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TEMPORARY QUARK CONFINEMENT
IN FIELD THEORETICAL BAGS

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ABSTRACT

The problem of confining quarks within hadronic bound states is studied in detail in the context of local relativistic field theories exhibiting spontaneous breakdown of chiral symmetry. In particular, interacting systems of scalar and spinor fields in one-space-one-time dimension and in three-space-one-time dimension with spherical symmetry are considered. For some such systems in the tree approximation bound state solutions are found. As we had conjectured earlier, the binding mechanism and the ensuing temporary confinement of quarks are seen to be intimately related to local vacuum excitations and the local restoration of manifest chiral symmetry (which is globally hidden). Properties of the solutions are discussed and various implications are drawn on aspects of the dynamics of reactions in which such bound states could participate as in-going or out-going particles.

1. INTRODUCTION

A bag is a region of space to which quarks are confined. The confinement may be permanent, in the sense that quarks can only exist within bags, with the ordinary quantum numbers of hadrons; or it may be temporary if quark quantum numbers can be isolated, at relatively high energies. Thus, our definition of a "bag" is general enough so that hadrons, if they are in any way made out of quarks, are bags by definition.

In the following we shall be concerned with particular containment mechanisms or "bag models". Our concern is motivated by the logical need to reconcile the successes of the quark model and of the parton model (where it works) with the present day unobservability of quarks. Our goal is to formulate a dynamical theory which accounts for these facts and is, of course, consistent with accepted principles such as relativistic invariance, locality, etc. We shall report here on some recent progress made along the lines that we had suggested in a previous article¹⁾. We find, in the tree approximation of a local relativistic field theory exhibiting spontaneous breakdown of chiral symmetry, that there exist bound states of quarks. These bound states have masses that are small as compared to the masses of the "free quarks". In fact, the masses of the latter, though finite, can be made arbitrarily large by a suitable choice of those parameters in the Lagrangian which monitor the spontaneous breakdown of chiral symmetry. Yet the effective masses of the "constituent quarks", inside the bag, remain vanishingly small, for in the binding process chiral symmetry is locally restored as a manifest (as opposed to hidden) symmetry. The bound state picture that emerges has many other desirable features of hadron dynamics, some of which we had conjectured earlier and about which we shall elaborate briefly.

Our revival of interest in our model was stimulated by recent work of various groups on the confinement problem. We shall compare our line of attack in particular with approaches that have been pursued by groups at MIT²⁾ and at SLAC-Standard^{3,4)}.

2. HOW MANY BAGS ARE THERE?

We would first like to note radical differences of principle between the so-called "MIT bag model" and what we refer to as "field theoretical bag models". The latter are the ones we are mainly interested in.

2.1 The MIT bag model

The starting point of the MIT bag model²⁾ may be traced to a covariant prescription for modifying the classical action of a system of local relativistic fields. If this system is conventionally described by the Lagrangian density $\mathcal{L}(x)$, the modified action is

$$W = \int_{t_1}^{t_2} dt \int_R d^3x (\mathcal{L}(x) - B) \quad (1)$$

where the spatial region of integration R extends over a closed, finite part of space (the bag). B is a constant, positive potential energy per unit volume. It plays the role of a fundamental constant. The action principle is, of course, also modified: the ordinary requirement that W be stationary with respect to variations of the "constituent fields" which enter the Lagrangian $\mathcal{L}(x)$, is supplemented by the requirement that W be stationary with respect to independent variations of the position of the surface of the bag. The equations of constraint that result from the latter variation define (implicitly) the geometrical variables associated with the surface of the bag in terms of the field degrees of freedom in the bag. Such geometrical variables are, therefore, not independent, dynamical quantities.

The prescription for the modification of the action, Eq. (1), amounts, manifestly, to a decree of permanent quark imprisonment within bags. Thus, in this approach, one does not attempt to "understand" quark containment in terms of something more fundamental. Rather, quark containment itself is taken to be a fundamental law⁵⁾.

2.2 Field theoretical bag models. Type I

The point of view adopted in the formulation of field theoretical bag models is quite different in this respect: temporary quark containment is here an output of the dynamics of a canonical field theory with spontaneously broken symmetries. We will show in detail later on how this is in fact realized in the context of semi-realistic models at the level of the tree approximation. At this stage we shall just introduce the very simple idea at the basis of these theories of quark confinement¹⁾. Attempts are now being made to implement this idea in different ways in order to obtain realistic models of hadrons, and we shall also comment on the differences that have emerged between these approaches. In this comparison, this author's own prejudices (favouring the particular implementation of his choice) will inevitably manifest themselves. We refer the reader to a forthcoming article by the Stanford-SLAC Collaboration for a more persuasive presentation of their approach³⁾.

As mentioned above, the basic idea^{1,6)} underlying both approaches is the same and can be understood in the following way by consideration of a simple semi-classical model. Let us take, for instance, just a Fermi field ψ (the "quark") in interaction with a neutral scalar field ϕ , a system characterized by the Hamiltonian density:

$$H(x) = \psi^\dagger \frac{\alpha \cdot \nabla}{i} \psi + \frac{1}{2} (|\nabla \phi|^2 + \dot{\phi}^2) + I(\phi, \psi) + V(\phi). \quad (2)$$

In Eq. (2) we have separated out an "interaction term", $I(\phi, \psi)$, as well as a "potential term" for ϕ , $V(\phi)$. There is *a priori* nothing in what follows that would prevent ϕ from being a composite field and $\mathcal{H}(x)$ from being only an effective Hamiltonian density. Thus (although this step is not strictly essential to the development of our idea) we shall not impose on the form of I and V any of the obvious requirements that would follow from criteria such as renormalizability, if ϕ were taken to be elementary and $\mathcal{H}$ to be the Hamiltonian density corresponding to the fundamental world Lagrangian. We now assume that $V(\phi)$ is such that spontaneous breakdown of chiral invariance occurs. This is the case if the absolute minimum of $V(\phi)$ corresponds to a value λ of the classical field different from zero:

$$V(0) - V(\lambda) = B \geq 0, \quad (3)$$

so that, in the ground state or vacuum $|0\rangle$, the expectation value of ϕ is different from zero:

$$\langle 0 | \phi | 0 \rangle = \lambda \neq 0;$$

and if, as a result of this, the quark ψ acquires, via the interaction term $I(\phi, \psi)$, a non-vanishing mass, proportional to (some power of) λ . Of course, this mechanism of generation of a quark mass term is not new and, in fact, it plays a crucial role in the construction of unified gauge theories of weak and e.m. interactions. The question we asked ourselves is what happens if such a quark mass term turns out to be quite large (say, of the same order or magnitude as the weak interaction gauge boson mass terms that result from the Higgs-Kibble mechanism): Will the bare massive quark then remain observable at low energies? Or would quarks then dress themselves with a coherent cloud of ϕ to locally erase the memory of the vacuum and of the broken (chiral) symmetry, to turn off their (effective) masses and bind into low mass hadrons? It is needless to stress how energetically convenient this (temporary) self-confinement could be, for the turn-off of the quark masses would provide a large contribution to the binding energy which could easily overwhelm the energy expenditure required by the creation of a coherent mesonic cloud.

