Gravitational Waves from Low-Mass X-ray Binaries: a Status Report

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Abstract. We summarize the observations of the spin periods of rapidly accreting neutron stars. If gravitational radiation is responsible for balancing the accretion torque at the observed spin frequencies of ≈ 300 Hz, then the brightest of these systems make excellent gravitational wave sources for LIGO-II and beyond. We review the recent theoretical progress on two mechanisms for gravitational wave emission: mass quadrupole radiation from deformed neutron star crusts and current quadrupole radiation from r-mode pulsations in neutron star cores.

I SPINS OF ACCRETING NEUTRON STARS

Gravitational wave emission from rapidly rotating neutron stars (NS) has attracted considerable interest in the past several years. In addition to radiation from the spindown of newborn NSs (see the review by B. Owen in this volume), it has long been suspected [1,2] that rapidly accreting NSs, such as Sco X-1, may be a promising class of gravitational wave (GW) emitters. However, firm observational evidence of fast spins of these neutron stars had been missing until recently.

NSs in low-mass X-ray binaries (LMXBs) have long been thought to be the progenitors of millisecond pulsars [3]. However, directly measuring their periods has proved elusive, probably because of their rather low magnetic fields.

1) Much theoretical progress has been made in understanding GW emission from LMXBs in the six months since the Amaldi conference in July 1999. Rather than just transcribe the talk given by one of us, we review the situation as of December 1999. Because of space limitations, this review is far from complete.
With the launch of the Rossi X-ray Timing Explorer, precision timing of accreting NSs has opened new threads of inquiry into the behavior and lives of these objects. RXTE observations [4] have finally provided conclusive evidence of millisecond spin periods of NSs in about one-third of known Galactic LMXBs. These measurements are summarized in Fig. 1a. Altogether, there are seven such NSs with firmly established spin periods, by either pulsations in the persistent emission (discovered by Wijnands & van der Klis in the millisecond X-ray pulsar SAX J1808.4-3658; [5]) or oscillations during type I X-ray bursts (burst QPOs, first discovered in 4U 1728–34 by Strohmayer et al. [6]). There are an additional thirteen sources with twin kHz QPOs for which the spin may be approximately equal to the frequency difference [4]. A striking feature of all these neutron stars is that their spin frequencies lie within a narrow range, 260 Hz < νs < 589 Hz. The frequency range might be even narrower if the burst QPOs seen in KS 1731–260, MXB 1743–29, and Aql X-1 are at the first harmonic of the spin frequency, as is the case with the 581 Hz burst oscillations in 4U 1636–536 [7]. These NSs accrete at diverse rates, from 10^{-11} M_\odot yr^{-1} to the Eddington limit, \dot{M}_{\text{Edd}} = 2 \times 10^{-8} M_\odot yr^{-1}. Since disk accretion exerts a substantial torque on the NS and these systems are very old [8], it is remarkable that their spin frequencies are so similar, and that none of them are near the breakup frequency of \nu = 1.5 kHz.

One possible explanation, proposed by White & Zhang [9], is that these stars have reached the magnetic spin equilibrium (where the spin frequency matches the Keplerian frequency at the magnetosphere) at nearly identical frequencies. This requires that the NS dipolar B field correlate very well with \dot{M} [9,10]. However, there are no direct B field measurements for LMXBs, and in the strongly magnetic binaries, where the B field has been measured directly, such a correlation is not observed. More importantly, for 19 out of 20 systems, there must be a way of hiding persistent pulses typically seen from magnetic accretors. These difficulties led Bildsten [11] to resurrect the conjecture originally due to Papaloizou & Pringle [12] and Wagoner [1] that gravitational radiation can balance the torque due to accretion. The detailed mechanisms will be discussed in the following sections.

