In particular, by choosing the residual slope parameter β^2 to be real we are making the explicit assumption that the phase variation of the amplitude is only influenced significantly by the two nearest zeros of the amplitude. Another motivation for

this form is that if the π - α amplitude is calculated from the π nucleon in single scattering approximation in this energy region, where only s, p and d waves are important, this leads to a quadratic in q^2 times a Gaussian helium form factor. One could hope that at these energies the multiple scatterings might change the positions of the zeros a bit without destroying the functional form. In fact the ansatz for $f(q)$ fits the new IISN-IPN measurements over their whole energy range, and angular range from the Coulomb nuclear interference region to near backward scattering¹²⁾. In Fig. 1 is shown the result of this fit for the trajectory of the first complex zero $q_1^2(T)$, together with the lower energy points obtained from phase shift analyses. As expected the trajectory is a smooth function of energy. The imaginary part of q_1^2 passes through zero at a little above the resonance energy, and the variation of $\text{Im}(q_1^2)$ with energy has a rather similar allure to that of the forward real part ρ . It has been argued by Nichitiu¹⁶⁾ that the sign of $\text{Im}(q_1^2)$ is determined by continuity from the lowest energy data where the alternative choice is ruled out by unitarity.

Also to be found in the figure is the prediction for the trajectory based upon a standard Glauber multiple scattering calculation taking for the four-body density the trivial form

$$\rho(r_1, r_2, r_3, r_4) \approx \exp(-\alpha^2 \sum r_i^2) - C \exp(-\alpha'^2 \sum r_i^2) \quad (3.3)$$

with $\alpha^2 = 0.60 \text{ fm}^{-2}$, $\alpha'^2 = 3.91 \text{ fm}^{-2}$, $C = 1.25$, which fits the helium charge distribution¹⁴⁾ very well. One difficulty which we shall return to later is that if the π -N amplitudes are parametrized just in terms of s, p and d waves,

$$f_{\pi N} = a + bq^2 + cq^4 \quad (3.4)$$

then the corresponding helium form factor does not damp the high momentum contributions sufficiently (one has integrands behaving as badly as q^{16} times a rather weak Gaussian) and the resulting cross-section can be arbitrarily large. Clearly we must insert into a Glauber calculation amplitudes which do not violate the basic assumptions too grossly, and the f of Eq. (3.4) corresponds to a potential which is extremely singular at the origin, and which is not smeared sufficiently over space by the helium density (3.3). The amplitudes must decrease for large q , and we have

obtained the predictions of Fig. 1 by damping f with Gaussian factors outside the physical region. When this is done, the single scattering dominates at low energies and the position of the first zero is quite close to that of the isoscalar π -N. At higher energies though the multiple scatterings ensure that $\text{Im}(q_1^2)$ passes through zero much before that of the π -N.

The situation is far less clear for the trajectory of the second zero, primarily because at low energies the second minimum is outside the physical region, and Glauber calculations provide no guide at all at such large scattering angles. We choose the branch of solutions which at 50-70 MeV give the better fit to the data with a reasonable value of the pion electromagnetic radius¹⁶⁾. The values of the total cross-sections were determined in a separate experiment, and using these the values of the relative real part ρ deduced from Coulomb nuclear interference. These are not very different from the predictions of the forward dispersion relations^{18),19)}. Numerical values and the associated errors will be found in the experimental article of the IISN-IPN group¹⁴⁾. One slight difficulty is that the new measurements are almost entirely with negative pions, and no attempt has been made to correct for the Coulomb distortion, which can be quite large at low energy. Although we shall mainly use the fitted amplitudes in a calculation for negative pion scattering from carbon, it would be ingenious to expect that the distortion would cancel completely between the two nuclei.

