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INCOHERENCE AND CLUSTER EMISSION

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ABSTRACT

It is shown that the generating function of the independent cluster emission model can be derived from the assumption that the number of modes in which the secondaries are produced is very large. A mechanism for the suppression of the quantum mechanical interference corrections to the classical cluster emission model is discussed.

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1. INTRODUCTION

Recent studies of two-particle correlation data on the emitted secondaries in high-energy proton-proton collisions indicate that the secondaries in the central region tend to come out in clusters, each cluster containing about four particles. This led to the introduction of cluster emission models ¹⁾, in which a two-step mechanism is assumed. In the first step one or several objects, called clusters, are produced, which then decay independently of each other into observable secondary particles. As is well known ²⁾, the generating function $\Lambda(z)$ defined as

$$\Lambda(z) = \sum_{n=0}^{\infty} z^n \sigma_n, \quad (1)$$

where σ_n for $n \geq 1$ is the probability for producing n secondaries, $\sigma_0 = 1 - \sum_{n=1}^{\infty} \sigma_n$, can be expressed as

$$\Lambda(z) = L(\ell(z)) \quad (2)$$

Here

$$L(z) = \sum_{n=0}^{\infty} z^n P_n,$$

$$\ell(z) = \sum_{n=0}^{\infty} z^n \lambda_n,$$

where P_n is the normalized probability distribution of cluster production and λ_n is the normalized probability distribution of the cluster decay, i.e., $L(z)$ is the generating function of the cluster formation and $\ell(z)$ is the generating function of the cluster decay ³⁾. Usually the clusters are assumed to be produced according to a Poisson distribution ¹⁾, i.e.,

$$L(z) = e^{-\langle n \rangle_c (1-z)}$$

with $\langle n \rangle_c$ being the mean number of clusters.

Introducing this into Eq. (2), one obtains the generating function for independent cluster emission model

$$\Lambda(z) = e^{-\langle n \rangle_c (1-\ell(z))} \quad (3)$$

The decay distribution is usually assumed to be narrow, with $\ell(z)$ typically a polynomial.

The reasoning that led to the generating function as given above, is classical. In the present paper we wish to investigate if there is a quantum mechanical structure that can give such a multiplicity distribution and examine its consequences. We shall show that one can derive the generating function of the independent cluster emission model from the assumption that, for a given value of the nucleon impact parameter b , the number of modes in which the mesons are produced is very large. On the basis of this, we shall discuss under what conditions the quantum mechanical interference corrections to the inclusive distributions remain small.

2. MAIN DISCUSSION

Our starting point is the observation that at large c.m. energy $\sqrt{s}$ the scattering operator S becomes diagonal in the nucleon impact parameter b , provided the total c.m. energy carried off by the produced secondaries is small ⁴⁾. Confining our attention to the central region (for the purposes of our qualitative discussion, we may disregard the fragmentation region pions), we write that

$$S = S(Y, b) \quad (4)$$

where Y is the difference of the incoming nucleon rapidities and $S(Y, b)$ is a unitary operator in the meson Fock-space. We normalize the meson states as

$$|y_1, \vec{q}_1, \dots, y_n, \vec{q}_n\rangle = \prod_{j=1}^n a^\dagger(y_j, \vec{q}_j) |0\rangle, \quad (5)$$

$$[a(y, \vec{q}), a^\dagger(y', \vec{q}')] = 2(2\pi)^3 \delta(y' - y) \delta(\vec{q}' - \vec{q}) \quad (6)$$

where y is the meson rapidity and $\vec{q}$ the transverse momentum. With this normalization one has that

$$\sigma_n = \int d^2 b \sigma_n(b), \quad n \geq 1, \quad (7)$$

where

$$\sigma_n(b) = \int d\psi_n |\langle y_1, \vec{q}_1, \dots, y_n, \vec{q}_n | S(Y, b) | 0 \rangle|^2. \quad (8)$$

