ANOMALOUS DIMENSIONS IN BACKGROUND FIELD GAUGES

S. Sarkar and H. Strubbe

CERN - Geneva

A B S T R A C T

Background field gauges, recently proposed for calculation of anomalous dimensions of operators in Wilson expansions in non-Abelian gauge theories, require the evaluation of extra diagrams. These diagrams are calculated and are found to give a correct gauge-invariant mixing (unlike in Fermi-type gauges), but nevertheless the anomalous dimensions of the operators, owing to interesting cancellations, remain unchanged from those found in the Fermi gauges.

INTRODUCTION

Non-Abelian asymptotically free field theories¹⁾ hold out the possibility of being useful in deep inelastic scattering²⁾. As usual a central role in the analysis of such phenomena has been played by the Wilson (or short-distance) and related expansions³⁾. However, writing down the Wilson expansion in such theories poses a problem. It is necessary to decide whether to include only naively gauge-invariant operators or not^{4,5)}. A related question is whether, even if we are forced in some circumstances to include non-gauge-invariant operators, the calculations in asymptotically free theories ignoring such subtleties are correct.

In Ref. 4 the necessity of non-gauge-invariant operators was argued on general grounds for a large class of gauges. However, gauges were also proposed in which only gauge-invariant operators need be considered. These latter gauges are the background field gauges^{6,7)} and differ in a non-trivial way from the usual Lorentz-type gauges in which calculations so far have been done²⁾. New diagrams occur in the calculation of anomalous dimensions. In this note we will examine by explicit calculation the role of these terms. *A priori* the extra terms may change the value of the anomalous dimensions of the operators occurring in the short distance expansion of two currents.

In order to be self-contained we shall give in Section 1 a very brief description of the background field gauge. In Section 2 we shall report on our calculations in this gauge.

1. BACKGROUND FIELD GAUGES

Given a Lagrangian L depending on a field A we can perturb around the classical solution A^{cl} (say). L then becomes a functional of A^{cl} and Q , which denotes $(A - A^{cl})$. So far we have just made a substitution, with nothing really gained. If A^{cl} satisfies the field equations then L has no linear term in Q . The Q part of A is the quantum field and it is this which flows in closed loops. A^{cl} , however, gives rise to tree diagrams; for clarity we shall give below a simple diagram in which the A^{cl} s and Q s are marked:

Whether L is a gauge theory or not, the background field split is a useful one especially for one loop calculations; but the method takes on a particularly important significance in gauge theories. This is because a gauge condition has to be specified. Such a term in the usual Fermi-

type gauges gives rise to the gauge invariance of the theory changing under renormalization. This is a familiar behaviour, and, when the original Lagrangian

L contains gauge-invariant composite operators (introduced via sources), the gauge invariance of the theory gets renormalized at the one-loop level in a source-dependent way⁴). This implies that the naively gauge-invariant operators mix with non-gauge-invariant operators. If a gauge-breaking term can be devised which breaks a gauge invariance but nonetheless permits L to retain some gauge invariance, then there is a possibility that gauge-invariant operators will mix only with gauge-invariant operators. The background field split into non-Abelian gauge theories coupled with the new gauge condition

$$(D_\mu Q_\mu)^a = C^a(x) \quad (1)$$

(where $D_\mu^{ac} = \partial_\mu \delta^{ac} - gf^{acb} A_\mu^{cb}$, $C^a(x)$ is some arbitrary C-number function, f^{abc} are the structure constants of the gauge group and A_μ^{cla} is the classical gauge field) gives rise to what is known as the background field gauge quantization⁶). This gauge condition is not equivalent to the usual Fermi-type gauges. Of course for condition (1) to have any meaning as a gauge fixing condition, it is necessary to specify how the Q and A^{cl} transform. Now the gauge transformation

$$A_\mu^a(x) \rightarrow A_\mu^a(x) - g f^{acb} A_\mu^c(x) \Lambda^b(x) - \partial_\mu \Lambda^a(x) \quad (2)$$

can be regarded in two different ways, firstly as

$$\begin{aligned} A_\mu^{cla}(x) &\rightarrow A_\mu^{cla}(x) - g f^{acb} A_\mu^{clc}(x) \Lambda^b(x) - \partial_\mu \Lambda^a(x) \\ Q_\mu^a(x) &\rightarrow Q_\mu^a(x) - g f^{acb} Q_\mu^c(x) \Lambda^b(x) \end{aligned} \quad (3)$$

and secondly as

$$\begin{aligned} A_\mu^{cla}(x) &\rightarrow A_\mu^{cla}(x) \\ Q_\mu^a(x) &\rightarrow Q_\mu^a(x) - g f^{acb} Q_\mu^c(x) \Lambda^b(x) - (D_\mu \Lambda(x))^a \end{aligned} \quad (4)$$

