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SPIN EFFECTS IN HADRON SCATTERING AT VERY HIGH ENERGIES

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A B S T R A C T

In this paper a spin-dependent diffractive model is studied for πN and NN elastic scattering. We find non-trivial results for spin correlation parameters which persist at high energies and have very distinct features in the dip region. It is stressed that these parameters provide good tests of Pomeron factorization.

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1. INTRODUCTION

In this paper we study the predictions of a spin-dependent diffractive model ^{1),*)} at high energies for the processes $\pi N \rightarrow \pi N$ and $NN \rightarrow NN$. There are several reasons for interest in polarization effects at high energies, which we now briefly summarize.

- i) Polarized proton beam and target experiments have already been carried out ²⁾ at 6 GeV/c and find appreciable spin correlation effects. It is obviously desirable to obtain theoretical understanding of the main features of this kind of data.
- ii) Polarization effects at high energy are also a possible testing ground of Pomeron factorization. In the spin-dependent diffractive model for $NN \rightarrow NN$ presented earlier, it was found that experimental measurements near the forward direction would not provide very sensitive tests of factorization. Here we follow on a suggestion that measurements of C , K , P parameters away from the forward direction provide more sensitive tests, and we examine these quantities in the framework of the model ¹⁾.
- iii) It is very interesting to study how much spin effects may realistically contribute to the filling in of the dip in proton-proton elastic scattering, and how important spin effects are at large values of t for $d\sigma/dt$.

In Section 2 we consider what can be learnt in πN scattering and we will also outline the essential features of the model which is used for spin-dependent pp elastic scattering in Section 3. We discuss the results in Section 4.

2. πN scattering

In this section we describe briefly our model for the case of πN elastic scattering.

In the eikonal model, the helicity non-flip amplitude f_{++} is given by ^{**)}

$$f_{++}(t) = \frac{k}{i} \int_0^\infty J_0(b\sqrt{-t}) (e^{i\chi_0(b)} - 1) b db \quad (1)$$

*) The model presented in Ref. 1) was studied only in the forward direction because of interest in factorization tests. Here we are studying a completely different region away from $t=0$.

**) For a review see, for example, Ref. 3).

while the helicity flip amplitude is

$$f_{+-}(t) = k \int_0^{\infty} J_1(b\sqrt{-t}) e^{i\chi_0(b)} \chi_1(b) b db \quad (2)$$

where k is the centre-of-mass momentum, t is the momentum transfer, and $\chi_{0,1}(b)$ are eikonal phase shifts dependent on the impact parameter b .

The Born terms f_{++}^B , f_{+-}^B are identified by

$$\begin{aligned} f_{++}^B(t) &= k \int_0^{\infty} J_0(b\sqrt{-t}) \chi_0(b) b db \\ f_{+-}^B(t) &= k \int_0^{\infty} J_1(b\sqrt{-t}) \chi_1(b) b db \end{aligned} \quad (3)$$

In the spinless Chou-Yang model, the Born term is taken as proportional to the square of the electromagnetic form factor. A natural generalization of this to the case of particles with spin is to take the Born term proportional to the product of the pion and nucleon currents (denoted j_{π}^{μ} and j_N^{μ} , respectively), i.e., proportional to $j_{\pi}^{\mu} j_N^{\mu}$ where

$$j_{\pi}^{\mu} = F_{\pi}(q_1 + q_2)^{\mu} \quad \text{and} \quad j_N^{\mu} = \bar{u} \left(\gamma_{\mu} F_1 + i \frac{\mu_A}{2M} \sigma_{\mu\nu} q^{\nu} F_2 \right) u \quad (4)$$

Here q_1 and q_2 are, respectively, the incoming and outgoing pion momenta, q is the momentum transfer between the nucleons of mass M and anomalous moment μ_A , and F_{π} , $F_{1,2}$ are the usual pion and nucleon form factors. Just as in the Chou-Yang model, we take a purely imaginary diffractive Born term.

