Archive

CERN LIBRARIES, GENEVA

CM-P00060590

Ref.TH.1942-CERN

SUPERSYMMETRY AND THE VACUUM

Bruno Zumino
CERN -- Geneva

ABSTRACT

It is shown that, in a supersymmetric theory, the sum of all vacuum diagrams vanishes identically as a consequence of compensations among contributions involving different fields of the supermultiplet. This means that the zero point energy momentum density is zero to all orders in the supersymmetric interaction. In an external gravitational field no cosmological term is induced through the vacuum polarization of the matter fields. The situation when supersymmetry is spontaneously broken, or softly broken is also discussed.

1. Introduction

Supersymmetric (supergauge invariant) field theories exhibit remarkable cancellations of divergences among Feynman diagrams [1]. These cancellations have been studied in detail for tadpole, self-energy, and vertex diagrams [2,3,4]. In this paper we examine the vacuum diagrams, usually considered as uninteresting in renormalization theory, and show (in sect. 2) that the sum of all vacuum diagrams of a given order vanishes exactly in a supersymmetric theory. Although this result could be easily obtained with the technique of superfields [5], we find it more enlightening and easier for the physical interpretation to work with the more traditional field multiplets. The physical meaning of our result is that the vacuum expectation value of the energy momentum tensor vanishes identically to all orders in a supersymmetric theory, without the introduction of the *ad hoc* subtraction constant necessary in each order for a generic theory. This is true in particular for the sum of the zero point energy densities of the free fields of a supermultiplet. As explained in sect. 3, our result means that, in the presence of an external gravitational field, the cosmological term which is usually induced because of the vacuum polarization due to the matter fields vanishes in the case of a supersymmetric theory. As shown in the appendix, the vanishing of the sum of all vacuum graphs does not by itself imply that the vacuum-vacuum amplitude equals unity when the volume of the system tends to infinity. Rather, a study of the volume dependence of the vacuum-vacuum amplitude, shows that the vacuum energy grows like the three-volume to the power one-third for a supersymmetric theory, to be contrasted with the usual proportionality to the volume itself for a generic theory. On the other hand, the scalar product between the unperturbed and the perturbed vacuum vanishes exponentially with the volume as in the generic field theory.

The vanishing of the induced cosmological term in a supersymmetric theory is interesting in view of the experimental smallness of that term in Einstein's equation, a fact which in a generic field theory must be considered as a peculiar accident. Unfortunately, when supersymmetry is broken, even only spontaneously broken, the result is no longer valid. For a supersymmetric field theory, the basic anticommutation relation

$$\{Q_i, Q_j\} = -2(\gamma^\lambda \gamma^0)_{ij} P_\lambda \quad (1)$$

can be considered as arising by integration from the local relation*

$$\{Q_i, J_j^\mu(x)\} = 2(\gamma^\lambda \gamma^0)_{ij} T_\lambda^\mu(x) + \text{o.s.t.}, \quad (2)$$

*This relation gives the transformation law of the supercurrent under supertransformations. The detailed form of the o.s.t. is not relevant to the argument presented here. It will be given in a future publication.

where o.s.t. denotes "operator Schwinger terms" which, for $\mu = 0$, integrate to zero, $T_{\lambda\mu}$ is the energy momentum tensor, and the spinor supercharge Q_i is the integral of the conserved supercurrent J_i^μ

$$Q_i = \int J_i^\mu d^3x, \quad \partial_\mu J_i^\mu = 0. \quad (3)$$

If the supercharge annihilates the vacuum, (2) implies that the vacuum expectation value of the energy momentum tensor is zero

$$\langle T_{\lambda\mu}(x) \rangle = 0, \quad (4)$$

since the vacuum expectation value of the operator Schwinger terms vanishes. However, when supersymmetry is spontaneously broken, (4) ceases to be true, already in the tree approximation. The left-hand side of (2) does not vanish in the vacuum because of the zero mass singularity (Goldstone spinor) in the vacuum expectation value of the anticommutator of two supercurrents and the supercharge does not annihilate the vacuum. At the same time, in the right-hand side of (2) the energy momentum tensor, through the vacuum expectation value of a scalar field, acquires a constant term^{*} proportional to the Minkowski metric $\eta_{\mu\nu}$. The local relation (2) remains true, but it cannot be integrated, because the integral would be infinite. [One could, of course, redefine $T_{\lambda\mu}$ so that it satisfies (4), but then an additional constant term shows up in the right-hand side of (2)]. Therefore (1) is no longer valid, at any rate for the vacuum^{**}. These general statements can be easily checked in the example of spontaneously broken supersymmetry given by Fayet and Iliopoulos [7] using the field theory first described in ref. [4]. It is amusing that (2) fixes the energy momentum tensor, as far as its constant term goes, to be that of the symmetric theory.

