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SHORT-DISTANCE ANALYSIS OF THE TRIANGLE ANOMALY
IN QUANTUM ELECTRODYNAMICS

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A B S T R A C T

The connection between the existence of a non-zero triangle anomaly and the short-distance properties of Green's functions in quantum electrodynamics (QED) is re-examined. We find that part of a previous analysis must be corrected, and we record some additional observations on the internal consistency of finite theories of QED.

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In the past, conflicting statements have emerged concerning the short-distance behaviour of the Green's function $\langle 0 | T J_\alpha J_\beta J_{5\gamma} | 0 \rangle$ constructed from the electromagnetic current J_μ and the gauge invariant axial vector current $J_{5\mu}$ in spinor quantum electrodynamics (QED) ¹⁾⁻⁴⁾. The problems arise when it is assumed that the theory possesses a Gell-Mann - Low eigenvalue ⁵⁾, making the charge renormalization finite ^{*}). As a consequence ^{7),3)} of this assumption, Wightman functions of gauge invariant operators (of which at least one is J_μ) must be less singular than the result of applying the dimensional rules of broken scale invariance ⁸⁾. The question is whether this softening of the asymptotic behaviour of $\langle 0 | T J_\alpha J_\beta J_{5\gamma} | 0 \rangle$ can be reconciled with Wilson's analysis ^{8),9)} of the axial vector current anomaly ^{10),11)} and with the fact ^{12),13)} that this anomaly (taken to second order in the photon momenta) is not renormalized in perturbation theory.

In Refs. 3) and 4) (denoted by CSY and KN below), a short-distance analysis of the triangle anomaly was carried out by means of the Callan-Symanzik ¹⁴⁾ equation, and it was found that the dimension d of $J_{5\mu}$ is presumably anomalous. In both references, it was noted that Wilson's analysis, which was given for $\pi^0 \rightarrow 2\gamma$ decay ^{8),9)}, requires modification when applied to QED. KN state that these modifications prevent one from deriving a connection between the short-distance behaviour of $\langle 0 | T J_\alpha J_\beta J_{5\gamma} | 0 \rangle$ and the existence of a non-zero anomalous constant. According to CSY, a connection does exist, with the consequence that consistency with the various constraints on the anomaly is obtained only for $d > 3$. Neither CSY nor KN gave detailed analysis to support their positions adequately. The appropriate analysis will be considered below. We find that the argument used by CSY is not sufficient to demonstrate the desired connection. We also consider whether, with additional input, a connection can be obtained, but this does not seem to be possible using acceptable assumptions. Thus it appears that the opinion expressed by KN is justified, and the bound $d > 3$ claimed by CSY remains unproved. Of course, the central conclusion of KN and CSY, that the existence of an eigenvalue does not contradict the Adler-Bardeen theorem ^{12),13)}, remains valid. We have more to say about this conclusion at the end of this paper.

^{*}) Reference 6) includes a review of the literature on finite theories of QED (except for problems concerning $J_{5\mu}$).

We begin with the definition of the anomalous constant

$$S = - \frac{\pi^2}{12} \varepsilon^{\mu\nu\alpha\beta} \iint d^4x d^4y x_\mu y_\nu \langle 0 | T J_\alpha(x) J_\beta(0) J_5(y) | 0 \rangle^{\text{prop.}} \quad (1)$$

which has no radiative corrections ^{(12), (13)} so that, to all orders in the fine structure constant α , we have

$$S = 1 \quad (2)$$

The symbol J_5 represents the soft pseudoscalar current $2m_0 i : \bar{\psi} \gamma_5 \psi :$, where ψ is the spinor field and m_0 is its bare mass. [The combination $m_0 : \bar{\psi} \gamma_5 \psi :$ is cut-off independent; see Appendix A of Ref. 15).] The superscript "prop." means that only proper (one-particle irreducible) graphs are included ^{*}). In order to proceed, we must consider the Ward identity ^{(10), (16), (17)} (Fig. 1)

