RESONANCES, FORM FACTORS AND THE HADRON SPECTRUM
IN INELASTIC ELECTRON-PROTON SCATTERING ^{*)}

W. Konetschny ⁺⁾ and F. Steiner ^{x)}

CERN -- Geneva

A B S T R A C T

We investigate the general resonance model for the deep inelastic structure functions incorporating spin and the Hagedorn degeneracy of hadron levels. The important quantities are then the transition form factors, which as a function of the excitation mass have to drop off exponentially. It is argued that scale breaking effects are likely to occur. The connection of our work with the statistical bootstrap is discussed.

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- *) Work supported in part by Deutsche Forschungsgemeinschaft.
- +) Address after October 1, 1974 : Institut für Theoretische Physik II, Technische Hochschule Wien, Austria.
- x) On leave from the II. Institut für Theoretische Physik, Universität Hamburg, Germany.

Deep inelastic lepton-nucleon scattering has shown the phenomenon of scaling. Possible violations of scaling behaviour are likely to be small. If the scale breaking is parametrized in terms of some additional form factor, the cut-off mass Λ turns out to be $\Lambda \gtrsim 10 \text{ GeV}^{1,2}$.

The popular explanation of scaling assumes pointlike constituents of the nucleon, i.e. partons³). However, the parton model has met considerable difficulties in explaining the unexpectedly high cross-sections for e^+e^- annihilation⁴) and for lepton production at large p_T in pp-collisions⁵).

In order to circumvent these difficulties one might (among other alternatives) try resonance models which have been successfully employed in hadron physics. In the context of deep inelastic scattering, resonance formation and scaling were first tied together by Bloom and Gilman⁶). Explicit resonance models for the deep inelastic structure functions have been written down by several authors⁷). However, up to now it has not yet been clarified under which conditions scaling can be achieved if spin and the physical hadron spectrum are taken into account.

In this note we formulate the general resonance model for the absorptive part of the Compton amplitude $W_{\mu\nu}$ including both spin and the Hagedorn spectrum of hadronic states. Such a model should be reasonable for the non-diffractive part of the amplitude (ω small). It will turn out that the requirement of scaling places rather stringent conditions on the form factors of the resonances: as a function of the excitation mass they have to fall off exponentially. This is a direct consequence of the exponential nature of the hadronic spectrum.

We start from the absorptive part of the off-shell (spin-averaged) Compton amplitude in forward direction ($-p^2 = M^2 = 1$) *)

$$W_{\mu\nu} = \frac{1}{4\pi} \sum_{\lambda} \int d^4x e^{-iqx} \langle p\lambda | [j_{\mu}(x), j_{\nu}(0)] | p\lambda \rangle \quad (1)$$

$$= (g_{\mu\nu} - \frac{q_{\mu}q_{\nu}}{q^2}) W_1 + (\rho_{\mu} - \frac{p_{\mu}q_{\nu}}{q^2})(\rho_{\nu} - \frac{p_{\nu}q_{\mu}}{q^2}) W_2$$

The basic ingredient of our s-channel resonance model is the assumption that the above matrix element of two electromagnetic currents between protons can be saturated by one particle intermediate states of increasing mass M_n . We assume the increase of M_n^2 to be given by a linear leading trajectory with slope α' . Each level n consists of a tower of states with spins j ranging from $1/2$ up to $j_{\max} = (1/2 - \alpha' M^2) + \alpha' M_n^2 = n + 1/2$, $n = 0, 1, 2, \dots$ ($M_0 = M = \text{proton mass} = 1$). Furthermore, each state (j, n) is still degenerate, the number of individual states

*) Metric: $g_{00} = -1$, $g_{ik} = \delta_{ik}$.

for a given j and n is denoted by $\rho(j,n)$. Clearly, such a form of the spectrum is inspired by our knowledge of dual models⁸⁾. Since the intermediate states are resonances, each state (j,n) makes a Breit-Wigner type contribution to $W_{\mu\nu}$:

$$\Delta_{jn}(s) \equiv \frac{1}{\pi} \frac{M_n \Gamma_{jn}}{(s - M_n^2)^2 + M_n^2 \Gamma_{jn}^2}, \quad s = -(p+q)^2$$

where Γ_{jn} is the total width^{*}). This is to be multiplied by the product of the matrix elements of j_μ between a proton of momentum p , helicity λ and the excited state of momentum P , $P^2 = -M_n^2$, spin j , spin projection j_z , normality N ($N = \pm 1$ for $1/2^+ \rightarrow 3/2^+$, $5/2^+$, ...) and isospin $I = 1/2$ or $3/2$ ^{**}).