4. - THE GLAUBER MULTIPLE SCATTERING CALCULATION

In terms of the scattering amplitude $f(q)$ on the α particle subunits fixed at positions $\vec{r}_1, \vec{r}_2, \vec{r}_3$, the Glauber theory for ^{12}C yields only single, double and triple scattering⁸⁾

$$\hat{F}_1(\vec{q}) = 3 f(q) e^{i\vec{q} \cdot \vec{r}_1} \quad (4.1a)$$

$$\hat{F}_2(\vec{q}) = \frac{3i}{2\pi R} \int d^2q' f(\frac{1}{2}\vec{q}-\vec{q}') f(\frac{1}{2}\vec{q}+\vec{q}') \exp[i\vec{q}' \cdot (\vec{r}_1 - \vec{r}_2) + \frac{1}{2}i\vec{q} \cdot (\vec{r}_1 + \vec{r}_2)] \quad (4.1b)$$

$$\hat{F}_3(\vec{q}) = -\frac{1}{(2\pi k)^2} \int d^2q' d^2q'' f(\vec{q}/3 - \vec{q}' - \vec{q}'') f(\vec{q}/3 + \vec{q}') \\ \times f(\vec{q}/3 + \vec{q}'') \exp[i\vec{q}' \cdot (\vec{r}_2 - \vec{r}_1) + i\vec{q}'' \cdot (\vec{r}_3 - \vec{r}_1)] \quad (4.1c)$$

where the integration variables q', q'' lie in the plane perpendicular to the mean momentum $\vec{k}$. Elastic scattering is found by averaging the operators $\hat{F}_i$ over the ^{12}C ground state density, and this introduces the form factors defined in Section 2.

$$F_1(q) = 3 f(q) S^1(q) \quad (4.2a)$$

$$F_2(q) = \frac{3i}{2\pi k} \int d^2q' f(\frac{1}{2}\vec{q} - \vec{q}') f(\frac{1}{2}\vec{q} + \vec{q}') S^2(\vec{q}', \vec{q}) \quad (4.2b)$$

$$F_3(q) = -\frac{1}{(2\pi k)^2} \int d^2q' d^2q'' f(\vec{q}/3 - \vec{q}') f(\vec{q}/3 + \frac{1}{2}\vec{q}' + \vec{q}'') \\ \times f(\vec{q}/3 + \frac{1}{2}\vec{q}' - \vec{q}'') S^2(\vec{q}'', 3\vec{q}') \quad (4.2c)$$

Using the Inopin form factors of Eq. (2.13), the expressions for the double and triple scattering are only simple if a single Gaussian is assumed for the α amplitude $f(q)$. Using such a parametrization ⁸⁾, the 1 GeV proton scattering from ^{12}C is reproduced at least as well as that achieved by the Glauber model calculations starting from the nucleon-nucleon amplitudes. Despite remarks to the contrary in the literature ⁸⁾, it is not simple to generalize this result to the case where the α amplitude is the sum of two Gaussian terms. For the form which we must consider here of a polynomial times a Gaussian, the algebra becomes very tedious and as a check we have only calculated the double scattering, and even that with only one zero in the amplitude $f(q)$. It is at this point that the Gaussian parametrization of the Inopin model [Eq. (2.9)] comes into its own. Although in this model the double and triple scattering could be evaluated with two (or more) zero's in $f(q)$, we have for simplicity confined ourselves to just using one zero here.

The second one has been suppressed, and the slope parameter adjusted accordingly. We would argue that for the calculation to be believable the multiple scatterings must be dominated by small q phenomena, for which this replacement is reliable. There is in addition the slight change of the effective value of β^2 coming from the transformation from the π - ^4He to the π - ^{12}C centre-of-mass. With this simplification, the integrands of Eq. (4.2) for the double and triple scattering amplitudes become

$$I_2 = f(0)^2 \exp[-\beta^2(q^2/4 + q'^2)] S^2(\vec{q}', \vec{q}) \times \left[1 - 2(q^2/4 + q'^2)/q_1^2 + ((q^2/4 + q'^2)^2 - (\vec{q} \cdot \vec{q}')^2)/q_1^4 \right] \quad (4.3a)$$