$$\begin{aligned} & \frac{\partial}{\partial y_\lambda} \langle 0 | T J_\alpha(x) J_\beta(0) J_{5\lambda}(y) | 0 \rangle^{\text{prop.}} \\ &= \langle 0 | T J_\alpha(x) J_\beta(0) J_5(y) | 0 \rangle^{\text{prop.}} - \frac{S}{2\pi^2} \varepsilon_{\mu\nu\alpha\beta} \frac{\partial}{\partial x_\mu} \delta(x) \frac{\partial}{\partial y_\nu} \delta(y) \\ &+ \frac{\alpha S}{4\pi} \langle 0 | T J_\alpha(x) J_\beta(0) \hat{N}[F, \tilde{F}](y) | 0 \rangle^{\text{prop.}} \end{aligned} \quad (3)$$

where the operator $\hat{N}[F, \tilde{F}]$ is the gauge invariant normal product ⁽¹⁷⁾ constructed from the field strength tensor $F_{\mu\nu}$ and its dual

$$\tilde{F}_{\mu\nu} = \varepsilon_{\mu\nu\alpha\beta} F^{\alpha\beta} \quad (4)$$

^{*}) Improper graphs do not contribute to S if the complete photon propagator is normalized to coincide with the free propagator at zero momentum. They can be incorporated in a Wilson analysis if the analogue of Eq. (3) for complete amplitudes is used.

Both Eq. (3) and the corresponding equation for $\pi^0 \rightarrow 2\gamma$ decay contain an anomalous term proportional to $\partial\delta(x)\partial\delta(y)$. However, Eq. (3) contains an additional non-trivial matrix element generated by $\hat{N}[\tilde{F}, \tilde{F}]$ which does not appear in the analysis of $\pi^0 \rightarrow 2\gamma$ decay. This term is responsible for the modifications to Wilson's analysis referred to previously.

The first step in the analysis is to modify Eq. (1) such that a small volume containing the origin in (x, y) space is excluded. For example, if we choose to restrict the volume of integration to

$$W(\epsilon) = \{x, y ; |x_0|^2 + |y_0|^2 \geq \epsilon^2\} \quad (5)$$

and the relation

$$\lim_{\epsilon \rightarrow 0} \epsilon^{10} \langle 0 | T J_\alpha(\epsilon x) J_\beta(0) J_5(\epsilon y) | 0 \rangle^{prop.} = 0 \quad (6)$$

holds, as is henceforth assumed ^{*}), only an $O(\epsilon)$ error is introduced :

$$S = - \frac{\pi^2}{12} \epsilon^{\mu\nu\alpha\beta} \iint_{W(\epsilon)} d^4x d^4y x_\mu y_\nu \langle 0 | T J_\alpha(x) J_\beta(0) J_5(y) | 0 \rangle^{prop.} + O(\epsilon) \quad (7)$$

When the Ward identity (3) is substituted in (7), the contact term $\propto \partial\delta(x)\partial\delta(y)$ does not contribute because the origin is not contained in $W(\epsilon)$. As shown by Wilson ⁸⁾, the contribution from the divergence $\partial_y^\gamma \langle 0 | T J_\alpha J_\beta J_{5\gamma} | 0 \rangle$ can be written as a surface integral which is necessarily $O(\epsilon)$ unless the condition

$$\lim_{\epsilon \rightarrow 0} \epsilon^9 \langle 0 | J_\alpha(\epsilon x) J_\beta(0) J_{5\gamma}(\epsilon y) | 0 \rangle \neq 0 \quad (8)$$

is satisfied. (The possibility that this limit is infinite, as in finite orders of perturbation theory, is not excluded here.) The contribution of the last term in (3) remains to be analyzed.

^{*}) This is the analogue of Wilson's assumption ⁸⁾ that the pionic axial-vector current $F_{5\mu}$ is partially conserved (dimension $\partial^\mu F_{5\mu} < 4$). In the presence of an eigenvalue, this condition follows from the standard renormalization group assumption that the logarithmic divergences of m_0 sum to something less singular than a linear divergence ; see Refs. 15), 18).