The phase-space element $d\varphi_n$ is given by ⁵⁾

$$d\varphi_n = \frac{1}{n!} \left(\frac{1}{2(2\pi)^3} \right)^n dy_1 d^2q_1 \dots dy_n d^2q_n. \quad (9)$$

We assume that for a given value of b , the scattering operator S is a function of N meson creation operators $w_j^+, j=1, 2, \dots, N$. Because of crossing symmetry in the meson variables ⁵⁾, S has to be a function of the sums $w_j + w_j^+$. We call this a model of N modes. Without loss of generality, one may assume the modes to be orthonormal, i.e.,

$$[w_i, w_j^+] = \delta_{i,j}. \quad (10)$$

The simplest models are the single mode ones, with $N=1$. All mesons for a given b are emitted with the same wave-function and therefore one has a high degree of coherence. Here we shall discuss the consequences of the opposite situation when N is very large. In this case, the emitted mesons are distributed in a large number of different modes, i.e., one has incoherent production. We assume that the mesons are emitted with equal and independent probability into the N different modes. This corresponds to putting

$$S = \tilde{S}(w_1 + w_1^+) \dots \tilde{S}(w_N + w_N^+), \quad (11)$$

where the operator $\tilde{S}(w + w^+)$ now contains only one single mode. Clearly, S is unitary if $\tilde{S}$ is unitary. Substituting this into Eq. (8), one obtains, using Eq. (10), that

$$\sigma_n(b) = \sum_{n_1=0}^{\infty} \dots \sum_{n_N=0}^{\infty} \delta_{n, \sum n_j} \tilde{\sigma}_{n_1}(b) \dots \tilde{\sigma}_{n_N}(b), \quad (12)$$

where

$$\tilde{\sigma}_n(b) = \int d\varphi_n |\langle y_1, \vec{q}_1, \dots, y_n, \vec{q}_n | \tilde{S}(w + w^+) | 0 \rangle|^2 \quad (13)$$

is the cross-section for emitting n mesons in any particular mode.

We now let $N \rightarrow \infty$. In order to have a finite mean multiplicity, the coupling constant of each mode has to go to zero. Thus we put

$$\tilde{S}(w + w^+) = e^{i \frac{1}{\sqrt{N}} A(w + w^+)} \quad (14)$$

where $A(w+w^+)$ is a Hermitean operator independent of N . This gives, to leading order in N , that

$$\begin{aligned}\tilde{G}_n(t) &= \frac{1}{N} a_n(t), \quad n > 0, \\ \tilde{G}_0(t) &= 1 - \frac{1}{N} a_0(t),\end{aligned}\tag{15}$$

where

$$a_n(t) = \int d\psi_n |\langle y_1, \vec{q}_1, \dots, y_n, \vec{q}_n | A(w+w^+) | 0 \rangle|^2$$

for $n > 0$ and

$$a_0(t) = \sum_{n=1}^{\infty} a_n(t) = \langle 0 | A^2 | 0 \rangle\tag{16}$$

provided $\langle 0 | A | 0 \rangle = 0$ in order to make the elastic amplitude purely imaginary. For simplicity, we take $a_n(t) = 0$ for $n > k$ where k is an arbitrary integer number (this will be the case if A is a polynomial in $w+w^+$).

The substitution of Eq. (15) into Eq. (12) gives that

$$\begin{aligned}G_n(t) &= \sum_{\lambda_1=0}^{\infty} \dots \sum_{\lambda_k=0}^{\infty} \delta_{n, \sum j \lambda_j} \frac{1}{\lambda_1! \dots \lambda_k!} \frac{1}{(N - \sum \lambda_j)!} \\ &\quad \left(\frac{1}{N}\right)^{\sum \lambda_j} \left(1 - \frac{1}{N} a_0(t)\right)^{N - \sum \lambda_j} (a_1(t))^{\lambda_1} \dots (a_k(t))^{\lambda_k}\end{aligned}$$

which in the limit as $N \rightarrow \infty$ becomes

$$\begin{aligned}G_n(t) &= e^{-a_0(t)} \sum_{\lambda_1=0}^{\infty} \dots \sum_{\lambda_k=0}^{\infty} \delta_{n, \sum j \lambda_j} \frac{1}{\lambda_1! \dots \lambda_k!} \\ &\quad (a_1(t))^{\lambda_1} \dots (a_k(t))^{\lambda_k}.\end{aligned}\tag{17}$$