It is likely that the results of the present paper can be understood as special cases from the point of view of a more general theory of gravitation in which the gravitational field $g_{\mu\nu}$ is a member of a supermultiplet together with particles of spin $\frac{3}{2}$ and lower [8].

*In this the situation is similar to that discussed by Veltman [6] for the case of spontaneously broken gauge invariance.

**The heuristic argument given in appendix 1 of ref. [3] to show that supersymmetry cannot be spontaneously broken is therefore invalid, being based on (1). Consequently it should not be necessary to have a theory with gauge fields, as argued in ref. [7], to find spontaneous breaking of supersymmetry.

2. Vanishing of the sum of vacuum diagrams

We begin with an example. We take the supersymmetric Lagrangian^{*}

$$\begin{aligned} L_f &= L_0 + f L_{\text{int.}} \\ L_0 &= -\frac{1}{2}(\partial A)^2 - \frac{1}{2}(\partial B)^2 - \frac{i}{2}\bar{\Psi}\gamma\cdot\partial\Psi + \frac{1}{2}F^2 + \frac{1}{2}G^2 \\ &\quad + m(FA + GB - \frac{i}{2}\bar{\Psi}\Psi) \end{aligned} \tag{5}$$

$$L_{\text{int.}} = FA^2 - FB^2 + 2GAB - i\bar{\Psi}\Psi A + i\bar{\Psi}\gamma_5\Psi B$$

whose renormalization was studied in detail in refs. [2] and [3]. Supersymmetric regularization terms can be added to L_f , for instance by using higher derivatives of the fields. Without writing everything out we assume that this has been done so that the Feynman rules for the unrenormalized but regularized quantities give rise to finite results satisfying the Ward identities appropriate to supersymmetry. All these questions were discussed in detail in ref. [3]. Performing now a calculation of the two-loop vacuum diagrams, one finds that the sum vanishes identically. For instance, the diagrams of fig. 1 (which would be each quartically divergent in the absence of regularization) cancel each other. The (quadratically divergent) diagrams of fig. 2 also cancel. The remaining two-loop vacuum diagrams, which are obtained by putting together two tadpole diagrams, also add up to zero.

As we shall see later, the cancellation occurs to all orders in the coupling constant f . This cancellation, however, does not mean that the vacuum-vacuum amplitude is identically equal to one. To see this, let us replace the coupling constant by a function $f(x)$ which vanishes smoothly in all directions in space-time (switch-off function) and which we shall let eventually tend to a constant (adiabatic switch-off). Graphically, this corresponds to replacing the diagrams of fig. 1 by those of fig. 3, where the double lines entering at the vertices carry the momentum of the "external" field $f(x)$. As the function $f(x)$ tends to a constant the external momenta tend to zero. Now, it is easy to check that, in the presence of space-time switch-off, the sum of the diagrams of fig. 3 does not vanish. Rather, one finds a contribution to the vacuum functional U (see below) equal to

^{*}The notation is slightly different from that of refs. [2] and [3]. In particular we denote the coupling constant by f , in order to avoid confusion with the determinant g of the $g_{\mu\nu}$ which occurs in sect. 3.

$$-\frac{4}{3} \iint [\Delta(x-x')]^3 \partial_\mu f(x) \partial^\mu f(x') d^4x d^4x', \quad (6)$$

where $\Delta(x)$ is the regularized Feynman propagator. The sum of the other two-loop vacuum diagrams (those corresponding to fig. 2 and those made of tadpole diagrams) vanishes even in the presence of space-time switch-off. In momentum space, (6) implies that the amplitude vanishes like the square of the external momenta, but not faster. As explained in the appendix this means that (6) tends to infinity like the square root of the four-volume over which $f(x) = f = \text{constant}$, in the adiabatic limit in which this four-volume tends to infinity in all directions. For a generic theory the amplitude would be proportional to the four-volume itself. The weaker volume dependence encountered in supersymmetric theories implies that the vacuum energy goes like the three-volume to the power one-third.