Here Λ is an infinitesimal group parameter. The specially useful feature of the condition (1) is that although it breaks the invariance (4), it nevertheless retains an invariance under (3). In particular, the classical field A^{cl} transforms under a naive gauge transformation. Since the quantum fields are the ones which flow in closed loops, the counter terms are functions of just A^{cl} , and consequently have the gauge invariance of the tree diagram Lagrangian.

As noted in Ref. 4 the gauge (1) leads to new interaction terms. Specifically these are

$$\frac{ig}{\xi} f^{abc} \left(-k_{\alpha} \delta_{\beta\gamma} + q_{\gamma} \delta_{\alpha\beta} \right) \quad (5)$$

(where the line with inflowing momentum p and gauge group and Lorentz indices given by b and β is classical and the other two lines quantum; ξ is a gauge parameter) and

$$-\frac{g^2}{\xi} \left(f^{hcb} f^{hda} \delta_{\mu\alpha} \delta_{\nu\beta} + f^{hdb} f^{hca} \delta_{\mu\beta} \delta_{\nu\alpha} \right) \quad (6)$$

(Here the top two lines are quantum and the remaining two classical.) The vector boson propagator in this gauge is

$$\frac{\delta^{ab}}{p^2 - i\epsilon} \left(\delta_{\alpha\beta} + (\xi - 1) \frac{p_{\alpha} p_{\beta}}{p^2 - i\epsilon} \right) \quad (7)$$

and so, apart from diagrams involving the new vertices, the situation is the same as in the Fermi-type gauges. Calculations in the literature have confined themselves to the ordinary Feynman gauge in which the vector boson propagator is just proportional to $\delta_{\alpha\beta}$.

From now on we will discuss the Lagrangian

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu}^a F_{\mu\nu}^a + i^{J-2} F_{\mu_1\nu}^a D_{\mu_2} \cdots D_{\mu_{J-1}} F_{\nu\mu_J}^a \chi_{\mu_1} \cdots \chi_{\mu_J} \quad (8)$$

where $F_{\mu\nu}^a = \partial_{\mu} A_{\nu}^a - \partial_{\nu} A_{\mu}^a + gf^{abc} A_{\mu}^b A_{\nu}^c$, χ_{μ} is a c-number source which satisfies $\chi_{\mu} \chi_{\mu} = 0$, D_{μ} is a covariant derivative, and J is the spin of the operator.

It has usually been assumed that the sum

$$\begin{aligned}
 & \left(\frac{1}{2}\right) \text{ (loop with black circle at top)} + \text{ (triangle with black circle at top)} + \left(\frac{1}{2}\right) \text{ (loop with black circle at top, left)} + \left(\frac{1}{2}\right) \text{ (loop with black circle at top, right)} \\
 & \alpha, \alpha, q \quad b, \beta, -q \quad \alpha, \alpha, q \quad b, \beta, -q \quad \alpha, \alpha, q \quad b, \beta, -q \quad \alpha, \alpha, q \quad b, \beta, -q
 \end{aligned} \tag{9}$$

(which gives the mixing of the source, denoted by the black circle) was gauge invariant by itself so that calculations in the Feynman gauge were adequate. Moreover, the assumption of gauge invariance encouraged the evaluation of only one term (namely, the $\delta_{\alpha\beta}$ term) in the sum.