These assumptions give

$$\begin{aligned} f_{++}^B(t) &\propto i\sqrt{s} F_{\pi} F_1 \\ f_{+-}^B(t) &\propto i\sqrt{s} \sqrt{-t} \left(\frac{\mu_A}{2M} \right) F_{\pi} F_2 \end{aligned} \quad (5)$$

which can be rewritten as

$$\begin{aligned} f_{++}^B(t) &= i\tilde{c}^2 F_{\pi} F_1 \\ f_{+-}^B(t) &= i\sqrt{-t} \tilde{\lambda} \tilde{c} F_{\pi} F_2 \end{aligned} \quad (6)$$

where $\tilde{c}$ is an over-all normalization parameter, and $\tilde{d}/\tilde{c}$ is given by $\mu_A/2M$, by comparison with Eqs. (5). We take as a first approximation $F_1 = F_2 = 1/(1 - t/\Lambda)^2$ and $F_\pi = 1/(1 - t/\Lambda)$, where $\Lambda = 0.71 \text{ GeV}^2$. On substituting Eqs. (6) in Eqs. (3), we obtain by inverting:

$$\begin{aligned}\chi_0(b) &= i \tilde{u} \tilde{k}_2(b\sqrt{\Lambda}) \\ \chi_1(b) &= i \tilde{u} \frac{\tilde{d}}{\tilde{c}} \frac{\Lambda}{2} b \tilde{k}_1(b\sqrt{\Lambda})\end{aligned}\quad (7)$$

where $\tilde{u} \equiv \tilde{c}^2 \Lambda / 4k$; $\tilde{K}_n(z) \equiv 2/\Gamma(n)(z/2)^n K_n(z)$ and $K_n(z)$ is the modified Bessel function. Using these expressions (7) for $\chi_{0,1}$, we find from Eqs. (1) and (2) the following expressions for f_{++} and f_{+-}

$$f_{++}(t) = -ik \int_0^\infty J_0(b\sqrt{-t}) \left(e^{-\tilde{u} \tilde{k}_2(b\sqrt{\Lambda})} - 1 \right) b db \quad (8a)$$

$$f_{+-}(t) = \frac{ik\tilde{u}}{2} \frac{\tilde{d}}{\tilde{c}} \Lambda \int_0^\infty J_1(b\sqrt{-t}) e^{-\tilde{u} \tilde{k}_2(b\sqrt{\Lambda})} \tilde{k}_1(b\sqrt{\Lambda}) b^2 db \quad (8b)$$

Equations (8a) and (8b) can be used to derive the following interesting relation

$$f_{+-}(t) = \frac{\tilde{d}}{\tilde{c}} \sqrt{-t} f_{++}(t) \quad (9)$$

This relation holds in any eikonal model, provided the following relation holds

$$\chi_1(b) = -\frac{\tilde{d}}{\tilde{c}} \frac{d}{db} (\chi_0(b)) \quad (10)$$

We have checked that the conditions for this relation to be true are that we have an input Born term which factorizes and which has the same t dependence for flip and non-flip amplitudes. In other words, it depends on our assumption that $F_1(t)$ is proportional to $F_2(t)$.

The relation (10) is reminiscent of what is normally called "b universality", which has been found valid phenomenologically for non-diffractive processes ⁴⁾.

We now have a model where everything is fixed except $\tilde{u}$ and the ratio $\tilde{d}/\tilde{c} = \mu_A/2M$. We regard the latter ratio as having some freedom since it depends on which anomalous moment is taken ¹⁾. Hence we regard $\mu_A/2M$ as a variable to be fitted to data, as outlined below. On the other hand, $\tilde{u}$ is fixed by the optical theorem

$$\sigma_{\text{tot}} = \frac{4\pi}{k} \text{Im} f_{++}(0)$$

The cross-section $d\sigma/dt$ is then given by

$$\frac{d\sigma}{dt} = \frac{\pi}{k^2} (|f_{++}|^2 + |f_{+-}|^2) = \frac{\pi}{k^2} (f_{++})^2 \left(1 + \left(\frac{\tilde{d}}{\tilde{c}}\right)^2 |t| \right) \quad (11)$$

while the polarization R parameter ^{*)} is given by

$$R = \frac{2 \text{Re}(f_{++} f_{+-}^*)}{|f_{++}|^2 + |f_{+-}|^2}$$

Hence:

$$R = 2\sqrt{-t} \left(\frac{\tilde{d}}{\tilde{c}}\right) \frac{1}{1 + \left(\frac{\tilde{d}}{\tilde{c}}\right)^2 |t|} \quad (12)$$

Of course, the A parameter is given by $A^2 = 1 - R^2$ since $P = 0$ in our diffractive model.