$$I_3 = f(0)^3 \exp[-\beta^2(q^2/6 + 3q'^2/4 + q''^2)] S^2(\vec{q}'', \vec{q}') \times \left\{ 1 - (q^2/3 + 3q'^2/2 + 2q''^2)/q_1^2 + [q^4/27 + q^2(q'^2/6 + 2q''^2/q) + (9q'^4/16 + \frac{5}{2}q'^2q''^2 + q''^4 - (\vec{q}' \cdot \vec{q}'')^2)]/q_1^4 - [q^6/729 + q^2(9q'^4/16 - 3q'^2q''^2/2 + q''^4 + 3(\vec{q}' \cdot \vec{q}'')^2)/q + q'^2(q'^4/16 + q'^2q''^2/2 + q''^4 - (\vec{q}' \cdot \vec{q}'')^2)]/q_1^6 \right\} \quad (4.3b)$$

where use has been made of the symmetry in q .

Now since in our model the S^2 is Gaussian, the integration over q' and q'' can be easily performed to give closed expressions for the double and triple scattering amplitudes.

As a check on the sensitivity to the large q behaviour, the calculation can also be done simply if $f(q)$ is taken as the sum of two Gaussians. All the integrations are then over simple Gaussians and their evaluation merely a matter of book-keeping. The parameters of $f(q)$ can be adjusted to give a plausible representation for small q while decreasing fast for large q .

5. - CONFRONTATION WITH EXPERIMENT

When comparing theory and experiment, we must first ask if the calculation depends sensitively upon the parametrization of the Inopin model, i.e., on whether the α 's are rigidly separated or have a smoother distribution. There is then the question of whether the large q behaviour of the scattering amplitudes influences the answer. Only after these questions are answered can we ask if the Inopin model [Eq. (2.3)] gives a reasonable description, and for this we can study the predictions of the form factor of Eq. (2.15) with its less rigid correlations.

Consider the integral (4.2b) for the double scattering amplitude in the forward direction. If it is evaluated with the Inopin form factor, it follows from Eq. (2.13) that $S^2(q', 0)$ only decreases like $1/q'$ for large q' , so that a parametrization of $f(q)$ with only s , p and d waves [i.e., two zeros, but without the Gaussian of Eq. (3.2)] would lead to a very ill-defined expression. Although the introduction of the extra Gaussian factor in $f(q)$ yields finite answers, they are very dependent upon the behaviour of $f(q)$ for q 's well beyond the physical region, and this result is meaningless in Glauber theory. In this energy range it is only reasonable to use the Inopin form factor with one zero in $f(q)$. The suppression of the extra zero reduces the power of q'^2 in the integration, and since we then increase the size of p^2 to give the correct slope at the origin, the integrand is much reduced at large q'^2 . The first zero cannot be similarly removed since it influences crucially the small q behaviour. The sensitivity is even further reduced if the Gaussian form of the Inopin model [Eq. (2.9)] is employed. The constraint that the α 's be a constant distance apart being relaxed, the form factors decrease exponentially for large q . At 260 MeV the difference between the F_2 calculated with Eq. (2.6) or Eq. (2.9) is only a couple of per cent anywhere in the measured angular region, but at 67.5 MeV the situation is much worse. The lower the energy is, the smaller the value of the first zero, q_1^2 , and the larger the relative coefficient of q'^4 in the integrand of Eq. (4.2). For our purpose the Gaussian Jastrow factor is more reliable, and from this point onwards we shall stick to it. The predictions for the total cross-sections are shown in Fig. 2. It is probably purely accidental that if the theoretical curve is displaced downwards in energy by some 15 MeV it passes through almost all the experimental points. At low energies Glauber theory is no longer unitary, so that it is there a little dangerous to use the optical theorem to calculate the total cross-section, and at the higher energies

the agreement is on the 5% level. The forward real parts of the π - ^{12}C scattering amplitudes are compared in Fig. 3 with those measured by Coulomb nuclear interference ^{1),21)} and the predictions of forward dispersion relations ^{18),19)}. Again the dispersion relation curve seems displaced with respect to the Glauber curve by about the same energy, but the major discrepancy is the too large magnitude of $\text{Re}(F(0))$ at low energies. (This is of course consistent with having a too rapidly varying total cross-section in this region.)