$$\langle P, j, j_z, N | j_\mu(0) | p, \lambda \rangle = \left[\frac{M_n^2}{(P^2 + M_n^2)(P^2 + 1)} \right]^{\frac{1}{4}} \psi_{j_z}^{\alpha_1 \dots \alpha_{j-\frac{1}{2}}} (P)$$

$$\begin{aligned} & \cdot q_{\alpha_2} \dots q_{\alpha_{j-\frac{1}{2}}} \left[q_{\alpha_1} (p_\mu - \frac{p_\mu q_\mu}{q^2} q_\mu) G_1^{jn} + 2 S_{\alpha_1 \mu} G_2^{jn} + \right. \\ & \left. + M_n q_{\alpha_1} \epsilon_{\mu\sigma\tau} \gamma_5 \gamma_\sigma p_\tau q_\tau (G_2^{jn} + G_3^{jn}) \right] \cdot \\ & \cdot \left[(1 + \gamma_5)/2 + N(1 - \gamma_5)/2 \right] u_\lambda(p) \end{aligned} \quad (2)$$

with

$$S_{\mu\nu} = p q (p_\mu q_\nu + p_\nu q_\mu) + p^2 (g_{\mu\nu} q^2 - q_\mu q_\nu) - [(p q)^2 g_{\mu\nu} + p_\mu p_\nu q^2]. \quad (3)$$

In Eq. (2), $\psi^{\alpha_1 \dots \alpha_{j-\frac{1}{2}}}$ is a Rarita-Schwinger spinor, $G_{1,2,3}^{jn} \rightarrow G_{1,2,3}^{jn}(q^2)$ are invariant form factors. For $j = 1/2$, $G_1 = G_E$, $G_2 = 0$, $G_3 = G_M$ with G_E , G_M the Sachs form factors. We do not make a universality (i.e. independence of j or universal fall-off in q^2) assumption for $G_{1,2,3}^{jn}$.

Carrying out the necessary sums over polarizations⁹⁾ we obtain for a given isospin the following expressions ^{***}) for W_1 and W_2 :

$$W_1(s, q^2) = E \sum_{n=0}^{\infty} M_n^5 B_1(M_n^2, s, q^2) \quad (4a)$$

^{*}) We take Γ_{jn} to be the same for each degenerate state (j,n) .

^{**}) Our normalizations and conventions are as in Ref. 9.

^{***}) $E^2 = 1 + q^{*2}$, $q^* =$ CM-momentum of the proton, $q^{*2} = (v^2 + q^2)/s$, $v = -p \cdot q$.

$$W_2(s, q^2) = \frac{q^2}{sq^{*2}} W_1 + \frac{E}{q^{*2}} \sum_{n=0}^{\infty} M_n B_L(M_n^2, s, q^2) \quad (4b)$$

$$\equiv \frac{q^2}{sq^{*2}} (W_1 + W_L),$$

where we have defined

$$B_1 = \sum_{j=\frac{1}{2}}^{j_{\max}} \bar{\rho}(j, n) \Delta_{jn}(s) \frac{(2j+1)!!}{(2j)!!} \left(\frac{q^{*2}}{2}\right)^{j+\frac{1}{2}} \left[\frac{2j+3}{2j-1} (G_2^{jn})^2 + (G_3^{jn})^2 \right] \quad (5a)$$

$$B_L = \sum_{j=\frac{1}{2}}^{j_{\max}} \bar{\rho}(j, n) \Delta_{jn}(s) \frac{(2j+1)!!}{(2j)!!} \left(\frac{q^{*2}}{2}\right)^{j+\frac{1}{2}} (G_1^{jn})^2 \quad (5b)$$