We now form the generating function

$$\mathcal{L}(z, t) = \sum_{n=0}^{\infty} z^n G_n(t)$$

and obtain that

$$\mathcal{L}(z, t) = e^{-a_0(t)(1 - \ell(z, t))}\tag{18}$$

where

$$\ell(z, b) = \frac{1}{a_0(b)} \sum_{j=1}^{\infty} a_j(b) z^j \quad (19)$$

If we now assume that $\ell(z, b)$ is independent of b , which will be the case if the operator A in Eq. (14) is of the form

$$A(w + w^+) = f(b) \bar{A}(w + w^+)$$

where $f(b)$ is some b dependent coupling constant and $\bar{A}(w + w^+)$ is an operator which does not explicitly depend on b , we obtain the final formula that

$$\Lambda(z, b) = e^{-a_0(b)(1 - \ell(z))} \quad (20)$$

Comparing this with Eq. (3), one sees that the derived generating function is the same as if one had assumed independent cluster emission at each value of b with $\langle n(b) \rangle_c = a_0(b)$ and with a decay distribution $\ell(z)$ which is independent of the particular value of b where the cluster was formed.

We shall now show that any classical cluster decay distribution $\ell(z)$ can be reproduced by a suitable choice of the Hermitean operator $A(w + w^+)$. To do this we solve Eq. (16) for A where we assume the set of decay probabilities $a_n(b) \geq 0$, $n = 0, 1, 2, \dots$, to be given. We write $A(w + w^+)$ as

$$A(w + w^+) = R(x) \Big|_{x=w+w^+} \quad (21)$$

where $R(x)$ is a real function for real x . We use Eq. (33) of Ref. 5) which in the present context says that

$$a_n(b) = \frac{1}{2^n} \frac{1}{n!} \left| \frac{1}{\sqrt{\pi}} \int_{-\infty}^{+\infty} dx e^{-x^2} H_n(x) R(\sqrt{2}x) \right|^2, \quad n > 0, \quad (22)$$

where $H_n(x)$ are the suitable normalized Hermite polynomials. We expand $R(\sqrt{2}x)$ in $H_n(x)$:

$$R(\sqrt{2}x) = \sum_{n=0}^{\infty} \alpha_n(b) H_n(x), \quad (23)$$

where $\alpha_n(b)$ are undetermined real coefficients. Substituting this into the previous equation and using the orthogonality properties of $H_n(x)$ one

obtains the solution

$$\alpha_n(b) = \pm \frac{1}{\sqrt{2^n n!}} \sqrt{a_n(b)}, \text{ for } n > 0 \quad (24)$$

and $\alpha_0(b) = 0$ if the elastic amplitude is required to be purely imaginary. This is via Eqs. (19), (21) and (23) the explicit connection between $A(w+w^+)$ and $\ell(z)$.

The main empirical evidence for the validity of the cluster emission model comes from the study of two-particle inclusive correlation data. Though the quantum mechanical model discussed here does reproduce the generating function of the independent cluster emission model, it contains quantum mechanical interference effects which, in general, will show up in the inclusive n-particle distributions, spoiling the simple classical picture of the cluster emission model. We shall now investigate the conditions which the operator A has to satisfy in order to make the interference terms negligible.