In the background field gauge it is necessary to evaluate not only the sum in (9) but in addition

$$\begin{aligned}
 & \text{ (triangle with black circle at top, left)} + \text{ (triangle with black circle at top, right)} + \left(\frac{1}{2}\right) \text{ (loop with black circle at top, left)} \\
 & + \left(\frac{1}{2}\right) \text{ (loop with black circle at top, right)} + \text{ (triangle with black circle at top, bottom)} + \left(\frac{1}{2}\right) \text{ (loop with black circle at top, bottom)}
 \end{aligned} \tag{10}$$

From the argument given above we know that the sum of the expressions in (9) and (10) are gauge invariant; so *a priori* (9) and (10) may be both independently gauge invariant. Also (10) could, if non-zero, affect the value of the anomalous dimension.

2. THE ANOMALOUS DIMENSIONS

For clarity we shall give the results of (9) and (10) separately. This will allow us to see the role of the new interaction terms in the background field gauge.

Owing to the algebraic complexity of the calculation, we used the analytic computer language SCHOONSCHIP⁸⁾. (The details of the source vertices that we have used are included in an appendix; the Pauli metric is used in order to be compatible with SCHOONSCHIP.)

Let $I_1^{(J)}$ denote the logarithmic divergent contribution of (9) for insertion of an operator of spin J and similarly $I_2^{(J)}$ for (10). $I^{(J)}$ will be the sum of $I_1^{(J)}$ and $I_2^{(J)}$. For even values of J from 2 to 8 these are given in Appendix 2. A check on our calculations is that they satisfy the Ward identities (i.e. the expressions I_1 and I_2 are orthogonal to q). We note that neither (9) nor (10) is gauge invariant by itself, but as expected on general grounds the sum is gauge invariant (which is a reasonably stringent test of the correctness of the calculation). In fact, apart from an over-all factor of $[\pi^2/(4-n)] g^2 C_{ab}^{\delta}$ (Bare vertex) (where $C_{ab}^{\delta} = f^{cda} f^{cdb}$ and n is the continued dimension of space-time), the logarithmic divergent parts of the sum of all the graphs in (9) and (10) is the following:

$$\begin{aligned} I^{(2)} &= \frac{22}{3} \\ I^{(4)} &= \frac{236}{15} \\ I^{(6)} &= \frac{403}{21} \\ I^{(8)} &= \frac{968}{45} \end{aligned} \tag{11}$$

In dimensional regularization the anomalous dimension⁹⁾ of a multiplicatively renormalized operator O is given by γ_O

$$\gamma_O = \lim_{n \rightarrow 4} \left\{ \frac{1}{2} g^{(n-4)} \frac{\partial}{\partial g} \log Z_O \Big|_{\mu} \right\} \tag{12}$$

where g is the bare coupling constant and μ can be regarded as a subtraction point in the dimensional regularization scheme. Z_O is the multiplicative renormalization factor for O , i.e.

$$O_{\text{bare}} = Z_O O_{\text{renormalised}} \tag{13}$$

In order to extract Z_O from (11) we have to take into account the wave function renormalization of the external lines (which is ξ independent). Applying (12) we obtain for the anomalous dimension, apart from an over-all factor $\pi^2 g^2 C$,

$$\begin{aligned}
 \gamma^{(2)} &= 0 \\
 \gamma^{(4)} &= \frac{42}{5} \\
 \gamma^{(6)} &= \frac{83}{7} \\
 \gamma^{(8)} &= \frac{638}{45}
 \end{aligned}
 \tag{14}$$

The anomalous dimension of the energy momentum tensor is zero from general arguments, and our calculation gives this too. Moreover, the remaining numbers agree with those given in the literature [taking into account a difference of $(2\pi)^4$ in the conventions]. Hence we see that the extra terms in the background field gauges fix up the gauge invariance of the mixing (cf. Appendix 2), but remarkably enough the wave function renormalization in the background field gauge is such as to wipe out any numerical change in the anomalous dimensions.

Acknowledgements

We are grateful to G. 't Hooft and R.J. Crewther for useful discussions as well as for showing us some of their calculations, and M. Rochon who did some of the computing.