In order to give a proof that the above cancellations occur to all orders in the coupling constant f , we consider the regularized connected vacuum functional $U[f(x)]$ in the presence of switch-off. It can be defined, for instance, by the functional integral

$$e^{iU} = \frac{\int e^{i \int L_{f(x)} d^4x} \mathcal{D}[\Phi]}{\int e^{i \int L_0 d^4x} \mathcal{D}[\Phi]} \quad (7)$$

normalized so that U vanishes in the absence of coupling. Here $\Phi = (A, B, F, G, \psi)$ stands for all the fields in the supermultiplet. It follows immediately from (7) that

$$U[x; f(x)] \equiv \frac{\delta U}{\delta f(x)} = \langle L_{\text{int.}}(x) \rangle_{f(x)}, \quad (8)$$

where the right-hand side denotes the vacuum expectation value in the presence of switch-off. From the expansion

$$U[f(x)] = \int f(x) U_1 d^4x + \frac{1}{2!} \iint f(x) f(x') U_2(x, x') d^4x d^4x' + \dots \quad (9)$$

we see that

$$U[x; f(x)] = U_1 + \int f(x') U_2(x, x') d^4x' + \dots \quad (10)$$

Therefore

$$U[f(x)] = \int f(x) \int_0^1 U[x; \lambda f(x)] d\lambda d^4x. \quad (11)$$

In the adiabatic limit $f(x) \rightarrow f = \text{constant}$

$$\frac{U[f(x)]}{\int f(x) d^4x} \rightarrow \int_0^1 U[x; \lambda f] d\lambda = \frac{1}{f} \int_0^f U[x; f] df. \quad (12)$$

This expression gives the sum of all vacuum diagrams and, in general, equals a finite constant when the theory is sufficiently regularized in the ultraviolet.

For our supersymmetric theory, (8), and therefore (12), vanishes in the adiabatic limit. This can be seen by means of the following Ward identities for the three-point functions, which are special cases of the general Ward identities of ref. [3]:

$$\begin{aligned} 4 \langle A' A'' F \rangle - i \langle A' \bar{\psi} \psi'' \rangle - i \langle A'' \bar{\psi} \psi' \rangle &= 0 \\ 4 \langle B' B'' F \rangle + i \langle B' \bar{\psi} \gamma_5 \psi'' \rangle + i \langle B'' \bar{\psi} \gamma_5 \psi' \rangle &= 0 \\ 4 \langle A' B'' G \rangle - i \langle A' \bar{\psi} \psi'' \rangle + i \langle B'' \bar{\psi} \gamma_5 \psi' \rangle &= 0. \end{aligned} \quad (13)$$

In these equations the products of fields are time-ordered products (not explicitly indicated); the primes denote the points at which the fields are calculated and sums over spinor indices are taken. These Ward identities are valid in the adiabatic limit, of course. Taking all points equal (which is allowed in the regularized theory) and combining the equations (13) one obtains

$$\langle L_{\text{int.}}(x) \rangle_f = 0 \quad (14)$$

Observe that, in the above proof, we had to be very careful with the volume dependence of the quantities involved. For instance, it would be incorrect to conclude that $U[f(x)]$ vanishes in the adiabatic limit, only the density given by (12) does.

Equation (14) can be understood quite simply if one remembers that $L_{\text{int.}}(x)$ is a component of a supermultiplet, more precisely the F component. Now, if the vacuum state is supersymmetric, the F component of a supermultiplet must have a vacuum expectation value equal to zero. This follows immediately from the infinitesimal transformation law of the ψ component of the same multiplet

$$[\bar{\alpha} Q, \psi] = \partial_\mu (A - \gamma_5 B) \gamma^\mu \alpha + (F + \gamma_5 G) \alpha. \quad (15)$$

Taking the vacuum expectation value of this equation, one obtains

$$\langle F \rangle = 0, \quad (16)$$

since the supercharges Q_i annihilate the vacuum and since the vacuum expectation values of derivatives of fields vanish and that of G vanishes by parity. This proof of (14) shows clearly that the result obtained above, namely the vanishing of the sum of all vacuum graphs, is valid for any supersymmetric theory and not only for the particular example considered in this section.