While postponing our quantitative arguments -- the actual solution of a bound state problem at the level of the tree approximation -- until the later portions of this article, at this point we should still make our qualitative picture more precise in order to make the reader more aware of what we are driving at. The picture of the hadron that we envisioned is represented in Fig. 1a. This is a "snapshot" of the hadron in its rest frame in the tree approximation; plotted against a radial coordinate measuring the distance from the centre of mass are the distribution of (charged) quark constituents and the value of the classical mesonic field, the "skin" of the hadron. Notice how the quark distribution and the classical field amplitude ϕ repel and avoid one another. The field

ϕ is essentially zero inside the hadron (the bag), and equal to λ , the expectation value of the quantum field in the vacuum, outside of it. Thus, in loose, intuitive language, the interior of the hadron appears to the quark as a (locally) "displaced vacuum", displaced by $-\lambda$. Correspondingly, the effective quark mass (as exhibited by the linearization of the field equations in the approximation of small oscillations) is vanishingly small inside the bag, where the quarks behave as quasi-free particles. Temporary quark confinement arises by virtue of the large value of the effective mass of the quark outside the bag, "in the true vacuum".

Here we should emphasize the obvious existence of many analogies between our picture and the MIT bag model²⁾. Thus we expect the latter model to provide, in many cases, a simple and good approximation to our approach, which we regard as more fundamental. This is not to say that our model and the MIT model would be phenomenologically indistinguishable, as there exist crucial differences between the two. One possible difference (although this is yet to be established) could be locality: while our bound states, which are solutions to a local field theory, would presumably carry local currents, this property might not be shared by the MIT states. The fact that there is a skin to our bags which corresponds to independent, dynamical degrees of freedom, in contrast to the MIT bag model, could also have "very observable" consequences, about which we have speculated⁷⁾.

2.3 The SLAC bag

The implementation of our idea of field-theoretical confinement which has been pursued by a group at Stanford-SLAC, differs from ours and leads to a picture of hadrons that, in contrast to ours, bears little resemblance to the MIT bag model²⁾. This picture is represented in Fig. 1b. The classical field amplitude ϕ is now changing rapidly from $\phi = -\lambda$, inside the hadron, to $\phi = +\lambda$ outside of it. As $\phi \neq 0$ inside the hadron, the quarks can no longer move as free massless quanta within the bag and are, instead, confined to the surface of the bag, the region of transition for the ϕ field. Thus, this is a "thin shell" rather than a "full bag" picture, i.e. a "surface solution" rather than a "volume solution".

The question at first seems to be which of the two solutions, if any, is the right one. The answer to this question, as we understand it, is that it depends on the "choice" of potential, $\mathcal{V}(\phi)$, and interaction term, $I(\phi, \psi)$, in the Hamiltonian density, Eq. (2). Our choice is different, slightly more complicated, than the one apparently made by the Stanford-SLAC group. (We shall further discuss these differences later on.) We feel, however, that our solutions have a better chance of being realistic, which, of course, is ultimately the only relevant criterion. What the potential and the interaction term in the effective Hamiltonian should be, and whether or not our choice is consistent at deeper levels we cannot answer here.

3. THE ROLE OF COLOUR

The binding mechanism for quarks that we have proposed is not in itself a complete theory of hadronic bound states. Our mechanism is in fact not capable of simply accounting for the structure and properties of the observed low-lying hadronic spectrum, as it would predict, for instance, the existence of low-lying hadrons with quark quantum numbers. It would, therefore, appear that some missing ingredient is necessary if we are to make direct contact with nature. It is now fashionable to attempt to identify such a missing ingredient with an (approximate) symmetry, associated with a label or "colour" carried by the quarks. This can immediately serve two useful purposes: a) by antisymmetrization with respect to the colour index, the global symmetry offers a solution to the puzzle of quark statistics, enabling one to bind three quarks in a completely symmetric state in space, spin, and SU(3) quantum numbers to make baryons; b) the interaction that results from the local extension of the symmetry, obtained by gauging the colour degrees of freedom, has the effect of raising the mass of colour non-singlet states and may account for the fact that such states have not yet been observed.

We wish to remark that

- i) the embedding of our binding mechanism in such a symmetry scheme can be realized in different ways;
- ii) its implementation would parallel lines of development which have already been discussed in the literature;
- iii) it is *a priori* possible, and it has been suggested that, thanks to the (obscure) infrared structure of non-Abelian gauge theories, non-Abelian colour interactions might lead to quark imprisonment. However, the size of this prison, if it exists, and if Bjorken scaling as observed in electroproduction is to be accounted for, would presumably have to be much greater than the typical hadronic size (1 fermi). In fact, the effective colour coupling constant is constrained to be weak at momentum transfers q^2 , of the order of 2 GeV^2 by "precocious" scaling, and its dependence on q^2 is only logarithmic! It thus appears that our confinement mechanism, which would, of course, already intervene at distances typical of the hadronic size, would shield strong colour interactions completely in the bound-state problem;
- iv) our primary interest, in this article, is the binding of unseen quarks and not the unbinding of unseen hadrons. The interplay between the two is, in our case, rather limited.

Because of these reasons, we shall ignore both global and local colour in what follows. One last comment: temporary confinement of colour is, of course, obtainable within our scheme as a trivial generalization of our considerations leading to the temporary confinement of quarks.

4. THE QUANTUM FIELD THEORETICAL BAG

Let us now attempt to construct quantum states that realize the intuitive picture of hadrons as bound states previously introduced. We start by constructing the mesonic "skin of the hadron" that we shall later fill with quark fields. In our present description of the hadronic skin, we restrict ourselves to the consideration of a singlet neutral scalar field $\phi(x)$. Although, as argued in our introductory statements, we should generally regard $\phi(x)$ as a composite field, in this section we shall also assume for ϕ canonical equal-time commutation relations:

$$\begin{aligned} [\phi(f_1), \dot{\phi}(f_2)] &= i(f_1, f_2), \\ [\phi(f_1), \phi(f_2)] &= [\dot{\phi}(f_1), \dot{\phi}(f_2)] = 0, \\ \phi(f) &= \int d^3x f(x) \phi(x), \quad (f_1, f_2) = \int d^3x f_1(x) f_2(x) \end{aligned} \quad (5)$$

where $f = f(x)$ are square integrable functions. We remind the reader that, thanks to the postulated non-trivial structure of the effective potential, the expectation value of ϕ in the ground state will be non-vanishing, as in Eq. (4).