REFERENCES

- 1) G. 't Hooft, Remarks (unpublished) at Colloquium on Renormalization of Yang-Mills Fields and Applications to Particle Physics, Marseille, 1972.
H.D. Politzer, Phys. Rev. Letters 30 (1973) 1346.
D.J. Gross and F. Wilczek, Phys. Rev. Letters 30 (1973) 1343.
- 2) D.J. Gross and F. Wilczek, Phys. Rev. D9 (1974) 980.
H. Georgi and H.D. Politzer, Phys. Rev. D9 (1974) 416.
- 3) K. Wilson, Phys. Rev. 179 (1969) 1499.
See also the lectures given by R.A. Brandt and G. Preparata, and Y. Frishman, at the Coral Gables Conference on Fundamental Interactions at High Energy, Miami, 1971 (Gordon and Breach, New York, 1971) Vol. 2, pp. 43 and 61.
- 4) S. Sarkar, Mixing of operators in Wilson expansions, CERN TH-1889 (Nuclear Physics B, to be published).
- 5) J.C. Taylor, Renormalization of Wilson operators in gauge theories, Oxford preprint 1974.
W. Kainz, W. Kummer and M. Schweda, Nuclear Phys. B79 (1974) 484.
H. Kluberg-Stern and J.B. Zuber, Ward identities and some clues to the renormalization of gauge invariant operators, Saclay preprint DPh-T/74-56, 1974.
- 6) J. Honerkamp, Nuclear Phys. B48 (1972) 269.
G. 't Hooft, Nuclear Phys. B62 (1973) 444.
- 7) B.S. De Witt, Phys. Rev. 162 (1967) 1195, 1239.
- 8) M. Veltman, SCHOONSHIP, A CDC 6600 program for symbolic evaluation of algebraic expressions, CERN preprint 1967.
- 9) G. 't Hooft, Nuclear Phys. B61 (1973) 455.
M.J. Holwerda, W.L. Van Neerven and R.P. Van Royen, Nuclear Phys. B75 (1974) 302.
S. Sarkar, Dimensional regularization and broken conformal Ward identities, CERN TH-1831 (Nuclear Phys. B, to be published).
D.A. Akyeampong and R. Delbourgo, Nuovo Cimento 19 A (1974) 219.

APPENDIX 1

Source-dependent vertices required in the calculation:

$$- \delta_{ab} (1 + (-1)^J) \left\{ (\chi \cdot l)^{J-1} (\chi_\alpha l_\beta + \chi_\beta l_\alpha) - l^2 (\chi \cdot l)^{J-2} \chi_\alpha \chi_\beta - (\chi \cdot l)^J \delta_{\alpha\beta} \right\}$$

$$- i g f^{abc} (1 + (-1)^J) \{ \theta(J-3) X + Y \}$$

where

$$\begin{aligned} X = & \sum_{s=2}^{J-1} (-1)^s \left[(\chi \cdot l)^{s-2} (-\chi \cdot q)^{J-s-1} \chi_\beta (\chi \cdot l \delta_{\alpha\beta} - l_\beta \chi_\alpha) (-q_\beta \chi_\delta + \chi \cdot q \delta_{\beta\delta}) \right. \\ & + (\chi \cdot (q-l))^{s-2} (\chi \cdot l)^{J-s-1} \chi_\delta (\chi \cdot (q-l) + (l-q)_\beta \chi_\beta) (l_\beta \chi_\alpha - \chi \cdot l \delta_{\beta\alpha}) \\ & \left. + (-\chi \cdot q)^{s-2} (\chi \cdot (q-l))^{J-s-1} \chi_\alpha (-\chi \cdot q \delta_{\beta\delta} + q_\beta \chi_\delta) ((q-l)_\beta \chi_\beta + \chi \cdot (l-q) \delta_{\beta\beta}) \right] \end{aligned}$$

and

$$\begin{aligned} Y = & \left((\chi \cdot l)^{J-2} \theta(J-3) + \delta_{J2} \right) (l_\beta \chi_\alpha - \chi \cdot l \delta_{\alpha\beta}) (\chi_\beta \delta_{\beta\delta} - \chi_\delta \delta_{\beta\beta}) \\ & + \left((\chi \cdot (q-l))^{J-2} \theta(J-3) + \delta_{J2} \right) ((q-l)_\beta \chi_\beta + \chi \cdot (l-q) \delta_{\beta\beta}) (\chi_\delta \delta_{\beta\alpha} - \chi_\alpha \delta_{\beta\delta}) \\ & + \left((-\chi \cdot q)^{J-2} \theta(J-3) + \delta_{J2} \right) (-q_\beta \chi_\delta + \chi \cdot q \delta_{\beta\delta}) (\chi_\alpha \delta_{\beta\beta} - \chi_\beta \delta_{\beta\alpha}) \end{aligned}$$

(Our conventions are such that all momenta are taken flowing into the vertices, and $\theta(x) = 1$ for $x \geq 0$ but zero otherwise.)