In order to give a precise proof, the vacuum functional U was normalized above so as to vanish in the absence of coupling. However, it is worth noticing that the sum of the zero point energy densities of the free fields of a supermultiplet also vanishes. This can be seen most simply if one remembers that the zero point energy density of a free field is given by the divergent expression

$$\frac{N}{2(2\pi)^3} \int \sqrt{\vec{p}^2 + m^2} d^3p, \quad (17)$$

where N is positive for tensor fields and negative for spinor fields, and $|N|$ denotes the number of independent one-particle states of given momentum ($N = 1$ for a real scalar field, $N = 2$ for a complex scalar, $N = 2$ for a real massless vector, $N = 3$ for a real massive vector, $N = -2$ for a Majorana field, $N = -4$ for a complex massive spin- $\frac{1}{2}$ field, etc.). The various fields are, of course, quantized according to the usual connection between spin and statistics, but this still leaves a vacuum energy, which is usually just redefined away by normal ordering. However, using (17), one sees immediately that the vacuum energies compensate among the various physical fields of a supermultiplet (auxiliary fields do not contribute). For instance, for a "scalar" supermultiplet (left-handed superfield), consisting of a scalar, a pseudoscalar, and a Majorana spinor, one has respectively $N = 1, 1, -2$, and the sum vanishes; for a massless "vector" supermultiplet consisting of a real massless vector and a massless Majorana field, one has respectively $N = 2, -2$, and the sum vanishes; for a massive "vector" supermultiplet, consisting of a real vector, two Majorana fields and a real scalar, all massive, $N = 3, -2, -2, 1$. The result is also valid if internal symmetries are introduced, obviously when the internal symmetry commutes with supersymmetry but also in the less trivial case, considered in refs. [1], [9], and [10], when the supercharges carry an internal symmetry index. Summing the zero point energy over all fields of the multiplet, one obtains zero. It may be worth mentioning that, in adding the various contributions (17) for a supermultiplet, there is a cancellation already in the integrand, for each value of the momentum.

3. The vacuum in an external gravitational field

In order to clarify the physical meaning of the results of the preceding section, we consider the vacuum functional in the presence of an external gravitational field $X[g_{\mu\nu}, f]$, which can be defined for instance by the functional integral

$$e^{iX} = \frac{\int e^{i\int L[g_{\mu\nu}, f] d^4x} \mathcal{D}[\Phi]}{\int \mathcal{D}[\Phi]}, \quad (18)$$

normalized so that X vanishes for $g_{\mu\nu} = 0$. $L[g_{\mu\nu}, f]$ is the Lagrangian obtained from (5) by introducing the gravitational field $g_{\mu\nu}$ in the standard way and is a density under Einstein transformations (actually, because of the presence of spinor fields, we should introduce a vierbein field, instead of the field $g_{\mu\nu}$; this would present no difficulty but, in order to use a notation more familiar to most readers, we pass over this point). The gravitational field is assumed to vanish smoothly at infinity in all directions in space time. The vacuum functional U of sect. 2 is related to X by

$$U[f(x)] = X[\eta_{\mu\nu}, f(x)] - X[\eta_{\mu\nu}, 0], \quad (19)$$

where $\eta_{\mu\nu}$ denotes the Minkowski metric. However, now we take f constant since the switch-off is provided by the external field $g_{\mu\nu}$.