Consider the case

$$\lim_{\vartheta \rightarrow 0} \vartheta^9 \langle 0 | J_\alpha(\vartheta x) J_\beta(0) J_{S\gamma}(\vartheta y) | 0 \rangle = 0 \quad (9)$$

which, according to CSY, is inconsistent with $S \neq 0$. [Because of causality, the ordering of operators in (9) is immaterial ; see Appendix A of CSY.] Equations (3), (7) and (9) imply

$$S = \frac{\pi^2}{12} \varepsilon^{\mu\nu\alpha\beta} \iint_{W(\varepsilon)} d^4x d^4y \ x_\mu y_\nu \left(\frac{\alpha S}{4\pi} \right) \langle 0 | T J_\alpha(x) J_\beta(0) \hat{N}[F, \tilde{F}](y) | 0 \rangle^{\text{prop.}} + O(\varepsilon) \quad (10)$$

It has been observed that, at the origin in momentum space, the last term in Eq. (3) is cubic in its external momenta. Therefore, Eq. (10) may be rewritten in terms of the short-distance region

$$V(\varepsilon) = \{ x, y ; |x_0|^2 + |y_0|^2 < \varepsilon^2 \} \quad (11)$$

which is complementary to $W(\varepsilon)$:

$$S = - \frac{\pi^2}{12} \varepsilon^{\mu\nu\alpha\beta} \iint_{V(\varepsilon)} d^4x d^4y \ x_\mu y_\nu \left(\frac{\alpha S}{4\pi} \right) \langle 0 | T J_\alpha(x) J_\beta(0) \hat{N}[F, \tilde{F}](y) | 0 \rangle^{\text{prop.}} + O(\varepsilon) \quad (12)$$

Most regions of $V(\varepsilon)$ contribute $O(\varepsilon)$ to the integral. Indeed, the full short-distance region $x, y \rightarrow 0$ [for all $(x, y) \neq (0, 0)$] contributes $O(\varepsilon)$, because Eqs. (3) and (9) imply constraints of the form

$$\lim_{\vartheta \rightarrow 0} \vartheta^{10} \langle 0 | J_\alpha(\vartheta x) J_\beta(0) \hat{N}[F, \tilde{F}](\vartheta y) | 0 \rangle = 0 \quad (13)$$

on the absorptive parts of the integrand. However, since the integral is over a volume $V(\varepsilon)$ rather than a surface, we must also consider what happens at the point $(x, y) = (0, 0)$. The origin cannot contribute to the integral unless a contact term $\partial\delta(x)\partial\delta(y)$ appears when Eq. (13) is time-ordered. CSY assumed that contact terms must involve at least three

derivatives of $\delta(x)\delta(y)$ (corresponding to a cubic or higher degree momentum polynomial), and so they concluded that the right-hand side of (12) is $O(\epsilon)$. Certainly, it is possible ^{10),12)} to extract a cubic momentum factor from the amplitude

$$\begin{aligned} & \langle 0 | T J_\alpha(x) J_\beta(0) \hat{N}[F, \tilde{F}](y) | 0 \rangle^{\text{prop.}} \\ &= \rho^{-3} \partial_x^\mu \partial_y^\nu (\partial_x + \partial_y)^\lambda B_{\mu\nu\lambda\alpha\beta}(\rho x, \rho y) \end{aligned} \quad (14)$$

However, to determine the contact terms generated by applying $\rho \rightarrow 0$ to the left-hand side, it is not sufficient to examine the contact terms produced by $\rho^7 B_{\mu\nu\lambda\alpha\beta}^{(u)}$ for $\rho \rightarrow 0$. The trouble is that the unordered products $B^{(u)}$ constructed from B can satisfy the equations

$$\begin{aligned} \lim_{\rho \rightarrow 0} \rho^7 B_{\mu\nu\lambda\alpha\beta}^{(u)} &\neq 0 \\ \frac{\partial}{\partial y_\nu} \lim_{\rho \rightarrow 0} \rho^7 B_{\mu\nu\lambda\alpha\beta}^{(u)} &= 0 \end{aligned} \quad (15)$$