In Eqs. (5) we have introduced the number of states $\bar{\rho}(j, n)$ for given j, j_z, n ; hence $\rho(j, n) = (2j+1) \bar{\rho}(j, n)$. The usual ratio $R = \sigma_L/\sigma_T$ can be written as

$$R = \frac{W_L}{W_1} \quad (6)$$

For the discussion of scaling it is convenient to introduce the dimensionless variables $\xi = M_n^2/s$, $x = q^2/s = (\omega' - 1)^{-1}$, and to replace the sum over n by an integral

$$\sum_n \rightarrow \int dn = \int \frac{dn}{dM_n^2} dM_n^2 = \alpha' s \int d\xi.$$

(By assumption, $dn/dM_n^2 = \alpha'$ is the constant slope of the leading trajectory.)

Then, in order to have scaling for W_1 and W_2 we see that B_1 and B_L must satisfy the following conditions ($b_j : s \rightarrow \infty, \xi, x$ fixed) ^{*}

$$B_1(M_n^2, s, q^2) \xrightarrow{b_j} \frac{1}{s^p} \beta_1(\xi, x) \quad (7)$$

$$B_L(M_n^2, s, q^2) \xrightarrow{b_j} \frac{1}{s^{2+p}} \beta_L(\xi, x), \quad p \geq 0 \quad (8)$$

^{*} The asymptotic form (8) could be modified by powers of $\ln s$.

Indeed, these conditions give

$$\begin{aligned} W_1(s, q^2) &\xrightarrow{b_j} F_1(\omega') \\ \sqrt{s} W_2(s, q^2) &\xrightarrow{b_j} F_2(\omega') \\ W_L(s, q^2) &\xrightarrow{b_j} \frac{1}{\sqrt{s}} F_L^{(p)}(\omega') \end{aligned} \quad (9)$$

with

$$\begin{aligned} F_1(\omega') &= \frac{\alpha'}{2} \frac{\omega'}{\omega'-1} \int_0^\infty d\xi \xi^{5/2} \beta_1(\xi, \frac{1}{\omega'-1}) \\ \frac{\omega'}{2} F_2(\omega') &= F_1(\omega') + \begin{cases} F_L^{(p=0)}(\omega') & p=0 \\ 0 & p>0 \end{cases} \quad (10) \\ F_L^{(p)}(\omega') &= \frac{\alpha'}{2^{p+1}} \frac{\omega'^{p+1}}{(\omega'-1)^p} \int_0^\infty d\xi \sqrt{\xi} \beta_L(\xi, \frac{1}{\omega'-1}) \end{aligned}$$

Obviously, for $p > 0$ one gets the well-known Callan-Gross relation¹⁰⁾, $F_1(\omega') = (\omega'/2) F_2(\omega')$, together with an asymptotically vanishing ratio R , since R behaves as

$$R(s, q^2) \xrightarrow{b_j} \frac{1}{\sqrt{s}} \frac{F_L^{(p)}(\omega')}{F_1(\omega')} \quad (11)$$

On the other hand, for $p = 0$ R tends to a non-zero scaling function and $F_L(\omega')$ measures the deviation from the Callan-Gross relation. While in the parton model a non-vanishing R can be interpreted as the scaling contribution from spin 0 constituents inside the proton³⁾, in a resonance model W_L and R are governed by the "electric" form factors $G_1^{jn}(q^2)$. An interesting example is provided by the following hypothetical "proportionality law":

$$\left(\frac{G_2^{jn}(q^2)}{G_1^{jn}(q^2)} \right)^2 \sim (q^2)^8 a \left(\frac{M_n^2}{q^2} \right), \quad \left(\frac{G_3^{jn}(q^2)}{G_1^{jn}(q^2)} \right)^2 \sim (q^2)^8 b \left(\frac{M_n^2}{q^2} \right) \quad (12)$$

(a may also be zero), which generalizes the approximate empirical proportionality¹¹⁾ of the elastic form factors: $G_M/G_E \approx \text{const.}$ [see Fig. 37 of Ref. 11) for the experimental situation]. Scaling for W_1 leads to $p = 2 + \gamma$. Since as a first guess one would assume $\gamma \approx 0$, this implies $R \sim 1/\sqrt{s}$, i.e. a remarkably strong suppression of the longitudinal cross-section.