It follows immediately from (18) that

$$\frac{d}{df} X[g_{\mu\nu}, f] = \int \langle L_{\text{int.}}(x) \rangle_{g_{\mu\nu}} \sqrt{g} d^4x, \quad (20)$$

where $g = -\det g_{\mu\nu}$ and the expectation value of $L_{\text{int.}}$ is taken in the interacting vacuum in the presence of the external field $g_{\mu\nu}$. On the other hand, like any translationally invariant functional, X can be expanded in a series of terms involving derivatives of $g_{\mu\nu}$ of increasing order:

$$X[g_{\mu\nu}, f] = c(f) \int \sqrt{g} d^4x + d(f) \int R \sqrt{g} d^4x + \dots \quad (21)$$

Differentiating this series with respect to f , going to the limit $g_{\mu\nu} \rightarrow \eta_{\mu\nu}$, and comparing with (20) one finds

$$\frac{dc}{df} = \langle L_{\text{int.}}(x) \rangle_{\eta_{\mu\nu}}. \quad (22)$$

Looking back at (8) and (12) we see that $c(f) - c(0)$ gives the sum of all vacuum diagrams. From sect. 2 we know that it vanishes in a supersymmetric theory. On the other hand, $-c(0)$ gives the total zero point energy density of the free fields of the supermultiplet, which was also shown to vanish in sect. 2. Therefore we have identically

$$c(f) = 0 . \quad (23)$$

Equation (21) shows that $c(f)$ is the induced cosmological term coming from the vacuum polarization due to the matter fields. If one takes the functional derivative of (21) with respect to $g_{\mu\nu}$ and then sets $g_{\mu\nu} = \eta_{\mu\nu}$, one obtains the vacuum expectation value of the energy momentum tensor. All the terms with derivatives of $g_{\mu\nu}$ give zero. Therefore our result (23) can be simply stated as

$$\langle T_{\mu\nu}(x) \rangle = 0 . \quad (24)$$

As mentioned in sect. 1, this equation is automatically satisfied, without any subtractive constant.

Since $c(f)$ vanishes identically, the first term in the expansion (21) remains zero in the adiabatic limit $g_{\mu\nu} \rightarrow \eta_{\mu\nu}$. The volume dependence of the vacuum functional X comes from the terms with derivatives of $g_{\mu\nu}$. The second term in the expansion (21) gives the leading volume dependence, and it can be seen indeed to behave like the square root of the four-volume, in agreement with the result of sect. 2 and of the appendix, where the adiabatic switch-off was introduced in the coupling constant instead of in the $g_{\mu\nu}$. The fact that the coefficient $d(\lambda)$ of the second term does not vanish in a supersymmetric theory means that there is a renormalization of the gravitational constant*.

Helpful discussions with J. Bell, H. Epstein, R. Haag and V. Glaser are gratefully acknowledged.

*The results obtained in the present section are confirmed by explicit calculations of the vacuum polarization in an external gravitational field. For instance, DeWitt [11] has calculated the functional $X[g_{\mu\nu}, 0]$ for a real scalar field and for a Majorana field, by the manifestly covariant proper-time method. His results

for $c(0)$ can be written as

$$c(0) = -\frac{N}{2} \frac{\partial}{\partial \sigma} \Delta^{(1)}(\sigma) \Big|_{\sigma \rightarrow 0}$$

$$= -\frac{N}{8\pi^2} \left[\frac{1}{\sigma^2} - \frac{m^2}{2\sigma} - \frac{m^4}{4} \left(\frac{1}{2} \log |2m^2\sigma| + \gamma - \log 2 - \frac{5}{4} \right) \right]_{\sigma \rightarrow 0}.$$

Here N is the same as in sect. 2, $\sigma = \frac{1}{2}x^2$ is a covariant cut-off, and

$$\Delta^{(1)}(\sigma) = \frac{1}{(2\pi)^3} \int e^{ik_\mu x^\mu} \delta(k^2 + m^2) d^4k.$$

This is in perfect agreement with (17). Observe that, if the masses of the field of the supermultiplet are different, only the quartic divergence of $c(0)$ cancels. If certain mass relations are satisfied, the quadratic divergence also cancels and $c(0)$ is only logarithmically divergent. For the example of sect. 2 this happens if

$$m_A^2 + m_B^2 = 2m_\psi^2.$$

Appendix

In the interaction picture the functional $U[f(x)]$ of sect. 2 is given by

$$e^{iU} = \left(\Psi_0, T e^{i \int f(x) L_{int.}(x) d^4x} \Psi_0 \right), \quad (A.1)$$

where Ψ_0 is the unperturbed vacuum state and the time dependence is that given by the free Hamiltonian. We take the contribution (6) to U , which we write as

$$\frac{4}{3} \iint f(x) \square [\Delta(x-x')]^3 f(x') d^4x d^4x'. \quad (A.2)$$