In Figs. 4, 5 are illustrated the predictions for the differential cross-sections at 260, 180 and 120 MeV. At the highest energy the accord is very satisfactory and this is relatively stable to the choice of the large q behaviour of $f(q)$. Multiple scatterings calculated with f a sum of Gaussians or a polynomial times a Gaussian give cross-sections which hardly differ on the scale of Fig. 4. As the energy is lowered the agreement with experiment gets steadily worse, but so does the agreement between the two theoretical predictions. In this context the rise in the cross-section at the backward angles at 120 MeV almost certainly comes from the large unphysical contributions. The relative importance of single, double and triple scattering can be judged from Fig. 4, where a relatively small change in the phase or magnitude of the triple scattering can destroy any agreement. Another aspect of the data is to be found in Fig. 6, where the predictions for the trajectory of the first zero q_1^2 in the scattering from ^{12}C is given. The main disagreement there is in the imaginary part, essentially the depths of the diffraction minima. The differential cross-sections do not determine the sign of the imaginary part of q_1^2 , but the possibility that the 69.5 and 87.5 MeV points ²²⁾ have negative $\text{Im}(q_1^2)$ is probably ruled out by unitarity. If this were not the case, at some energy between 90 and 120 MeV, the first diffraction minimum would get much deeper.

Apart from the uncertainty in the large q behaviour, there is also the discrete ambiguity in $f(q)$ associated with the sign of the imaginary part of the second zero, which in our approximation influences only the single scattering. Taking the sign opposite to that we argued for in Section 3 makes the agreement worse at low energies, without changing noticeably the 260 MeV results.

If we take the attitude that the scattering theory is working reasonably well at this highest energy, then at this point we can ask if the nuclear model for the α - α correlation function makes any difference. That it does give a small effect can be seen from Fig. 4, where the result of using $S_{NC}^2(\vec{q}', \vec{q})$ of Eq. (2.15) is shown. By construction this two-body form factor gives the same charge form factor as that of Inopin (or the Gaussian version employed in this paper) but there is no correlation between the directions of $\vec{q}'$ and $\vec{q}$.

6. - CONCLUSIONS

We have calculated medium energy pion carbon elastic scattering in the naïve α particle model using Glauber theory. As the energy decreases the agreement with experiment gets poorer. It is universally accepted that this multiple scattering formalism must get less reliable at lower energies, but there is no simple criterion at what energy we should stop using it, and so people keep using it to as low an energy as there are data ! There are, however, certain obvious limitations. If a rigid triangle model is taken for ^{12}C and the π - α amplitudes parametrized with just s and p waves, the integral for the double scattering amplitude [Eq. (4.2b)] diverges. This divergence is primarily due to the scattering theory rather than the nuclear model, in that if we use the Brueckner multiple scattering scheme²³⁾ with non-overlapping α scatterers, then all answers are finite and unitarity is preserved. There is no a priori guarantee that Glauber theory will not even predict negative total cross-sections when we are outside its real domain of validity. If because of its great simplicity we want to apply the Glauber prescription, then we must at least ensure that the input amplitudes $f(q)$ and nuclear form factor $S(q)$ are well behaved at large q so that all the integrals are defined. In some ways this minimum condition is equivalent to demanding that the eikonal approximation be good for the nucleus as a whole, but not for the π - α input as well. The one-body optical potential in the α cluster model is