According to Ref. 6) where an amplitude analysis was carried out for isoscalar contributions in the t channel, a ratio $f_{+-}/\sqrt{-t} f_{++}$ was found equal to -0.17 ± 0.02 at 6 GeV/c and -0.14 ± 0.03 at 16 GeV/c, so we take this ratio as -0.15 , and hence obtain [from Eq. (9)] $\tilde{d}/\tilde{c} = \mu_A/2M = -0.15$. We regard this as an estimate of an "effective magnetic moment" ¹⁾.

The predicted cross-section is shown in Fig. 1, where it is noted that there is a dip at $t \approx -3.4 \text{ GeV}^2$ and a second maximum at $t = -5.2 \text{ GeV}^2$. This result was obtained by normalizing to $\sigma_{\text{tot}} = 31 \text{ mb}$. However, if a total cross-section $\sigma_{\text{tot}} = 26 \text{ mb}$ is used, the dip is very far out. It is worth noting that spin effects vanish also at the dip in this case.

It is remarkable that the R parameter is quite large, e.g., at $t = -1 \text{ GeV}^2$, $R = -30\%$; and that it has such a simple form [see, e.g., Eq. (12) and Fig. 4].

*) Chou and Yang have also discussed the R parameter in their spin-dependent geometrical velocity profile description inside the hadron ⁵⁾.

3. NN ELASTIC SCATTERING

The essential features of the model for NN scattering are the same as those of Section 2 for πN scattering. The model for NN scattering has been described in full detail in Ref. 1) to which we refer the reader. In that paper ¹⁾, however, we were only interested in the forward direction. We now consider the model away from the forward direction; the results are obtained as follows.

The expressions for the amplitudes φ_1 to φ_5 are given in Ref. 1) by Eq. (5) in terms of five functions of impact parameters χ_+ , χ_- , δ_0 , δ_1 and ϵ . The latter five functions are given in terms of the Born terms ϕ_1^B to ϕ_5^B in Eq. (7) of Ref. 1). The Born terms are given by Eq. (8) of Ref. 1) and we use dipole form factors, as in Section 2, to obtain the following expressions

$$\begin{aligned}
 \chi_+ &= iu \left[\tilde{k}_3(b\sqrt{\Lambda}) + \frac{d^2}{4c^2} \wedge \left(\tilde{k}_2(b\sqrt{\Lambda}) - \frac{\Lambda b^2}{2} \tilde{k}_1(b\sqrt{\Lambda}) \right) \right] \\
 \delta_0 &= iu \left[\tilde{k}_3(b\sqrt{\Lambda}) - \frac{d^2 \Lambda}{2c^2} \left(\tilde{k}_2(b\sqrt{\Lambda}) - \frac{\Lambda b^2}{4} \tilde{k}_1(b\sqrt{\Lambda}) \right) \right] \\
 \delta_1 &= iu \left[\tilde{k}_3(b\sqrt{\Lambda}) + \frac{d^2}{c^2} \frac{\Lambda^2 b^2}{8} \tilde{k}_1(b\sqrt{\Lambda}) \right] \\
 \chi_- \sin 2\epsilon &= -iu \frac{d^2}{4c^2} \wedge \tilde{k}_2(b\sqrt{\Lambda}) \\
 \chi_- \cos 2\epsilon &= i \frac{d}{2c} u \wedge b \tilde{k}_2(b\sqrt{\Lambda})
 \end{aligned} \tag{13}$$

Here $d/c \equiv \mu_A/2M$ (and hence equal to $\tilde{d}/c$ of Section 2), and u is an over-all normalization constant ^{*)}. We take $\mu_A/2M = -0.15$ as in the

*) Here u is determined by the optical theorem from the total NN cross-section

$$\sigma_{tot} = \frac{4\pi}{k} \operatorname{Im} \left(\frac{\varphi_1 + \varphi_3}{2} \right) \Big|_{t=0}$$

previous Section. We now use the expression given above [Eq. (13)] to obtain, as just explained, the amplitudes ^{*}) $\varphi_1(t)$ to $\varphi_5(t)$.