Remembering that Δ is the regularized Feynman propagator we can use the representation

$$\frac{4}{3} [\Delta(x-x')]^3 = \int_{M^2}^{\infty} da \rho(a) \Delta_a(x-x'), \quad (A.3)$$

where Δ_a is the Feynman propagator for mass $\sqrt{a}$ and M is the (three-particle) threshold. In terms of the Fourier transform $\hat{f}(p)$ of $f(x)$, (A.3) takes the form

$$\int_{M^2}^{\infty} da \rho(a) \int d^4p |\hat{f}(p)|^2 \frac{p^2}{p^2 + a - i0}, \quad (A.4)$$

up to numerical factors. In order to go to the adiabatic limit, in (A.2) we replace $f(x)$ by $f(\epsilon x)$ and we shall let $\epsilon \rightarrow 0$. This corresponds to replacing $\hat{f}(p)$ by $\epsilon^{-4} \hat{f}(p/\epsilon)$ in (A.4). Introducing the variable $q = p/\epsilon$ we write

$$\begin{aligned} & \frac{1}{\epsilon^2} \int_{M^2}^{\infty} da \rho(a) \int d^4q |\hat{f}(q)|^2 \frac{q^2}{\epsilon^2 q^2 + a - i0} \\ & \sim \frac{1}{\epsilon^2} \int_{M^2}^{\infty} da \frac{\rho(a)}{a} \int d^4q |\hat{f}(q)|^2 q^2. \end{aligned} \quad (A.5)$$

This (real) expression behaves like ϵ^{-2} , the square root of the four-volume ϵ^{-4} , which is proportional to the four-volume over which the function $f(\epsilon x)$ is different from zero. Observe that, in a non-supersymmetric theory, the factor p^2 in the integrand of (A.5) would be missing and the resulting behaviour would be like ϵ^{-4} instead of ϵ^{-2} .

In order to extract more information from the functional $U[f(x)]$, we must take a function $f(x)$ which is constant in time between t_1 and t_2 and zero outside the interval, while it vanishes smoothly for $\vec{x}$ going to infinity in all directions. It is well known that, as $t_2 \rightarrow \infty$, or $t_1 \rightarrow -\infty$, or both,

$$\begin{aligned}
 & (\Psi_0, T e^{i \int_{t_1}^{t_2} \int f(\vec{x}) L_{\text{int.}}(t, \vec{x}) dt d^3x} \Psi_0) \\
 & \rightarrow |(\Psi, \Psi_0)|^2 e^{-i(t_2 - t_1) \Delta E} ,
 \end{aligned} \tag{A.6}$$

where Ψ is the perturbed (physical) vacuum state and ΔE the energy difference between the two vacua. For the contribution (A.2) to U we obtain now, up to numerical factors,

$$\int_{M^2}^{\infty} da \rho(a) \int d^3p \int dp^0 \frac{2 - e^{-ip^0\tau} - e^{ip^0\tau}}{(p^0)^2 (p^0 + \omega)(p^0 - \omega)} [(p^0)^2 - (\vec{p})^2] |\hat{f}(\vec{p})|^2, \tag{A.7}$$

where $\hat{f}(\vec{p})$ is the Fourier transform of $f(\vec{x})$, $\tau = t_2 - t_1$, and the p^0 integral goes around the singularities $\pm\omega = \pm \sqrt{\vec{p}^2 + a}$ in the usual Feynman way (observe that there is no singularity at $p^0 = 0$). The integration can be performed by suitably closing the contour at infinity in the upper or lower half plane with a result proportional to

$$\int_{M^2}^{\infty} da \rho(a) \int d^3p \left(i \frac{1 - e^{-i\omega\tau}}{\omega^3} a + \tau \frac{\vec{p}^2}{\omega^2} \right) |\hat{f}(\vec{p})|^2. \tag{A.8}$$