$$2\mu V = -\pi A f(q) S^1(q) \quad (6.1)$$

and, neglecting correlations, the eikonal solution to the scattering from this potential is just the Glauber ansatz. But if we take $f(q)$ as constant (just s waves) together with the Inopin form of a Bessel function for $S^1(q)$, then the eikonal approximation is very bad for the resulting δ shell potential until the energy is very high. The introduction of linear terms in $f(q)$ yields derivatives on this shell, and the potential becomes quite singular. If we want to use Glauber theory we must have a gentle behaviour of $V(q)$ at large q , and in the parametrization we have used we have enforced this. It is possible that a more realistic model which fits the π - α cross-sections would give a more reasonable q behaviour, but this is a difficult problem in strong interaction dynamics.

It has often been argued ⁴⁾ that the use of an optical potential avoids the eikonal problem, but in the ^{12}C case the number of α 's is only three, and these are strongly correlated. The correlation corrections to the $V(q)$ of Eq. (6.1) would therefore be very large, and so it might be more appropriate to try this method first for, say, ^{40}Ca . Much more promising is the Brueckner model approach ^{23),24)}, which like the Glauber scheme first considers the scattering from fixed centres and averages afterwards. If the scatterers do not overlap, it is in principle possible to solve the scattering problem exactly in terms of on-shell π - α scattering amplitudes. With many partial waves possible in the input, the resulting algebraic complications become enormous, but we are presently investigating this technique especially with respect to the low energy data.

Is it worth while taking the α model so seriously? In our opinion it is since despite its obvious drawbacks, lack of over-all antisymmetrization, it is the only microscopic model which can at present incorporate the pion absorption within a multiple scattering scheme in a simple and plausible way.

Let us end by emphasizing once again the usefulness of studying the zero's of the scattering amplitudes. It is at the very least a practical way of summarizing the energy dependence of a certain property of the scattering amplitude, allowing us to interpolate in energy and possibly in nucleon number also, in much the same way as is done for the total cross-section. (Compare the similarities between Fig. 1 and Fig. 6.) It might also provide a stable method for extrapolating the amplitudes to lower energies.

ACKNOWLEDGEMENTS

We are extremely grateful to the members of the IISN-IPN group, especially F. Binon and J. Jansen for many valuable discussions, and permission to use their data prior to publication. We also thank C. Rogers for supplying us with a copy of his thesis.

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FIGURE CAPTIONS

- Figure 1 Trajectory of the first complex zero in π - α elastic scattering. For energies above 100 MeV the points have been obtained by fitting formula (3.2) to the new IISN-IPN data ¹²⁾. The lower energy points come from the phase shift analysis ¹⁶⁾ of older measurements ¹⁵⁾. Also shown is the Glauber prediction for this quantity.
- Figure 2 The α cluster model prediction for the π - ^{12}C total cross-section compared with the measurements of Refs. 19) (triangles) and 20) (crosses).
- Figure 3 The α cluster model prediction ²⁾ for the real part of the forward π - ^{12}C amplitude compared with the predictions ¹⁾ based on forward dispersion relations ^{18),19)} and measurements using Coulomb nuclear interference of Ref. 1) (crosses) and 21) (circles).
- Figure 4 Predictions for single, single + double, and single + double + triple scattering, compared with the 260 MeV points of Ref. 1). The change in the subsidiary maximum when the angular correlation is relaxed [Eq. (2.15)] is illustrated (NC).
- Figure 5 Predictions of the α particle model at (a) 180 MeV, and (b) 120 MeV compared to the measurements of Ref. 1). The difference between taking the π - α input as a polynomial times a Gaussian (P) or the sum of two Gaussians (E) is clearly seen.
- Figure 6 Trajectory of the first complex zero of the π - ^{12}C scattering obtained by fitting the function (3.2) to the data of Refs. 1) and 22). The α cluster model predictions were obtained using a sum of two Gaussians for the π - α input.

Fig. 1

Fig. 2

Fig. 3

Fig. 4

Fig. 5a

Fig. 5b

Fig. 6