APPENDIX 2

$$\begin{aligned} I_1^{(2)} &= \chi_\alpha \chi_\beta q^2 \left(\xi - \frac{19}{3} \right) \\ &+ (\chi_\alpha q_\beta + q_\alpha \chi_\beta) \chi \cdot q \left(-\xi + \frac{16}{3} \right) \\ &+ \delta_{\alpha\beta} (\chi \cdot q)^2 \left(\xi - \frac{13}{3} \right) \end{aligned}$$

$$\begin{aligned} I_2^{(2)} &= \chi_\alpha \chi_\beta q^2 (-\xi - 1) \\ &+ (\chi_\alpha q_\beta + q_\alpha \chi_\beta) \chi \cdot q (\xi + 2) \\ &+ \delta_{\alpha\beta} (\chi \cdot q)^2 (-\xi - 3) \end{aligned}$$

$$\begin{aligned} I_1^{(4)} &= \chi_\alpha \chi_\beta (\chi \cdot q)^2 q^2 \left(\xi - \frac{196}{15} \right) \\ &+ (\chi_\alpha q_\beta + q_\alpha \chi_\beta) (\chi \cdot q)^3 \left(-\xi + \frac{387}{30} \right) \\ &+ \delta_{\alpha\beta} (\chi \cdot q)^4 \left(\xi - \frac{191}{15} \right) \end{aligned}$$

$$\begin{aligned} I_2^{(4)} &= \chi_\alpha \chi_\beta (\chi \cdot q)^2 q^2 \left(-\xi - \frac{40}{15} \right) \\ &+ (\chi_\alpha q_\beta + q_\alpha \chi_\beta) (\chi \cdot q)^3 \left(\frac{85}{30} + \xi \right) \\ &+ \delta_{\alpha\beta} (\chi \cdot q)^4 (-\xi - 3) \end{aligned}$$

$$\begin{aligned}
 I_1^{(6)} &= \chi_\alpha \chi_\beta (\chi \cdot q)^4 q^2 \left(\xi - \frac{1714}{105} \right) \\
 &+ (\chi_\alpha q_\beta + q_\alpha \chi_\beta) (\chi \cdot q)^5 \left(-\xi + \frac{1707}{105} \right) \\
 &+ \delta_{\alpha\beta} (\chi \cdot q)^6 \left(\xi - \frac{1700}{105} \right)
 \end{aligned}$$

$$\begin{aligned}
 I_2^{(6)} &= \chi_\alpha \chi_\beta (\chi \cdot q)^4 q^2 \left(-\xi - \frac{301}{105} \right) \\
 &+ (\chi_\alpha q_\beta + q_\alpha \chi_\beta) (\chi \cdot q)^5 \left(\frac{308}{105} + \xi \right) \\
 &+ \delta_{\alpha\beta} (\chi \cdot q)^6 (-3 - \xi)
 \end{aligned}$$

$$\begin{aligned}
 I_1^{(8)} &= \chi_\alpha \chi_\beta (\chi \cdot q)^6 q^2 \left(\xi - \frac{11707}{630} \right) \\
 &+ (\chi_\alpha q_\beta + q_\alpha \chi_\beta) (\chi \cdot q)^7 \left(-\xi + \frac{23369}{1260} \right) \\
 &+ \delta_{\alpha\beta} (\chi \cdot q)^8 \left(\xi - \frac{833}{45} \right)
 \end{aligned}$$

$$\begin{aligned}
 I_2^{(8)} &= (\chi \cdot q)^6 q^2 \chi_\alpha \chi_\beta \left(-\frac{41}{14} - \xi \right) \\
 &+ (\chi_\alpha q_\beta + q_\alpha \chi_\beta) (\chi \cdot q)^7 \left(\xi + \frac{83}{28} \right) \\
 &+ \delta_{\alpha\beta} (\chi \cdot q)^8 (-\xi - 3)
 \end{aligned}$$