Our picture of hadrons as quark bound states is based upon the identification of the "interior of the hadron", the region to which quarks are confined, with a local excitation of the symmetric, unphysical vacuum", $|0'\rangle$ (for which $\langle 0' | \phi(x) | 0' \rangle \equiv 0$). Although $|0'\rangle$ is, of course, not a state within the physical Hilbert space, its local excitations or "skin states", $|\text{skin}\rangle$, are. To construct such states, as local displacements of the physical vacuum ($|0\rangle$), we introduce vacuum displacement operators $D(f)$. The action of these operators on the field ϕ and its time derivative $\dot{\phi}$,

$$\begin{aligned} D^{-1}(f) \phi(x) D(f) &= \phi(x) + f(x), \\ D^{-1}(f) \dot{\phi}(x) D(f) &= \dot{\phi}(x), \end{aligned} \quad (6)$$

has an obvious character. Corresponding to infinitesimal displacements $\epsilon f(x)$ ($\epsilon \ll 1$), Eqs. (6), in the light of the commutation relations Eqs. (5), yield the identification:

$$D(\epsilon f) = 1 - i\epsilon \dot{\phi}(f),$$

or, upon integration,

$$D(f) = \exp [-i \dot{\phi}(f)] \quad (7)$$

Thus skin states,

$$|\text{skin}\rangle = D(f) |0\rangle, \quad (8)$$

are defined as such "coherent states"⁸⁾ of the mesonic field ϕ . Note, in particular, that:

$$\langle \text{skin} | \phi(x) | \text{skin} \rangle = \lambda + f(x).$$

Skin states, corresponding to which

$$\langle \text{skin} | \phi(x) | \text{skin} \rangle \simeq \begin{cases} 0, & |x| < R - D/2 \\ \lambda, & |x| > R + D/2 \end{cases}, \quad (9)$$

are coherent, localized meson clouds, and the main building block in our construction of hadronic states at rest.

Next we introduce the quark field ψ , with canonical equal-time anticommutation relations:

$$\{ \psi(u_1), \psi^+(u_2) \} = (u_1, u_2),$$

$$\{ \psi(u_1), \psi(u_2) \} = \{ \psi^+(u_1), \psi^+(u_2) \} = 0,$$

$$\psi(u) = \int d^3x u^+(x) \psi(x), \quad (u_1, u_2) = \int d^3x u_1^+(x) u_2(x),$$

where $u(x)$ are normalized spinor functions. We can now proceed to fill the skin states with quarks and antiquarks:

$$\begin{aligned} |B\rangle &= \psi(v_m) \cdots \psi(v_1) \psi^+(u_n) \cdots \psi^+(u_1) | \text{skin} \rangle, \\ (u_i, v_j) &= (u_i, u_j) = (v_i, v_j) = 0, \end{aligned} \quad (10)$$

"Bag states" $|B\rangle$ are thus obtained. Among such states, the ones that minimize the energy functional

$$\mathcal{E}(f, u_i, v_j) = \int d^3x \langle B | \mathcal{H}(x) | B \rangle \quad (11)$$

and are, therefore, solutions of the Euler-Lagrange equations

$$\delta \mathcal{E} / \delta f = \delta \mathcal{E} / \delta u_i = \delta \mathcal{E} / \delta v_j = 0, \quad (12)$$

under the constraints

$$(u_i, u_i) = (v_j, v_j) = 1,$$

will be referred to as "bag bound-states". They are, obviously, of particular interest to us: these states could, in fact, provide an approximate description of hadrons at rest. This description is a natural extension to the quantum level of our semi-classical picture of hadrons as vacuum excitations.

In the remainder of this paper we shall investigate the bag bound-state problem in the tree approximation. This certainly constitutes an oversimplification of the actual physical problem at hand. However, the binding mechanism we have

conjectured is intrinsically classical in character, and the classical approximation should at least tell us whether or not we may be going in the right direction. In the tree approximation, the energy functional $\mathcal{E}$ reduces to

$$\mathcal{E}(f, u_i, v_j) = \int d^3x \mathcal{H}(f(x), u_i(x), v_j(x)) \quad (13)$$

as the quantum fields are everywhere replaced by the classical fields. Correspondingly, the Euler-Lagrange equations reduce to the semi-classical time-independent field equations of interacting spinor and scalar fields.

4. THE SLAC QUARK-BOUND-STATE IN ONE + ONE DIMENSION

To begin with, let us identify in the Hamiltonian density Eq. (2), the interaction term $I(\phi, \psi)$ with an ordinary Yukawa coupling term,

$$I(\phi, \psi) = (\mu/\lambda) \bar{\psi} \psi \phi, \quad (14)$$

and the potential term for ϕ , $V(\phi)$, with:

$$V(\phi) = H(\phi^2 - \lambda^2)^2 / \lambda^4. \quad (\text{See Fig. 2a.}) \quad (15)$$

In this case, the analogue of the parameter B in Eq. (3) is H , and there exist two degenerate ground states corresponding to the field values $\phi = -\lambda$ and $\phi = \lambda$. If the latter is the physical ground state, then the "free quark" mass is μ .

It is instructive to first study this system in one-space-one-time dimension³⁾. Then, in the one-quark sector, the semi-classical bag bound-state equations are simply:

$$\left(\frac{d}{dx}\right)^2 f(x) - 4H f(f^2 - \lambda^2) / \lambda^4 = \frac{\mu}{\lambda} \bar{u} u(x), \quad (16)$$

$$\left(\frac{\alpha}{2} \frac{d}{dx} + \frac{\mu}{\lambda} f \beta\right) u(x) = \varepsilon u, \quad (17)$$

where ε is the spinor energy eigenvalue. Of course, as we are in one-space dimension, u is a spinor without spin and, therefore, with an upper and a lower component only. The Dirac matrices α and β may be identified with the Pauli matrices σ_1 and σ_3 .

Ignoring for a moment the source term on the r.h.s. of Eq. (16), this equation admits the "soliton" solution⁹⁾:

$$f(x) = \lambda \tanh [(2H)^{1/2} (x - x_0) / \lambda]. \quad (18)$$

The SLAC group observes³⁾ that there exist exact solutions to the coupled equations (16) and (17), for which $f(x)$ is still given by Eq. (18), as the source term on the r.h.s. of Eq. (16) for these solutions vanishes identically. In fact, these authors find that

$$u(x) \propto \left\{ \cosh \left[(2H)^{1/2} (x-x_0)/\lambda \right] \right\}^{-\frac{4\lambda}{\sqrt{2H}}} \begin{pmatrix} 1 \\ i \end{pmatrix}, \quad (19)$$

which implies

$$\bar{u} u(x) \equiv 0, \quad (20)$$

is a solution of Eq. (17) with the "input potential", $f(x)$, given by Eq. (18).

The Bardeen-Chanowitz-Drell-Weinstein-Yan two-dimensional semi-classical bound state, exhibited by Eqs. (18) and (19) and in Fig. 1b, has remarkable properties. The quark wave function is, in the limit $(\mu\lambda/\sqrt{2H}) \rightarrow \infty$, confined to a narrower and narrower region. Yet the quark contribution (ϵ) to the bound-state mass is zero, because of the vanishing of the interaction term, implied by Eq. (20), and of the kinetic energy term, implied by:

$$u^\dagger \propto \frac{d}{dx} u(x) \equiv 0. \quad (21)$$

Thus the bound-state mass is due entirely to the scalar field $f(x)$, and is given by³⁾:

$$M = \frac{8}{3} \sqrt{2H} \lambda. \quad (22)$$

Since in the physical ground state ($f = \lambda$) the quark mass is μ , strong binding is present providing that $\mu \gg M$.