We define the b-dependent n-particle inclusive distribution by

$$\rho_n(b, q_1, \dots, q_n) = \int d\psi_{n'} \frac{d\sigma_{n+n'}(b, q_1, \dots, q_n, q'_1, \dots, q'_{n'})}{d\psi_{n+n'}} \quad (25)$$

where the q_i denote the meson three-momenta. The ρ_n are normalized as

$$\int d\psi_n \rho_n = \frac{1}{n!} \left. \frac{\partial^n \mathcal{L}(z, b)}{\partial z^n} \right|_{z=1} \quad (26)$$

Using Eq. (11) and the formalism developed in Ref. 5), it is straightforward, if somewhat laborious, to calculate ρ_n . We begin by quoting the result for the single-particle distribution. One gets that

$$\begin{aligned} \rho_1(b, q) &= \frac{1}{4\pi^2} \frac{1}{N} \sum_i |W_i(q)|^2 \left. \frac{\partial \mathcal{L}(z, b)}{\partial z} \right|_{z=1} \\ &\quad - \frac{1}{4\pi^2} \frac{1}{N} \frac{1}{2\sqrt{2\pi}} |\alpha_1(b)|^2 \sum_{i \neq j} W_i(q) W_j(q) \end{aligned} \quad (27)$$

where $W_i(q)$ are the orthogonal wave-functions of the modes in which the mesons are produced (for simplicity we assume W_j to be real), i.e.,

$$w_j^\dagger = \frac{1}{4s(2\pi)^3} \int d^4 q \, W_j(q) a^\dagger(q, \vec{q}),$$

$$\int d^4 q \, W_i W_j = 8s^2 (2\pi)^3 \delta_{i,j}. \quad (28)$$

The coefficient $\alpha_1(b)$ is as defined by Eq. (23).

The first term in Eq. (27) is essentially classical, whereas the second term is due to quantum mechanical interference. Remembering Eq. (24), one sees that the interference term in ρ_1 is going to vanish if one requires that

$$\left. \frac{d \ell(z)}{dz} \right|_{z=0} = 0 \quad (29)$$

where $\ell(z)$ is again the "cluster decay" generating function. This result is exact in the sense that it holds to all orders of $1/N$.

The situation is similar for the higher inclusive distributions. One can show that all interference terms in ρ_n are going to vanish, at least to leading order in N , if

$$\left. \frac{d^j \ell(z)}{dz^j} \right|_{z=0} = 0 \quad (30)$$

for all $j \leq n$. To be more precise, vanishing to leading order in N implies, e.g., that if $W_j(q)$ are bounded for all j and q then one can show that the interference term remains bounded for all values of the meson momenta in the limit as $N \rightarrow \infty$. This is a non-trivial result since if the condition given by Eq. (30) is not satisfied, then the interference term would blow up at $q=0$ in the limit $N \rightarrow \infty$ if, e.g., all $W_j(0)=1$. The bound is obtained by replacing in a sum containing an infinite number of terms, in general, with different signs, by a sum of the absolute values. For this reason, it grossly overestimates the actual magnitude of the interference. Therefore we do not bother to exhibit it. To summarize, if condition (30) is satisfied, then the interference corrections to the classical expression in ρ_n are strongly suppressed and it seems that they can be safely neglected in all practical applications.

This remarkable suppression is due to the fact that our model satisfies approximate multiparticle unitarity and crossing, which leads to cancellations when summing over an infinite number of states. No comparable suppression is expected to occur for the semi-inclusive distributions.

Another, trivial, way of making the interference terms vanish is to require the meson wave-functions to be non-overlapping, i.e., $W_i W_j \equiv 0$ for $i \neq j$.

To show that the quantum mechanical model discussed here does exhibit the same type of correlations as one is accustomed to in the classical cluster emission model, we give the formula for the inclusive two-particle distribution (neglecting interference)

$$Q_2(b, q_1, q_2) = Q_1(b, q_1) Q_1(b, q_2) + \frac{1}{N} \frac{1}{(4\sigma^2)^2} \left(\sum_i |W_i(q_1)|^2 |W_i(q_2)|^2 a_0(b) \frac{d^2 l(z)}{dz^2} \right) \Big|_{z=1}. \quad (31)$$

The first term can give rise to long-range correlations due to b integration. The second term gives rise to positive short-range correlations typical of the cluster emission models. The explicit form of this short-range correlation will of course depend on the explicit form of the meson wave-functions W_j .