which are consistent with (13) and with the possibility

$$\frac{\partial}{\partial y_\nu} \lim_{\rho \rightarrow 0} \rho^7 B_{\mu\nu\lambda\alpha\beta} = (\text{constant}) \delta(x) \delta(y) \quad (16)$$

for the time-ordered amplitude. We are therefore unable to exclude a contribution $\propto \partial\delta(x)\partial\delta(y)$ in the limit $\rho \rightarrow 0$ of Eq. (14). Equation (12) (with $S \neq 0$) is satisfied if the formula

$$\begin{aligned} & \lim_{\rho \rightarrow 0} \rho^{10} \langle 0 | T J_\alpha(x) J_\beta(0) \hat{N}[F, \tilde{F}](y) | 0 \rangle^{\text{prop.}} \\ &= \frac{2}{\pi\alpha} \epsilon_{\mu\nu\alpha\beta} \frac{\partial}{\partial x_\mu} \delta(x) \frac{\partial}{\partial y_\nu} \delta(y) \end{aligned} \quad (17)$$

is valid. Indeed, Eq. (17) can be deduced directly from Eqs. (3) and (9), without recourse to a Wilson analysis.

As a further check, we note that Eq. (17) is entirely consistent with the CSY discussion of skeleton expansions. The decomposition of $\langle 0 | T J_{\alpha} J_{\beta} \hat{N}[F, \tilde{F}] | 0 \rangle^{\text{prop.}}$ represented by the last diagram of Fig. 1 does not constitute a legitimate skeleton expansion. Therefore, the formula ⁶⁾

$$\lim_{\rho \rightarrow 0} \rho^{12} \langle 0 | T J_{\alpha}(\rho x) J_{\beta}(\rho y) J_{\gamma}(\rho z) J_{\delta}(0) | 0 \rangle^{\text{prop.}} = 0 \quad (18)$$

does not imply that the left-hand side of (17) must vanish.

The conclusion is that the requirement $S \neq 0$ seems to be consistent with either (8) or (9) in quantum electrodynamics. This means that the CSY bound $d > 3$ on the dimension d of $J_{5\mu}$ must be abandoned, because its derivation supposes that Eq. (9) is impossible. Despite the emphasis placed on this bound by CSY, the effects of its retraction are relatively unimportant : the remainder of the CSY discussion (Schwarz inequalities, skeleton expansions, possible violations of asymptotic scale invariance, etc.), and the KN and CSY conclusion that $S \neq 0$ is consistent with the presence of an eigenvalue, are still valid. Of course, the weaker bound $d \geq 2$ (obtained by CSY) is not affected, because it follows directly from positivity for the two-point function $\langle 0 | J_{5\alpha} J_{5\beta} | 0 \rangle$. We also note that the conformal Ward identities for QED ¹⁹⁾ imply that $\langle 0 | J_{5\alpha} J_{5\beta} | 0 \rangle$ is conformal invariant at the eigenvalue ^{*}), so a theorem of Ferrara, Gatto and Grillo ²⁰⁾ implies that it must vanish unless $d \geq 3$. Therefore, in the range $2 < d < 3$, the above-mentioned softening of the short-distance behaviour of Wightman functions of gauge invariant operators ^{7), 3)} extends to cases where one of the operators is $J_{5\mu}$. Note that these arguments exclude the case $d = 2$.

Now we report some new observations on whether $S \neq 0$ is consistent with the presence of an eigenvalue in QED. These do not depend on the issues discussed above ; for example, either Eq. (8) or Eq. (9) may be valid.

^{*}) Extending the analysis of Ref. 19) to $\langle 0 | T J_{5\alpha} J_{5\beta} | 0 \rangle$ is straightforward. The conformal Ward identity contains extra sea-gull terms (as in the corresponding Callan-Symanzik equation ³⁾), but these are irrelevant for the short-distance behaviour of the unordered function $\langle 0 | J_{5\alpha} J_{5\beta} | 0 \rangle$.