5. A MECHANICAL ANALOGY

In attacking more complicated systems of semi-classical time-independent field equations, we shall find it particularly convenient to keep in mind a certain mechanical analogy. By introducing this analogy [which was also recently discussed by Lee and Wick¹⁰⁾] at this time and in the context of the bound-state model suggested by the SLAC group we will, in addition, gain better insight into the particular nature of their suggestion.

One can change the name of the variables

$$x \rightarrow t, \quad f(x) \rightarrow y(t), \quad \frac{H}{\lambda} \bar{u} u(x) \rightarrow S(t) \quad (23)$$

and cast Eq. (16) in the form:

$$\frac{d^2 y}{dt^2} = - \frac{dV}{dy} + S(t), \quad V = -U. \quad (24)$$

This is then immediately recognizable as the equation of motion of a classical point particle subject to a conservative field of force, deriving from the potential V , and to a time-dependent, non-conservative force $S(t)$. It is crucial to note that $V = -U$, i.e. the potential for the classical particle V is opposite in sign to the field potential U (see Fig. 2b). Setting $S(t) \equiv 0$, for the moment, one can then proceed to a classification of the different types of solutions to Eq. (24) in terms of the values of the constant of motion

$$W = \frac{1}{2} \left(\frac{dy}{dt} \right)^2 + V \quad (25)$$

which is the energy of the classical particle. It is clear in the case of Fig. 2b that there exist three static solutions: two of unstable equilibrium, at the points $y = \pm\lambda$ and energy $W = 0$, and one of stable equilibrium, at the point $y = 0$ and energy $W = -H$. There also exist three types of dynamical solutions: a) for all values of W , the particle motion can be unbounded; b) for $W < 0$, the particle motion can be oscillatory, bounded within the interval $(-\lambda, \lambda)$; c) finally, for $W = 0$, the particle motion can start, at $t = -\infty$, at $y = -\lambda$ and end, at $t = +\infty$, at $y = \lambda$ or vice versa. The latter dynamical solutions together with the static solutions $y(t) = \pm\lambda$ are the only solutions of interest in the field theoretical analogue model, as they are then clearly the only solutions of finite energy content. They correspond to the two degenerate ground states associated with the field potential, Eq. (15), and the "soliton" solution given in Eq. (18).

As we have seen in the previous section, it is on the use of the soliton solution that the two-dimensional bound-state model proposed by the SLAC group is based. We now realize how crucial to the existence of this model is the particular structure of the potential, Eq. (15). Any deviation from the condition of degeneracy

$$V(-\lambda) = V(\lambda) \quad (26)$$

would in fact destroy the soliton solution. It is true that a deviation from degeneracy could be compensated by a non-vanishing source term on the r.h.s. of Eq. (16), or, in terms of the mechanical analogue, by the work done by the force $S(t)$:

$$V(\lambda) - V(-\lambda) = \int_{-\lambda}^{\lambda} S(t(y)) dy. \quad (27)$$

Thus one could in principle retain a solution similar in form to Eqs. (18) and (19). However, it is apparent that, in the field theoretical case, such a solution would possess infinite energy and would therefore be unphysical. The latter objection does not apply to the generalization of the SLAC model to three-space-one-time dimension.

The mechanical analogy also sheds light on what is needed to construct bag-like solutions (of the type advocated by us) in one-space-one-time dimension. Indeed, there cannot be physical soliton solutions of this type involving only a self-interacting scalar field. In terms of the mechanical analogue, such solutions would require the particle to start off, at $t = -\infty$, at $y = \lambda$, make a transition to $y = 0$ for some finite t , and then rapidly approach again $y = \lambda$ as $t \rightarrow +\infty$. Now in absence of a non-conservative force $S(t)$, this is manifestly inconsistent with the requirement that $y = \lambda$ be the absolute maximum of the particle potential [or, equivalently, that $\phi = \lambda$ be the ground state, i.e. the absolute minimum of $V(\phi)$]. In fact, if this is the case, the particle leaving the point

$y = \lambda$ cannot obviously ever come back to it unless, again, it is subject to some force other than that deriving from the potential. Thus a non-vanishing source term on the r.h.s. of Eq. (16), i.e. a non-trivial interplay with the Fermi field, is needed if such a solution is to exist. Furthermore, it would be clearly advantageous if the point $y = 0$ were a relative maximum of $V(y)$ [i.e. if $\phi = 0$ were a relative minimum of $\mathcal{V}(\phi)$], since this would reduce the amount of work by the Fermi field which is necessary to brake the particle motion at $y = 0$ and enable the particle to return to $y = \lambda$. This strongly suggests the adoption of potentials $\mathcal{V}(\phi)$, such as the one represented in Fig. 3a.

Since the analogies between the bag bound-state problem in one-space-one-time dimension and the more realistic problem in three-space-one-time dimension are very limited, and our main interest rests with the latter, we shall now directly proceed to its analysis without further delay.

6. THE BOUND-STATE EQUATIONS IN THREE-SPACE-ONE-TIME DIMENSION

Following the arguments given at the end of the previous section, we shall take the potential $\mathcal{V}(\phi)$ to be of the type represented in Fig. 3a, and parametrize it in the form:

$$\mathcal{V}(\phi) = B + B \phi^2 (9\phi^2 - 16\lambda\phi + 6\lambda^2) / \lambda^4 + H (\phi^2 - \lambda\phi)^2 / \lambda^4 \quad (28)$$

where B , λ , and H will all be positive constants: B is the potential difference defined in Eq. (3); λ is the expectation value of ϕ in the vacuum, defined in Eq. (4); and H characterizes, in the limit $H \gg B$, the height of the barrier between the local minimum of the potential occurring at $\phi = 0$ and the ground state.

Next we take the interaction term $I(\phi, \psi)$ in Eq. (2) to be:

$$I(\phi, \psi) = (\mu / \lambda^2) \bar{\psi} \psi \phi^2, \quad (29)$$

where μ again defines the "free quark" mass. Notice that this term would lead to a non-renormalizable interaction. However, this is of no concern to us here since, as argued earlier, we only regard $\mathcal{H}(x)$ in Eq. (2) as an effective Hamiltonian, and we restrict our calculations to the level of the tree approximation. We postpone the motivation for the choice made in Eq. (29) until later.

We now wish to write down the "bag bound-state equations", Eqs. (12), corresponding to the Hamiltonian $\mathcal{H}(x)$, Eq. (2), with the identifications made in Eqs. (28) and (29). We restrict ourselves to the one-quark sector, to the semi-classical approximation, and to three-space-one-time dimension. We also project out and concentrate upon s-wave solutions. Thus we obtain the following system of radial equations:

$$\left(\frac{d}{dz}\right)^2 f(z) + \frac{2}{z} \frac{d}{dz} f(z) - \frac{dV}{dz} = \varepsilon \left(\mu/\lambda^2\right) (u_1^2 - u_2^2) f(z), \quad (30)$$

$$\left(\frac{d}{dz}\right) u_1(z) = \left(\mu/\lambda^2 f^2 + \varepsilon\right) u_2(z) - \frac{k+1}{z} u_1(z), \quad (31)$$

$$\left(\frac{d}{dz}\right) u_2(z) = \frac{k-1}{z} u_2(z) + \left(\mu/\lambda^2 f^2 - \varepsilon\right) u_1(z), \quad (32)$$

where $k = -1$, since the total and the orbital angular momentum are, respectively, $j = \frac{1}{2}$, $\ell = 0$ ¹¹⁾. Note that again ε denotes the spinor energy eigenvalue, and that $u_1(r)$ and $u_2(r)$ define the radial dependences of the upper and lower spinor components according to

$$u(z) = \begin{pmatrix} u_1(z) \chi_k^m \\ u_2(z) \chi_{-k}^m \end{pmatrix}, \quad (33)$$

where χ_k^m are ordinary spin-angular functions¹¹⁾.