We are now in a position to calculate the polarization parameters C , K , D , using the expressions given for them in Ref. 7). The C_{NN} , K_{NN} and D_{NN} parameters have already been presented in Ref. 8).

The results were calculated using a total cross-section 40.6 mb at p_{lab} 1500 GeV/c. In Fig. 2 are plotted C_{NN} , K_{NN} , D_{NN} ; in Fig. 3 C_{KK} , K_{KK} , D_{KK} ; in Fig. 4 C_{PK} , K_{PK} , D_{PK} and D_{PP} . We note that $C_{NN} = K_{NN}$, $C_{KK} = K_{KK}$ and $K_{PK} = -C_{PK}$, and this reflects the fact that φ_1 and φ_3 are almost equal. The same quantities as above were also calculated at $p_{lab} = 6$ GeV/c ($\sigma_{tot} = 40.6$ mb), and the results are shown in Fig. 5 (C_{KK} , K_{KK} , D_{KK}) and Fig. 6 (C_{PK} , K_{PK} , D_{PK} and D_{PP}). While the quantities C_{NN} , K_{NN} , D_{NN} do not depend on energy, the other quantities have a strong energy dependence.

The cross-sections $d\sigma/dt$ are plotted in Fig. 7 for $\sigma_{tot} = 40.6$ mb and for $\sigma_{tot} = 43.2$ mb ($d\sigma/dt$ is independent of p_{lab}). From the latter graph it will be seen that the dip is filled in somewhat by spin effects, but not sufficiently to explain the experimental data. We would expect real parts also to contribute to the filling of the dip. It is also to be noted that the larger the σ_{tot} , the smaller the $|t|$ value where the dip occurs, and the higher the second maximum.

We would like to remark that there is a lot of activity in the polarization parameters C , K , D around the position of the dip.

*) In particular we have

$$\phi_+(t) \equiv \frac{\varphi_1(t) + \varphi_3(t)}{2} = -i\mathcal{L} \int_0^\infty J_0(b\sqrt{-t}) \left(e^{-u\tilde{\kappa}_3(b\sqrt{\Lambda})} - 1 \right) b db$$

$$\varphi_r(t) = \frac{icd\Lambda^2}{24} \int_0^\infty J_1(b\sqrt{-t}) e^{-u\tilde{\kappa}_3(b\sqrt{\Lambda})} \tilde{\kappa}_2(b\sqrt{\Lambda}) b^2 db$$

together with non-trivial expressions for $\varphi_2(t)$, $\varphi_4(t)$ which we do not write down here ($\varphi_1 - \varphi_3$ turns out to be negligible). From these expressions it can be shown that

$$\varphi_r(t) = \frac{d}{dt} \sqrt{-t} \phi_+(t)$$

which is the analogous equation to Eq. (9) of Section 2. It is to be noted that $\tilde{\kappa}_2$ appears in φ_5 -- see footnote 14) of Ref. 9).

4. CONCLUSIONS

We have studied a diffractive model in which there are no real parts, and hence we regard our predictions for spin effects to provide a skeleton towards which experimental results should tend as energy increases. [For the feasibility of ISR polarized beam experiments, see Ref. 10)].

We have found in our model that spin effects fill in the dip in the proton-proton cross-section, but not sufficiently to agree with data ^{*)} for which real parts are necessary.

We have studied carefully the proton-proton polarization parameters C , K , D which persist at very high energy and find strong variation in the region of the dip in $d\sigma/dt$.

We also find that measurements of C , K , D parameters provide strong tests of Pomeron factorization for the following reason. In a simple factorizing normal parity Pomeron pole model, one finds all the C 's and all the K 's are zero ¹¹⁾, while we find large and presumably observable deviations from zero in the dip region (see Figs. 2-6). Factorization also implies ¹¹⁾ $D_{NN} = 1$, while we observe a drop to 0.89 at the dip (Fig. 2). Factorization ¹¹⁾ predicts D_{PP} is the same as the A parameter in πN scattering; while we observe D_{PP} can differ considerably from A , especially in the dip region (see Fig. 4). Similarly, factorization predicts D_{KP} is the same as the R parameter in πN scattering: again we find marked differences.