Consider the gauge dependent symmetry current $^{10),19),21)}$

$$J_{5\lambda}^S = J_{5\lambda} - \frac{\alpha S}{2\pi} [A^\mu \tilde{F}_{\lambda\mu}] \quad (19)$$

which is partially conserved,

$$\begin{aligned} & \frac{\partial}{\partial y_\lambda} \langle 0 | T J_\alpha(x) J_\beta(0) J_{5\lambda}^S(y) | 0 \rangle^{\text{prop.}} \\ &= \langle 0 | T J_\alpha(x) J_\beta(0) J_5(y) | 0 \rangle^{\text{prop.}} - \frac{S}{2\pi^2} \varepsilon_{\mu\nu\alpha\beta} \frac{\partial}{\partial x_\mu} \delta(x) \frac{\partial}{\partial y_\nu} \delta(y) \end{aligned} \quad (20)$$

and from which the generator Q_5 of γ_5 -transformations is constructed :

$$Q_5 = \int d^3x J_{50}^S(x) \quad (21)$$

Renormalization subtractions for the vertex $[A^\mu \tilde{F}_{\lambda\mu}]$, (where A^μ denotes the photon field), are performed such that its divergence is $\frac{1}{2} \hat{N}[F \cdot \tilde{F}]$, in accord with Eqs. (3) and (20). Equations (7) and (20) imply the formula

$$\begin{aligned} S &= - \frac{\pi^2}{12} \varepsilon^{\mu\nu\alpha\beta} \iint_{W(\varepsilon)} d^4x d^4y x_\mu y_\nu \frac{\partial}{\partial y_\lambda} \langle 0 | T J_\alpha(x) J_\beta(0) J_{5\lambda}^S(y) | 0 \rangle^{\text{prop.}} \\ &+ O(\varepsilon) \end{aligned} \quad (22)$$

first noted by KN. Wilson's analysis applied to (22) converts $\iint_{W(\varepsilon)}$ into a surface integral which emphasizes [to accuracy $O(\varepsilon)$] the short-distance properties of $\langle 0 | T J_\alpha(x) J_\beta(0) J_{5\lambda}^S(y) | 0 \rangle^{\text{prop.}}$ (instead of $\langle 0 | T J_\alpha J_\beta J_{5\gamma} | 0 \rangle$, as in $\pi^0 \rightarrow 2\gamma$ decay).

Since $J_{5\gamma}^S$ is partially conserved, its dimension is canonical. Therefore, the limit

$$\lim_{g \rightarrow 0} g^q \langle 0 | T J_\alpha(gx) J_\beta(0) J_{5\lambda}^S(gy) | 0 \rangle^{\text{prop.}} = F_{\alpha\beta\lambda}(x, y) \quad (23)$$

converges if there is an eigenvalue. As explained previously ¹⁹⁾, the usual arguments for asymptotic conformal invariance of Green's functions do not work for $\langle 0 | T J_\alpha J_\beta J_{5\gamma}^S | 0 \rangle$, because the relevant conformal Ward identity

contains an additional insertion $\int d^4x N_3[A^\mu \partial_\mu A](x)$ (Fig. 2) generated by the gauge fixing term in the Lagrangian. For example, in finite orders of perturbation theory with vacuum polarization insertions omitted, the function $F_{\alpha\beta\lambda}$ is definitely not conformal invariant. However, it can be argued that $F_{\alpha\beta\lambda}$ is conformal invariant in the Johnson-Baker-Willey (JBW) version ⁶⁾ of finite QED : the extra insertion is asymptotically negligible because the decomposition shown in Fig. 2 represents a legitimate skeleton expansion in the sense of CSY. It follows ²²⁾ that $F_{\alpha\beta\lambda}$ is proportional to

$$\Delta_{\alpha\beta\lambda}(x, y) = \frac{i}{4} \text{Tr } \gamma_\alpha \gamma_\lambda \gamma_\beta \gamma_\gamma \gamma_\lambda \gamma_\gamma (x-y) \gamma_5 / x^4 y^4 (x-y)^4 \quad (24)$$

with the constant of proportionality determined by computing the surface integral obtained from Eq. (22):

$$F_{\alpha\beta\lambda}(x, y)_{\text{JBW}} = \frac{S}{\pi^6} \Delta_{\alpha\beta\lambda}(x, y) \quad (25)$$