The bound-state equations [Eqs. (30)-(32)] must, of course, be solved with the boundary conditions that derive from the requirements of regularity at the origin and asymptotic fall-off at infinity:

$$\begin{array}{ll} (z \rightarrow 0) & (z \rightarrow \infty) \\ \left\{ \begin{array}{l} \frac{d}{dz} f(z) \rightarrow 0 \\ u_2(z) \rightarrow 0 \end{array} \right. & \left\{ \begin{array}{l} f(z) \rightarrow \lambda \\ u_1(z) \rightarrow 0 \\ u_2(z) \rightarrow 0 \end{array} \right. \end{array} \quad (34)$$

Note, in particular, that asymptotic fall-off for $f(r)$ means approaching λ sufficiently fast, i.e. the vacuum expectation value of ϕ .

In addition to these general requirements, our bound-state picture (represented in Fig. 1a) demands that

$$f(z) \approx 0, \quad z \ll R, \quad (35)$$

corresponding to the formal condition given in Eq. (8).

Finally, the normalization condition for the one-quark sector,

$$4\pi \int dz \, z^2 (u_1^2 + u_2^2) = 1, \quad (36)$$

must also be met.

7. ASYMPTOTIC ANALYSIS

Using standard techniques, asymptotic solutions to the system of Eqs. (30)-(32) with the appropriate behaviours at the origin or at infinity, as dictated by Eq. (34), are easily found:

$$\begin{aligned}
 & (\tau \rightarrow 0) \\
 \left\{ \begin{aligned} u_1(\tau) & \sim A j_0((\epsilon^2 - \mu_{\text{eff}}^2)^{1/2} \tau) \\ u_2(\tau) & \sim -A \frac{(\epsilon^2 - \mu_{\text{eff}}^2)^{1/2}}{\epsilon + \mu_{\text{eff}}} j_1((\epsilon^2 - \mu_{\text{eff}}^2)^{1/2} \tau) \\ f(\tau) & \sim \eta \frac{\sinh((m_0^2 + 2\mu A^2/\lambda^2)^{1/2} \tau)}{\tau} \end{aligned} \right. \quad (37)
 \end{aligned}$$

$$\begin{aligned}
 & (\tau \rightarrow \infty) \\
 \left\{ \begin{aligned} u_1(\tau) & \sim A' \frac{e^{-\tau}}{\tau} \\ u_2(\tau) & \sim -\frac{(\mu^2 - \epsilon^2)^{1/2}}{\epsilon + \mu} A' \frac{e^{-\tau}}{\tau} \\ f(\tau) & \sim \lambda - \eta' \frac{e^{-m\tau}}{\tau} \end{aligned} \right. \quad (38)
 \end{aligned}$$

Here, the j 's denote conventionally defined spherical Bessel functions; μ_{eff} is the effective quark mass "inside the bound state" [this will actually be zero unless a bare quark mass term is added to the Hamiltonian, Eq. (2)]; m_0 is the second derivative of the effective potential $V(\phi)$ [Eq. (28)] at the origin:

$$m_0 = \frac{d^2 V}{d\phi^2} \Big|_{\phi=0} = 2(H + 6B); \quad (39)$$

and m is the physical mass of the scalar field ϕ :

$$m = \frac{d^2 V}{d\phi^2} \Big|_{\phi=\lambda} = 2(H + 12B). \quad (40)$$

The constant coefficients A , A' , η , η' , and the spinor energy eigenvalue ϵ , are, of course, left unspecified by the "linearized" equations from which the asymptotic solutions (37) and (38) are derived. They will have to be determined from the complete solution and the normalization condition Eq. (36). Note, however, that we expect

$$\eta \ll 1$$

by virtue of Eq. (35).

8. THE MECHANICAL ANALOGUE OF OUR BOUND-STATE SYSTEM

Before directly attacking the problem of joining Eqs. (37) and (38) to obtain a complete solution, it will be useful to further develop our insight into this

problem by consideration of its mechanical analogue. While it is possible to discuss the mechanical analogue of the entire system of Eqs. (30)-(32), this is rather complicated. Here we prefer to concentrate on the scalar field equation alone and treat its source term, due to the coupling to the quarks, as an external force.

Changing the name of the variables, as in Eq. (23), we cast Eq. (30) in the form:

$$\frac{d^2 y}{dt^2} = - \frac{dV}{dy} - \frac{2}{t} \frac{dy}{dt} + y(t) \frac{2\mu}{\lambda^2} \bar{\Psi}\Psi(t), \quad V = -U. \quad (42)$$

We recognize this as the equation of (one-dimensional) motion of a classical point particle subject to a conservative field of force deriving from the potential V , a time- and velocity-dependent dissipative force $[-(2/t)(dy/dt)]$, and a position- and time-dependent external force $[y(t)(2\mu/\lambda^2)\bar{\Psi}\Psi(t)]$. Corresponding to the boundary conditions Eqs. (39), and the additional requirement given in Eq. (35), the motion starts at time 0 in the proximity of the point of coordinate $y = 0$, and should rapidly approach the point of coordinate $y = \lambda$ as $t \rightarrow +\infty$.

Thus, with reference to Fig. 3b, at early times the particle starts "falling" under the action of the potential and of the external force while dissipating energy because of attrition. Over-all energy conservation and consistency with our boundary conditions require that

$$\int_0^{\infty} dt \frac{dy}{dt} \left(y(t) \frac{2\mu}{\lambda^2} \bar{\Psi}\Psi(t) - \frac{2}{t} \frac{dy}{dt} \right) = V(+\infty) - V(0) \simeq B. \quad (43)$$

We observe that, although the coefficient of attrition $(2/t)$ is very large for small t , the particle velocity (dy/dt) is, for such t , so small that the actual power dissipation is then negligible. In fact, on the basis of Fig. 1a, we expect the power dissipation to peak very strongly for $t \sim R$. On the basis of the same picture, the approximate particle velocity for $t \sim R$ is $dy/dt \sim \lambda/D$. We therefore estimate the total energy dissipated in attrition to be

$$- \int_0^{\infty} dt \frac{dy}{dt} \frac{2}{t} \frac{dy}{dt} \simeq \frac{2}{R} \frac{\lambda}{D} \int_0^{\lambda} dy y = \frac{2\lambda^2}{RD} \quad (44)$$

Furthermore, again on the basis of Fig. 1a, we expect

$$\int_0^{\infty} dt \frac{dy}{dt} y(t) \frac{2\mu}{\lambda^2} \bar{\Psi}\Psi(t) \simeq \frac{2\mu}{\lambda^2} \bar{\Psi}\Psi(R) \int_0^{\lambda} dy y = \mu \bar{\Psi}\Psi(R). \quad (45)$$

Equation (43), together with Eqs. (44) and (45), yields the approximate relation:

$$\mu \bar{\Psi}\Psi(R) \simeq B + 2\lambda^2/RD. \quad (46)$$

This relation sheds useful light on the interplay of different parameters in the model.