We would also like to mention that though the model discussed here gives a generating function which, for a fixed value of b , is the same as that of the cluster emission model, one can have negative values for the Mueller correlation coefficients⁶⁾. This is so because the final generating function $\Lambda(z)$ is obtained by integrating, over all values of b , the generating function $\Lambda(z, b)$ given by Eq. (20). Whether or not this happens will depend on the explicit form of $a_0(b)$.

We have shown that the existence of a large number of modes in which the mesons are emitted, can give a distribution which simulates the classical distribution of the independent cluster emission model. In the present approach the clusters need not be physical objects. The clustering is a result of incoherent production. Incoherent production is symptomatic of the presence of a large number of independent emission centres. Therefore, one might try to give the model discussed here the following physical interpretation: The proton consists of a large number of weakly coupled constituents. The modes w_j^+ are the modes in which the mesons are emitted when two constituents hit each other in proton-proton collision. In the spirit of Ref. 7), one would then speak of collision probability between constituents (hits) and of bunches instead of the two-step concept of cluster emission and decay. More work is needed, however, before one could see whether this interpretation is really tenable.

The idea that the mesons in proton-proton collision are produced in many different modes ⁸⁾ helps to resolve a long-standing difficulty one had when trying to apply impact parameter models to proton-proton scattering. In models containing a single mode only, the effective coupling constant is of the order of the mean multiplicity. Therefore one was unable to account for the fact that, while experimentally the mean multiplicity is reasonably large, the proton is still rather transparent ⁹⁾. In models with a large number of modes of the type discussed here, the effective coupling constant turns out to be reduced to

$$\langle n \rangle / \langle \tilde{n} \rangle$$

where

$$\langle \tilde{n} \rangle = \left. \frac{d \ell(z)}{dz} \right|_{z=1}$$

is the mean multiplicity of the cluster decay. This again is very similar to what was found in Ref. 7).

3. SUMMARY AND CONCLUSIONS

We have shown that the generating function of the classical independent emission model can be obtained from the assumption that the number of modes in which the mesons are emitted is very large.

We have discussed interference corrections and have shown that due to unitarity and crossing, they are strongly repressed in the first n inclusive distributions if the cluster decay distribution is narrow and if the average number of decay products is larger than n . Judging by our results, such interference terms might, however, show up in the semi-inclusive distributions and in the higher inclusive distributions.

Insofar as the basic idea of our paper is correct, our results seem to point towards the following general conclusions.

- 1) The mesons in high-energy proton-proton collision are produced in a large number of different modes.
- 2) The clustering of secondaries is a result of incoherent production. There is no need to postulate the existence of clusters as physical objects.
- 3) Since incoherent production indicates the presence of a large number of emission centres, our results seem to imply that it might be useful

when discussing high-energy proton-proton scattering, to consider the proton as being composed of a large number of weakly coupling constituents ¹⁰⁾.

In order to apply the formalism discussed here to high-energy proton-proton scattering in a more quantitative manner, it is necessary to incorporate the isospin of the pion and diffractive excitation. Work in this direction is in progress.

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A. Białas, TH.1745 (1973).
E.L. Berger, TH.1800 (1974).
- 2) See, e.g., M. Le Bellac loc. cit.
- 3) For simplicity, we assume that all the secondaries are of the same kind. The content of the paper can easily be generalized to the case of several kinds of secondaries.
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See, R.P. Feynman, "Photon-Hadron Interactions" (W.A. Benjamin, Inc., Reading, Massachusetts, 1972).

INCOHERENCE AND CLUSTER EMISSION

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Please note on page 4 the equation before Eq. (17) should read:

$$G_n(t) = \sum_{\lambda_1=0}^{\infty} \dots \sum_{\lambda_k=0}^{\infty} \delta_{n, \sum \lambda_j} \frac{1}{\lambda_1! \dots \lambda_k!} \frac{N!}{(N - \sum \lambda_j)!} \left(\frac{1}{N}\right)^{\sum \lambda_j} \left(1 - \frac{1}{N} a_0(t)\right)^{N - \sum \lambda_j} (a_1(t))^{\lambda_1} \dots (a_k(t))^{\lambda_k}$$

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