This argument can be checked by observing that the skeleton expansion shown in Fig. 3 is legitimate, so that non-vanishing contributions to $F_{\alpha\beta\lambda}$ (in the JBW model) must arise from the one-fermion loop graphs which contain $\gamma_\lambda \gamma_5$, and hence ¹⁾ from the bare triangle graphs alone.

It is important to check that this new result is not in conflict with positivity requirements. Of course, positivity cannot be applied directly to Eq. (25) because $J_{5\lambda}^S(x)$ is not generally gauge invariant. However, it is obviously crucial for our conclusion that there be no smearing function $f(y)$ which makes

$$J_{5\lambda}^S(f) = \int d^4y f(y) J_{5\lambda}^S(y) \quad (26)$$

gauge invariant and satisfies

$$\int d^4y f(y) \Delta_{\alpha\beta\lambda}^{(+)}(x, y) \neq 0 \quad (27)$$

(where $\Delta_{\alpha\beta\lambda}^{(+)}$ is the unordered version of $\Delta_{\alpha\beta\lambda}$). To see that this is indeed the case, we consider a small change of gauge

$$\delta A_\mu = \partial_\mu \chi \quad (28)$$

which produces a change in $J_{5\lambda}^S(f)|0\rangle$ proportional to

$$|\psi\rangle = \tilde{F}_{\mu\lambda}(f\partial^\mu\chi)|0\rangle \quad (29)$$

Positivity requirements imply that $|\psi\rangle$ cannot vanish for $f\partial\chi \neq 0$. Hence $J_{5\lambda}^S(f)$ is gauge dependent for all functions f (except $f=0$), and so no contradiction is evident. [The smearing procedure does not include the formal expression (21) for the gauge invariant charge Q_5 . However, no problem can arise, because $\int d^3y \Delta_{\alpha\beta 0}^{(+)}(x,y)$ vanishes.]

One should, however, remain suspicious of the validity of the JBW or Adler summation procedures ^{6),23)} because of two difficulties listed by CSY :

- (i) the peculiar short-distance behaviour ¹⁾ of the one-fermion loop contributions to $\langle 0|T J_\alpha J_\beta J_\gamma J_\delta|0\rangle$;
- (ii) a contradiction ³⁾ obtained by applying these summation procedures to octet vector and axial vector currents in a singlet gluon model.

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FIGURE CAPTIONS

- Figure 1 The anomalous Ward identity (3) for one-particle irreducible graphs. The contact term $\propto \partial\delta(x)\partial\delta(y)$ is represented by the diagram which contains no blobs.
- Figure 2 The extra insertion term in the conformal Ward identity for $\langle 0 | T J_\alpha J_\beta J_{5\lambda}^S | 0 \rangle$. The blob with four legs is one-particle irreducible. The blobs with two legs represent complete photon propagators.
- Figure 3 Skeleton expansion for $\langle 0 | T J_\alpha J_\beta J_{5\lambda}^S | 0 \rangle$.

$$\frac{\partial}{\partial y_\lambda} \left(\text{Diagram 1} \right) = \text{Diagram 2} + \text{Diagram 3} + \text{Diagram 4}$$

Diagram 1: A vertex with a cross in a circle, labeled $J_{5\lambda}(y)$, connected to two wavy lines labeled J_α and J_β .

Diagram 2: A vertex with a cross in a circle, labeled $J_5(y)$, connected to two wavy lines.

Diagram 3: A vertex with a cross in a circle, connected to two wavy lines.

Diagram 4: A vertex with a cross in a circle, connected to two wavy lines, with the label $\frac{\alpha S}{4\pi} \hat{N}[\mathbf{F}, \tilde{\mathbf{F}}]$ next to it.

FIG.1

FIG. 2

FIG. 3