Notice that there would be no simple analogue to Eq. (45) if we were to substitute the more conventional Yukawa coupling term of Eq. (14) for the interaction term defined in Eq. (29). In fact, the power communicated to the particle by the external force would then no longer peak (strongly) near $t = R$, but rather it would peak near $t = 0$. This would make it difficult for our bound-state picture to come through, for, to that end, in terms of the mechanical analogue, it appears crucial that energy be supplied to the particle as it is being dissipated by friction. In more figurative language, the particle should get its kick when it needs it, not before! Otherwise, the structure of the bound state is radically different and its (manifest) properties do not appear to us to be as desirable. We had not appreciated this apparently important point when first proposing the bound-state model, when we implied the use of a conventional Yukawa coupling. In fact, this point does not emerge in a discussion limited solely to questions of balance among energy contributions to the bound-state mass, which makes no reference to the actual solution of the field equations¹²).

In conclusion, because of the possibility of an approximate cancellation between the second and third terms on the r.h.s. of Eq. (42), the particle can move effectively as if it were only subject to the conservative force deriving from the potential V , except for some extra "lift" provided by the external force which compensates exactly for the potential difference B . Indeed, B will, in most cases, be negligible with respect to the frictional term in the r.h.s. of Eq. (46).

9. NUMERICAL SOLUTIONS

We have numerically constructed complete, normalized solutions to the bound-state equations, Eqs. (30)-(32), with the boundary conditions prescribed by Eqs. (37) and (38). Corresponding to each solution we have also computed quantities such as the bound-state mass, the charge radius, etc. The procedure used is summarized in the Appendix. Here we concentrate on a discussion of the results.

Figure 4 shows a typical solution. We are looking at the semi-classical description of a one-quark bound state localized in configuration space: the bound state is at rest, and its centre of mass coincides with the origin of our coordinate system. Plotted against the radial coordinate r are the expectation value $\langle B | \phi(r) | B \rangle = f(r)$ of the scalar field ϕ , and the upper and lower components u_1 and u_2 , of the quark wave function, defined in Eq. (33).

Corresponding to the choice of parameters which led to this solution (which are indicated in the upper r.h. corner of the figure), the mass of the "free quark" μ is 4 GeV, while the bound-state mass turns out to be only 0.899 GeV. We therefore find, as expected, very strong binding.

We have isolated contributions to the bound-state mass M , corresponding to the quark kinetic energy Q , the kinetic and self-interaction energy of the scalar field S , and the quark-scalar interaction energy I :

$$Q = 4\pi \int dr r^2 \psi^\dagger \frac{\alpha \cdot \nabla}{i} \psi(r), \quad (47)$$

$$S = 4\pi \int dr r^2 \left(\frac{1}{2} |\nabla \phi|^2 + V(\phi) \right), \quad (48)$$

$$I = 4\pi \int dr r^2 \frac{M}{\lambda^2} \bar{\psi} \psi \phi^2(r), \quad (49)$$

$$M = Q + S + I. \quad (50)$$

In the case of Fig. 4 we find:

$$Q = 0.561 \text{ GeV}, \quad S = 0.277 \text{ GeV}, \quad I = 0.051 \text{ GeV}. \quad (51)$$

Thus a large fraction of the bound-state mass originates from the quark kinetic energy. This is relatively large in the particular case under discussion since the quark is confined within a radius $R \approx 3.3 \text{ GeV}^{-1}$, which is small in relation to the typical hadronic scale, set by the pion Compton wavelength ($\sim 7 \text{ GeV}^{-1}$).

In fact, from general principles we expect

$$Q \sim n \pi \hbar / 2R \quad (52)$$

for the lowest energy bound state, where n is the number of bound quarks (or antiquarks).

If we further split S , in Eq. (48), into its kinetic (or "surface") and potential (or "volume") contributions,

$$S_{\text{surf}} = 2\pi \int dr r^2 |\nabla \phi|^2, \quad (53)$$

$$S_{\text{vol}} = 4\pi \int dr r^2 V(\phi), \quad (54)$$

for the solution represented in Fig. 4 we find:

$$S_{\text{surf}} = 0.141 \text{ GeV} \quad S_{\text{vol}} = 0.137 \text{ GeV}. \quad (55)$$

Thus the surface energy is comparable to the volume energy in this particular solution. This conclusion agrees with the following rough estimates:

$$S_{\text{surf}} \sim \frac{1}{2} \left(\frac{\lambda}{D} \right)^2 4\pi R^2 D = 2\pi R^2 \lambda^2 / D, \quad (56)$$

$$S_{\text{vol}} \sim B \frac{4}{3} \pi R^3 + \frac{\pi}{30} 4\pi R^2 D, \quad (57)$$

where, in the case of Fig. 4, $D \approx 1.2 \text{ GeV}^{-1}$.

As the interaction energy I turns out to be (at least in the case of Fig. 4) a relatively small fraction of the total mass M , we may ignore it for a moment, and, on the basis of Eqs. (52), (56) and (57), write

$$M \sim n\pi k/2R + 2\pi R^2 \lambda^2/D + B \frac{4}{3} \pi R^3 + \frac{4}{30} 4\pi R^2 D. \quad (58)$$

Minimization of M with respect to D and R yields, then, the approximate relations

$$D \sim 4\lambda/\sqrt{H} \quad (59)$$

$$n/R^4 \sim 4\lambda\sqrt{H}/R + 8B. \quad (60)$$

Both of them are verified by the above-mentioned solution.

We have also studied numerically the response of characteristic bound-state parameters (R , D , M , Q , S , I) to small changes in the parameters which enter the Hamiltonian. The results of some of these "experiments" are reported and commented upon below. We warn the reader against attempting to extract from the following graphs anything more than general trends, since these graphs (which are direct, unedited computer outputs) contain "experimental" errors, mainly due to our self-imposition of a limit in the use of computer time.

9.1 Effective quark mass (Fig. 5)

The addition of a bare mass term

$$\mu_{\text{eff}} \bar{\Psi} \Psi \quad (61)$$

to the Hamiltonian density Eq. (2), which gives the confined quark, otherwise essentially massless, a small effective mass μ_{eff} , is seen to have no effect on the structure of the bound state (see also Fig. 6), except for a proportionally small increase in the bound-state mass. The contribution of the bare mass term has been included in a redefinition of I :

$$I + 4\pi \int dr r^2 \mu_{\text{eff}} \bar{\Psi} \Psi(r) \rightarrow I. \quad (62)$$

9.2 "Free-quark" mass (Fig. 7)

An increase in the mass of the "free quark" μ from 4 to 6 GeV is seen to cause a slight decrease in D , while leaving R essentially unchanged (see also Fig. 8). Such a response may be understood with the help of Eq. (46). What happens is that, as the coupling of the quarks to the scalar field becomes stronger, the overlap of the quark wave function with the scalar amplitude, measured by D , has to reduce accordingly. In fact, in terms of the mechanical analogue, the same amount of "work" is done on the "particle" by a stronger "external force" in a smaller amount of "time" ($\leftrightarrow D$).