Recent experiments indicate that νR tends to a constant, $\nu R \rightarrow r(\omega)$, at least for $\omega \lesssim 5^2$. [For larger ω , there is indication for a behaviour $\nu R \rightarrow r(\omega) + \nu r'(\omega)$.] Hence experimentally $p \approx 1$, which in turn implies $\gamma = -1$ in Eq. (12). Under the assumption that the behaviour (12) is really correct, one concludes that the inelastic magnetic form factors have to decrease somewhat faster in q^2 than the inelastic electric form factors.

We now turn to the main question: can the scaling conditions (7), (8) be satisfied at all if a realistic spectrum is inserted into (5)?

The level density $\bar{\rho}(j, n)$ can always be written as

$$\bar{\rho}(j, n) = \tau(M_n^2) \hat{g}(j, n), \quad (13)$$

where $\tau(M_n^2)$ is the total number of levels at a given mass M_n and $\hat{g}(j, n)$ is the fractional number of states with spin j , $|j_z| \leq j$, and mass M_n ($\sum_{j=1/2}^{\infty} (2j+1) \hat{g} = 1$)

Already in 1965, in the framework of the thermodynamical model Hagedorn¹²⁾ predicted that the total level density $\tau(M^2)$ should increase exponentially for large M :

$$\tau(M^2) \sim \frac{C}{M^a} e^{M/T_0}, \quad M \rightarrow \infty \quad (14)$$

where $a = 5/2$, and $T_0 \approx 160$ MeV is the maximum temperature for hadronic systems. Recent investigations of this so-called "statistical bootstrap" by Frautschi¹³⁾, Frautschi and Hamer¹⁴⁾ and Nahm¹⁵⁾ have confirmed the behaviour (14) with $a > 5/2$. The same behaviour of the level density is found in dual models^{8,16)}.

Finally we mention a third model which shows a similar degeneracy of hadron levels, namely the MIT "bag" model¹⁷⁾, which possesses many desirable features of hadron dynamics, e.g. a parton interpretation and infinitely rising Regge trajectories.

Thus there seems to be good evidence for the particle spectrum following an exponential law^{*)}.

Concerning the spin density $\hat{\rho}(j, n)$, it is to be expected on physical grounds that $\hat{\rho}(j, n)$ is peaked at the peripheral resonances, namely around $\langle j \rangle \sim \sqrt{\alpha'} M_n$ corresponding to a fixed impact parameter $b \sim \sqrt{\alpha'} \sim 2 \cdot 10^{-14}$ cm. This actually happens in dual models^{8,19)}.

*) For a discussion of the experimental situation, the reader is referred to a recent article by Frautschi¹⁸⁾.

For definiteness let us consider W_1 and denote by $[G^{jn}(q^2)]^2$ the combination of form factors appearing in square brackets in Eq. (5a). To keep the discussion as simple as possible let us furthermore assume that the widths Γ_{jn} do not change too drastically with j so that they can be replaced under the sum of Eq. (5a) by an average level width $\langle \Gamma_{jn} \rangle = \Gamma(M_n^2)$. Then we are left with

$$W_1 = \frac{E}{\pi} \sum_n \frac{M_n^6 \Gamma(M_n^2) \tau(M_n^2)}{(s - M_n^2)^2 + M_n^2 \Gamma^2} \sum_{j=1/2}^{j_{\max}} \hat{S}(j, n) \frac{(2j+1)!!}{(2j)!!} \left(\frac{q^2}{2}\right)^{j+\frac{1}{2}} (G^{jn}(q^2))^2 \quad (15)$$

$$\equiv \frac{E}{\pi} \sum_n \frac{M_n^6 \Gamma(M_n^2) \tau(M_n^2) V(M_n \rightarrow p\gamma)}{(s - M_n^2)^2 + M_n^2 \Gamma^2}.$$