In πN scattering we have a simple prediction [Eq. (12)] for R [which has no variation in the dip region (see Fig. 4)]. This should be easily tested experimentally. We have also a result for $d\sigma/dt$ in πN elastic scattering at very high energies with an exact zero at $t = -3.4$ GeV, if σ_{tot} is taken to be 31 mb.

In summary, we conclude that measurements of C , K , D parameters ^{**)} provide sensitive tests of factorization (especially for the Pomeron), the main features of which we would expect to be given by our spin-dependent model, even though we have not included real parts. This question of the inclusion of real parts is a subject which should be investigated carefully, especially at low energy.

*) Spin effects alone do not fill in the dip as much as one might expect. This is because ϕ_+ is proportional to ζ_5 in our model, and both amplitudes vanish at the dip. In turn, this happens because we have taken a model with $F_1(t)$ and $F_2(t)$, as remarked earlier. Models which do not make this latter assumption may fill the dip in more, and deserve further study.

**) We have been informed by A. Yokosawa that experiments will be carried out in the near future to measure most of the quantities we have calculated.

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Figure captions

- Fig. 1 : πN elastic differential cross-section versus t , normalized to $\sigma_{\text{tot}} = 31 \text{ mb}$.
- Fig. 2 : C_{NN} , K_{NN} and D_{NN} at $p_{\text{lab}} = 1500 \text{ GeV/c}$ normalized with $\sigma_{\text{tot}} = 40.6 \text{ mb}$.
- Fig. 3 : C_{KK} , K_{KK} and D_{KK} at $p_{\text{lab}} = 1500 \text{ GeV/c}$ normalized with $\sigma_{\text{tot}} = 40.6 \text{ mb}$.
- Fig. 4 : C_{PK} , K_{PK} , D_{PK} and D_{PP} in pp elastic and A , R parameters in πp elastic at $p_{\text{lab}} = 1500 \text{ GeV/c}$ normalized with $\sigma_{\text{tot}} = 40.6 \text{ mb}$ for pp .
- Fig. 5 : C_{KK} , K_{KK} , D_{KK} at $p_{\text{lab}} = 6 \text{ GeV/c}$ normalized with $\sigma_{\text{tot}} = 40.6 \text{ mb}$.
- Fig. 6 : C_{PK} , K_{PK} , D_{PK} and D_{PP} at $p_{\text{lab}} = 6 \text{ GeV}$ normalized with $\sigma_{\text{tot}} = 40.6 \text{ mb}$.
- Fig. 7 : Proton-proton differential cross-section versus t , including spin. The solid curve is normalized with $\sigma_{\text{tot}} = 43.2 \text{ mb}$ and the dashed curve with $\sigma_{\text{tot}} = 40.6 \text{ mb}$.

FIG. 1

$d/c = -0.15$ $p_{\text{lab}} = 1500 \text{ GeV/c}$ $\text{---} K_{\text{NN}} = C_{\text{NN}}$
 $\sigma_{\text{tot}} = 40.60 \text{ mb}$ $\text{---} D_{\text{NN}}$

FIG.2

$d/c = -0.15$ $p_{\text{lab}} = 1500 \text{ GeV}/c$ — $K_{KK} = C_{KK}$
 $\sigma_{\text{tot}} = 40.60 \text{ mb}$ - - - D_{KK}

FIG.3

$d/c = -0.15$ $p_{\text{lab}} = 1500 \text{ GeV/c}$ $\sigma_{\text{tot}} = 40.60 \text{ mb}$

$\cdots\cdots\cdots K_{PK} = -C_{PK}$
 $\cdots\cdots\cdots D_{PK}$
 $\cdots\cdots\cdots D_{PP}$
 $\cdots\cdots\cdots (-R)$
 $\circ\circ\circ\circ\circ A$

FIG. 4

$d/c = -0.15$ $p_{\text{lab}} = 6 \text{ GeV}/c$ — $K_{KK} = C_{KK}$
 $\sigma_{\text{tot}} = 40.60 \text{ mb}$ - - - D_{KK}

FIG. 5

$d/c = -0.15$ $p_{\text{lab}} = 6 \text{ GeV}/c$ $\sigma_{\text{tot}} = 40.60 \text{ mb}$

$\text{---+---} \quad K_{PK} = -C_{PK}$
 $\text{---} \quad D_{PK}$
 $\text{---} \quad D_{PP}$

FIG. 6

FIG. 7