The responses of Q , S , and I are reflections of the dependence of D on μ . The quark kinetic energy Q increases as the quark is confined to a smaller region:

this detects the (obvious) presence of a correction term involving D in Eq. (52). The slight increase in S results from the fact that the decrease in the volume energy S_{vol} , Eq. (57), only partially offsets the increase in the surface contribution S_{surf} , Eq. (56). Finally, the decrease in I simply mirrors the decrease in overlap of the quark wave function with the scalar amplitude.

Note that since changes in the interaction energy with μ are evidently appreciable, our neglect of I in the steps that led to Eq. (59) are clearly not justified in general.

9.3 Potential difference B (Fig. 9)

These plots are based on two solutions only, and the lines drawn by the computer are linear interpolations.

The declining trend of R with increasing B is consistent with Eq. (60) and reflects itself in the increase in the quark kinetic energy Q , according to Eq. (52). Larger B is also seen to imply larger volume energy S_{vol} , and larger S , as one would conclude on the basis of Eqs. (56), (57), (59), and (60).

9.4 Expectation value (vacuum) λ (Fig. 10)

A decrease in λ determines opposite responses in R (increasing) and D (decreasing) in accordance with Eqs. (59) and (60).

The behaviour of Q again mirrors the behaviour of R through Eq. (52). The surface energy S_{surf} decreases for decreasing λ , as implied by Eqs. (56), (59), and (60), and so does S_{vol} , in this régime, where the second term on the r.h.s. of Eq. (57) dominates over the first one.

9.5 Height of the potential barrier H (Fig. 11)

The observed decrease in both R and D for increasing H is consistent with Eqs. (59) and (60). Again, there are no surprises in the observed behaviour of Q , S , and I .

9.6 Quark wave function normalization (u,u) Fig. 12)

To learn about the response of the bound state to changes in the number of confined quarks, we have continuously changed the normalization of the quark wave function (u,u) from 1 to 3. The case $(u,u) = 3$ could simulate a baryon made of three coloured quarks in a completely symmetric state with respect to space and spin (Fig. 13).

The constancy of D and the dramatic change in R are consistent with Eqs. (59) and (60). The increase in the kinetic energy of the quarks Q is accounted for by Eqs. (52) and (60). Finally, the increases in S and I are induced by the increase in the bag radius R [see, for example, Eqs. (56) and (57)].

Needless to say, the fact that the three-quark bound state of Fig. 13 (whose parameters are exhibited in Fig. 12) cannot, because of its radius, mass, etc., represent even approximately the "average baryon" should not worry us: it simply means that the choice of parameters that led to this solution is not a realistic one. Figures 5, 7, 9, 10, and 11 provide a guide to how to go about searching for more "realistic" choices.

10. CREUTZ'S LIMIT

Creutz, in a recent paper¹³⁾, argues that there exists a limit of our model which yields the MIT bag model.

First, he evokes our argument that, corresponding to certain Higgs-type Lagrangians, there should exist a bound state of quarks, associated with local excitations of the vacuum, with a number of desirable properties. He points out the similarities between these conjectured properties and properties exhibited by the MIT bag states, and also identifies the main differences, having to do with the sharpness of the transition region at the boundaries of the bag and the infiniteness of the quark mass outside the bag.

Then he suggests how to eliminate such differences. This involves taking a limit of the parameters which appear in the original Lagrangian whereby only one (dimensional) constant is kept finite. This constant is the potential difference referred to, in this paper, as B , and which should correspond to the MIT bag constant B . The other Lagrangian parameters are to be taken to infinity or to zero in an appropriate way, which Creutz discusses in detail.

We believe that the findings of the work reported in this paper strongly support Creutz's conjecture of a connection between our model and the MIT model and his prescriptions for the implementation of such a connection.

In fact, starting from one of our solutions, which are indeed similar to the MIT bags, we have seen that it is possible to make the quark mass outside the bag larger and, at the same time, make the boundaries of the bag sharper by increasing μ (Fig. 7). We have also verified the theoretical predictions about the "beneficial" effects of decreasing λ and increasing H (Figs. 10, 11, and 12) with the aim of suppressing scalar field excitations and ultimately decoupling the scalar field, as an independent dynamical degree of freedom, from the bag.

However, we would like to emphasize strongly that, although we are glad to discover more flexibility in our model, we do not think one should make use of it to rederive, in a complicated way, the MIT model. There is no point in doing so, in our opinion, because of a number of apparent reasons. For one thing, the actual process of taking the limit of a Lagrangian where parameters go to infinity

is questionable, because of the likelihood of leaving behind desirable properties of the field theory, such as locality. Besides the danger of such losses, we also feel that the MIT model, although very attractive in its simplicity and originality and exciting in its phenomenological potential, is just too schematic and naive to have a chance of succeeding as a fundamental theory.

Thus we like to regard Creutz's connection and the MIT model as a way of providing a simple approximation to our field theoretical bound states, capable in some cases of accurate predictions.

11. DYNAMICS AND STATICS OF FIELD THEORETICAL HADRONIC BAGS

Having argued -- convincingly, we hope -- that field theoretical bags, of the type suggested by us, could provide a viable mechanism of quark confinement, the question to be asked is whether the end products of such a confinement mechanism could be successfully identified with ordinary hadrons. We believe there exists some evidence, although preliminary and only circumstantial, to this effect.

The first dynamical check that one may want to consider is (where else?) in the deep inelastic limit of lepto-production. There, our picture offers what we regard as a natural explanation for a temporary, precocious Bjorken scaling of the hadronic structure functions (it was with this motivation that we first suggested the model). In fact, quarks are confined in our model in such a way that they are effectively free and almost massless inside the hadron. The binding forces only intervene in a relatively small region at the boundaries of the bag. Thus, as long as hadronic weak currents are probed in this way at relatively short distances, a scale invariant and free-field type of structure, à la parton model, will be detected. Note that, although the skin-field ϕ is transparent to electromagnetism and will not contribute to the electromagnetic current, the presence of the hadronic skin will still manifest itself in electroproduction sum rules as the skin carries a sizeable fraction of the nucleon momentum. It is well known that the existence of such a "neutral glue" is indeed suggested by experiments.

If Bjorken scaling eventually breaks down, this could in our picture be attributed to a more dynamical role of the skin field. The latter could in fact possess a weak direct coupling to leptons as well as quarks, as it would in some gauge models. The implications of couplings of this type have been analysed elsewhere¹⁴⁾ and found to be consistent with present and past experiments. Such a coupling would affect the observed scaling behaviour of the structure functions as the typical momentum transferred to the hadron becomes comparable to the ϕ mass. This effect could in fact be manifesting itself already in the μ -p scaling experiments at FNAL if the relevant ϕ mass is ~ 10 GeV.