Comparing with (A.6) we find

$$\Delta E \propto - \int_{M^2}^{\infty} da \rho(a) \int d^3p |\hat{f}(\vec{p})|^2 \frac{\vec{p}^2}{\omega^2} \tag{A.9}$$

$$\log |(\Psi, \Psi_0)|^2 \propto - \int_{M^2}^{\infty} da \rho(a) \int d^3p |\hat{f}(\vec{p})|^2 \frac{a}{\omega^3}.$$

We now replace $f(\vec{x})$ by $f(\epsilon\vec{x})$ and $\hat{f}(\vec{p})$ by $\epsilon^{-3}\hat{f}(\vec{p}/\epsilon)$. We obtain

$$\begin{aligned}
 \Delta E & \propto - \frac{1}{\epsilon} \int_{M^2}^{\infty} da \rho(a) \int d^3q \frac{|\hat{f}(\vec{q})|^2 \vec{q}^2}{\epsilon^2 \vec{q}^2 + a} \\
 & \sim - \frac{1}{\epsilon} \int_{M^2}^{\infty} da \frac{\rho(a)}{a} \int d^3q |\hat{f}(\vec{q})|^2 \vec{q}^2
 \end{aligned} \tag{A.10}$$

and

$$\begin{aligned}
 \log |(\Psi, \Psi_0)|^2 &\propto -\frac{1}{\varepsilon^3} \int_{M^2}^{\infty} da \, \rho(a) \int d^3q \, \frac{|\hat{f}(\vec{q})|^2 a}{(\varepsilon^2 \vec{q}^2 + a)^{3/2}} \\
 &\sim -\frac{1}{\varepsilon^3} \int_{M^2}^{\infty} da \, \frac{\rho(a)}{\sqrt{a}} \int d^3q \, |\hat{f}(\vec{q})|^2 .
 \end{aligned}
 \tag{A.11}$$

Thus ΔE behaves like ε^{-1} , which is the cube root of the three-volume ε^{-3} , while for a generic theory it would go like ε^{-3} . On the other hand, $|(\Psi, \Psi_0)|^2$ vanishes exponentially with the three-volume, the same as in any theory.

References

- [1] For a general review on supersymmetry and an extensive list of references, see B. Zumino, Proc. 17th Int. Conf. on High-Energy Physics, London, 1974; also available as CERN preprint TH.1901.
- [2] J. Wess and B. Zumino, Phys. Letters 49 B (1974) 52;
S. Ferrara, J. Iliopoulos and B. Zumino, Nucl. Phys. B77 (1974) 413. For a compensation involving a finite quantity see S. Ferrara and E. Remiddi, CERN preprint TH.1935 (1974).
- [3] J. Iliopoulos and B. Zumino, Nucl. Phys. B76 (1974) 310.
- [4] J. Wess and B. Zumino, Nucl. Phys. B78 (1974) 1.
- [5] A. Salam and J. Strathdee, Nucl. Phys. B76 (1974) 477;
S. Ferrara, J. Wess and B. Zumino, Phys. Letters 61 B (1974) 239;
A. Salam and J. Strathdee, Trieste preprints IC/74/42 (1974), IC/74/85 (1974);
R. Debourgo, Imperial College, London, preprint ICTP/73/44 (1974);
D.M. Capper, Trieste preprint IC/74/66.
- [6] M. Veltman, Utrecht University preprint (1974).
- [7] P. Fayet and J. Iliopoulos, Phys. Letters 51 B (1974) 461.
- [8] See, for example, D.V. Volkov and V.A. Soroka, JETP Letters 18 (1973) 529.
- [9] A. Salam and J. Strathdee, Trieste preprint IC/74/16 (1974), to be published in Nucl. Phys. B.
- [10] P.H. Dondi and M. Sohnius, University of Karlsruhe preprint (1974), to be published in Nucl. Phys. B.
- [11] B.S. DeWitt, Dynamical Theory of Groups and Fields, pages 231 and 233 (Gordon and Breach publishers, New York, 1965).

Figure captions

Figure 1 Quartically divergent two-loop vacuum diagrams.

Figure 2 Quadratically divergent two-loop vacuum diagrams.

Figure 3 Quartically divergent vacuum diagrams with space-time dependent coupling constant.

Fig. 1

Fig. 2

Fig. 3