For scaling or even in the case of a weak (e.g. powerlike) scale breaking, the sums in Eq. (15) have to converge properly. In the framework of the statistical bootstrap Frautschi recently found¹⁸⁾ that the product $\Gamma \cdot \tau$ is proportional to the number N of open channels,

$$\Gamma(M_n^2) \tau(M_n^2) = \frac{1}{2\pi} N(M_n^2) \quad (16)$$

[a relation well-known in nuclear physics²⁰⁾] and has to increase exponentially. Assuming the exponential Hagedorn spectrum (14), one is then led to a constant behaviour for the average width, $\Gamma(M_n^2) \rightarrow \text{constant}^*)$, a result which has also been obtained in dual models²¹⁾. The data for Γ are inconclusive, but show at most a slow increase in the mass M_n ¹⁸⁾. Thus the Breit-Wigner-term in Eq. (15) does not provide an exponential damping, and the only way to compensate the exponential growth of the spectrum is therefore through the spin sum. A rough estimate of the terms in the j -sum of Eq. (15) at the peripheral value $j = \langle j \rangle \sim \sqrt{\alpha'} M_n$ yields $\hat{\rho}(j, n) \sim 1/M_n^2$ [see Ref. 19], $(2j+1)!!/(2j)!! \sim \sqrt{j} \sim \sqrt{\alpha'} M_n$, $(q^2/2)^{j+\frac{1}{2}} \sim e^{j \ln(q^2/2)} \sim e^{\sqrt{\alpha'} M_n \ln(q^2/2)}$ and one concludes that the form factor $(G^{jn})^2$ not only has to kill the exponential growth of the spectrum but also the exponentially increasing threshold factor [e.g. through a Dürr-Pilkuhn "penetration" factor²²⁾]. In other words, as a function of the excitation mass M_n the transition form factors G^{jn} have to go down like the inverse of the square root of the spectrum τ , i.e. exponentially.

Interestingly enough, a similar conclusion concerning the form factors G^{jn} is reached in the statistical bootstrap model: if a fireball of momentum P and mass M_n decays into a light particle with momentum k and another fireball, one obtains^{24, 25)} for the decay probability, which is nothing but G^2 on shell,

*) A power law growth $\tau(M_n^2) \sim M_n^\alpha$ results in an experimentally disfavoured exponential growth of $\Gamma(M_n^2)$ ¹⁸⁾.

$$g_{\text{on shell}}^2 \sim \frac{\tau(-(P-k)^2)}{\tau(M_n^2)} \quad (17)$$

Our analysis corroborates this result.

Of course, the exponential drop-off of the form factors does not guarantee scaling behaviour for the structure functions, since scaling depends crucially on the remaining power in front of the exponential. In order to obtain exact scaling, this power has to co-operate with the power α in the hadron spectrum (14) and with a possible power behaviour of the average width. To our knowledge, nobody has been able so far to construct a model in which such a wonderful conspiracy between form factors, spectrum and average width occurs. One is therefore tempted to conclude that the most probable situation is not the case of scaling but a power-like breaking of scaling. We note, however, that universal (dipole) form factors, as they have been assumed in the literature²³⁾, together with a realistic spectrum of hadronic states violate scaling exponentially.

At last we would like to remark that our resonance saturation scheme can be interpreted in a statistical manner. This is most transparent from Eq. (15): a heavy object of mass M_n (fireball, cluster?) propagates in a Breit-Wigner type fashion and is globally described by its spectrum and its total width, which together determine the number N of open channels, Eq. (16), and its coupling $V(M_n \rightarrow p\gamma)$ to the particular channel consisting of a proton and the photon. The connection to the statistical bootstrap becomes even closer if one assumes²⁶⁾ that the hadronic content of the photon is made up of fireballs in the sense of generalized vector meson dominance. Then the coupling $V(M_n \rightarrow p\gamma)$ is essentially determined by the decay probability (17) and the fireball-photon coupling, which in turn can be related to the total annihilation cross-section $\sigma(e^+e^- \rightarrow \text{hadrons})$ ²⁶⁾.

Acknowledgements

We are indebted to M. Böhm, C. Hamer, D.I. Olive and T. Walsh for discussions. We are especially grateful to R. Hagedorn for informing us about the results of the statistical bootstrap, and to J. Ellis and R. Hagedorn for reading the manuscript. One of us (F.S.) would like to thank D. Amati for hospitality at the Theoretical Studies Division of CERN.

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