The impact of our field-theoretical bag picture on $e^+e^- \rightarrow \text{hadrons}$ is even more dramatic. In fact, corresponding to such processes for which no hadronic bag is present in the initial state of the reaction, the skin of the bags which yield the particles in the final state must be "excited from the vacuum". Because of the very nature of the skin [see Eq. (7)], this is necessarily a very coherent phenomenon. Thus we should expect a breakdown of the applicability of the impulse approximation and of the whole parton picture to these processes in our model. This conclusion is to be contrasted with a different one which could be reached for processes such as electroproduction where a hadronic bag is already present in the initial state of the reaction. There, incoherent scattering from quasi-free constituents could be applicable to the key stages of the reaction. These would, of course, be followed by (hopefully inoffensive) final-state re-scattering, consisting mainly of bag stretching and fissioning.

In the light of the colliding beam data on $e^+e^- \rightarrow \text{hadrons}$, the above conclusion about the inapplicability of the impulse approximation to this process is certainly quite welcome. The total cross-section $\sigma(e^+e^- \rightarrow h)$ appears in fact to be approximately constant with energy at the higher energies at present available, in complete defiance of the conventional approach based on one-photon mediation and naive expectations of quasi-free partons or free-field light-cone singularities. The bag picture is, on the other hand, at least off to a good start in the difficult problem of attempting to reconcile this apparent lack of (conventional) scaling in the annihilation channel with the precocious character of scaling in the scattering channel, i.e. deep inelastic scattering as observed at SLAC.

We should stress that, if there exists a direct skin field-lepton coupling, virtual skin-field quanta may mediate the e^+e^- annihilation into hadrons. Notice that photon mediation of this process could be suppressed, as the coupling of the hadronic electromagnetic current outside a bag is dynamically suppressed by the large free-quark mass.

[This brings us to the question of how this process is handled in the context of the MIT bag model²⁾, where the (classical) hadronic electromagnetic current vanishes identically outside bags, as the free-quark mass is infinite. Is $\sigma(e^+e^- \rightarrow \text{hadrons}) \equiv 0$ in the MIT bag model? Of course, this would necessarily not be true if the MIT model had locality, since then this process would be related by analytic continuation to processes for which bags are already present in the initial state of the reaction and for which scattering amplitudes are certainly non-vanishing. But, is the MIT model local? It appears to us that we see here yet another indication that the MIT model may clash with locality.]

Many more questions await an answer. For instance, we are not prepared to say anything here about properties of high-energy hadron-hadron (bag-bag) scattering in our model. In particular, the Regge structure of the model is yet to be established.

Related to this question is the structure of excited hadronic states. Intuitive arguments would lead us to a possible connection with the "dual string" picture¹⁵⁾ by a road which parallels the vortex line approach of Nielsen and Olesen¹⁶⁾, Zumino¹⁷⁾, and Tassie¹⁸⁾.

Static properties of baryons and mesons are also to be investigated. So is SU(6), the relation between constituent and current quarks, etc.

12. CONCLUSION

In a world that "starts off" chiral invariant, in the sense that the world Lagrangian is chiral invariant, one expects the spontaneous breakdown of this symmetry to be intimately related to the formation of massive bound states. In this paper we have investigated in some detail our suggestion¹⁾ that the binding mechanism behind the formation of such massive bound states actually results from the local restoration of manifest chiral invariance. We have found certain implementations of this attractive idea to be feasible.

In conclusion, we feel that our model, and variations of it, are rich in exciting promises for the understanding of hadron dynamics.

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APPENDIX

Here we briefly sketch the procedure used in finding numerical solutions to the system of Eqs. (30)-(32) with the boundary conditions prescribed by Eqs. (34) and (35) and the normalization condition Eq. (36).

Equations (30)-(32) are rewritten as a system of four first-order differential equations. To avoid problems of stability, etc., the actual boundary value problem is simulated by an initial value problem via a "multiple shooting technique". Initial conditions for u_1 , u_2 , f , and df/dr are given, in accordance with the asymptotic structure of the solution near the origin, Eqs. (37), at a point r_0 in the neighbourhood of $r = 0$. This involves choosing values for A , η , and ϵ in Eqs. (37). With these initial conditions, using a standard routine, the computer integrates the differential system from r_0 out to some predetermined point r_{end} . In the neighbourhood of this point, an attempt is made to match the solution to the initial value problem just found with the asymptotic behaviour as $r \rightarrow \infty$ given by Eqs. (38) that is required of our "true solutions". In finding a "match" a pre-determined amount of "error" is, of course, allowed. To obtain a "true solution" the normalization condition, Eq. (36), must also be satisfied within a certain error. If a "true solution" is not found, ϵ , A , and η must be changed and the entire process is repeated.

The quantities ϵ , A and η are changed by adding or subtracting multiples of certain prearranged unit steps. Changes in ϵ are monitored by the value of u_1 , u_2 , and r_{end} . The direction of change is such as to minimize these quantities. Changes in A are monitored by the value of the integral in the l.h.s. of Eq. (36). Thus A is increased or decreased depending on whether the integral is smaller or greater than one. Finally, η is monitored by the value of $[\lambda - f(r_{\text{end}})]$ and it is changed to minimize this quantity.

Changes in ϵ , A , and η always occur in an appropriate sequence. Thus, ϵ is always re-adjusted to its best value after each step in A or η ; and A is always re-adjusted to its best value after each step in η .

The size of the steps in ϵ , A , and η has to be small enough to avoid blow-up which occur whenever the mismatch in the spinor energy eigenvalue ϵ causes the exponentially increasing root of the Dirac equation to couple in with a non-negligible coefficient. This technical problem becomes more serious as one progresses toward a strong coupling régime and, in particular, as μ is made larger. Of course, reducing the size of the steps ultimately makes the procedure of finding a "true solution" very lengthy and the computer time that is required non-trivial.

Wandering around in the space of "true solutions" by varying one (independent) parameter (μ_{eff} , μ , B , λ , H) at a time in order to obtain diagrams such as those in Figs. 5, 7, 9, 10, 11, 12, is done in a similar way, and the same sort of care must be exercised.

Figure captions

- Fig. 1a : Our picture of quark confinement: the continuous line represents the distribution of quarks and the dotted line the classical mesonic skin-field.
- Fig. 1b : The SLAC picture of quark confinement.
- Fig. 2a : Potential for the scalar field $\mathcal{V}(\phi)$ in the SLAC model.
- Fig. 2b : Particle potential in the mechanical analogue of the SLAC model.
- Fig. 3a : Potential for the scalar field $\mathcal{V}(\phi)$ in our model.
- Fig. 3b : Particle potential in the mechanical analogue of our model.
- Figs. 4, 6, 8, 13 : Bound-state solutions.
- Figs. 5, 7, 9, 10, 11, 12 : Dependence of bound-state parameters on Hamiltonian parameters.

FIG. 1

FIG.2

FIG.3

FIG.4

FIG.5

FIG.6

(a)

(b)

FIG. 7

FIG. 9

(a)

(b)

FIG. 10

FIG. 11

FIG. 12

FIG. 13