Regardless of the detailed mechanism for GW emission, if gravitational radiation balances the accretion torque, then it is easy to estimate the GW strength. As noted by Wagoner [1], in equilibrium the luminosities in GWs and in X-rays are both proportional to the mass accretion rate \dot{M}, so the characteristic strain amplitude h_c depends on the X-ray flux F_x at Earth and the spin frequency

\[ h_c = 4 \times 10^{-27} \frac{F_x^{3/4}}{\dot{M}^{1/4}} \left( \frac{300 \text{ Hz}}{\nu_s} \right)^{1/2} \left( \frac{F_x}{10^{-8} \text{ erg cm}^{-2} \text{ s}^{-1}} \right)^{1/2}. \tag{1} \]

In Fig. 1b we show h_c for Sco X-1 (marked with a star), a few other bright LMXBs with the spin inferred from kHz QPO separation (thick...
FIGURE 1. Left (a) The distribution of spins of NSs in LMXBs, summarizing Tables 1-4 of [4]. Right (b) Characteristic signal amplitude $h_c$ for several known LMXBs (symbols, see text) compared with the sensitivity $h_{3/yr}$ of LIGO-II in broadband (the dotted line) and narrowband (the solid line) configuration (provided by K. A. Strain on behalf of the LIGO Scientific Community). The dashed line shows LIGO-II sensitivity for a two-week integration (see text).
dots) and burst QPO frequency (triangles), and the millisecond X-ray pulsar SAX J1808.4-3658 (open diamond). The dotted line shows LIGO-II sensitivity $h_{3/yr}$ (i.e., $h_c$ detectable with 99% confidence in $10^7$ s, provided the frequency and the phase of the signal are known in advance [13]) in the broadband configuration, while the solid line shows $h_{3/yr}$ for the narrowband configuration. However, the frequency and the phase are known precisely only for the SAX J1808.4-3658 millisecond X-ray pulsar [14]. For other sources, Brady & Creighton [13] showed that the number of trials needed to guess the poorly known orbital parameters or to account for the torque noise due to $M$ variations lowers the effective sensitivity by roughly a factor of two.

While the average $\dot{M}$ certainly correlates with the X-ray brightness, current observations unfortunately do not let us robustly infer the instantaneous torque [4]. Even though $\dot{M}$ varies on a timescale of days, torque noise leads to frequency drift only on a timescale of weeks. The accretion torque is $N_a = \dot{M}(GMR)^{1/2}$, and the total time-averaged torque is zero due to equilibrium with GW emission. Assume that $N_a$ flips sign randomly on a timescale $t_s \approx$ few days. The spin frequency $\Omega$ will experience a random walk with step size $\delta\Omega = (N_a/I)t_s$, where $I$ is the NS moment of inertia. After an observation time $t_{\text{obs}}$, the drift is $\Delta\Omega = (t_{\text{obs}}/t_s)^{1/2}\delta\Omega$. This will exceed a Fourier frequency bin width, i.e., $\Delta\Omega \approx 2\pi/t_{\text{obs}}$ only after

$$t_{\text{obs}} = \frac{21 \text{ days}}{M_{1.4}^{1/3} R_6^{1/3}} \left( \frac{1 \text{ day}}{t_s} \right)^{1/3} \left( \frac{10^{-8} M_\odot \text{ yr}^{-1}}{\dot{M}} \right)^{2/3}.$$ 

(2)

Hence, on a timescale of tens of days, the intrinsic GW signal is coherent. The dashed line in Fig. 1b shows the LIGO-II sensitivity for a two-week integration in a narrowband configuration. This suggests that the way to detect GWs from LMXBs may be short integrations [13].

Currently, there are two classes of theories for GW emission from NSs in LMXBs. The presence of a large-scale temperature asymmetry in the deep crust will cause it to deform [11]. The resulting “mountains” will give the rotating star a time-dependent mass quadrupole moment. Alternatively, unstable $r$-mode pulsations (see a review by B. Owen in this volume) of a suitable amplitude in the NS liquid core can emit enough gravitational radiation to balance the accretion torque [11,15].

II DEFORMATIONS OF ACCRETING NS CRUSTS

The crust is a $\approx 1$ km layer of crystalline “ordinary” (albeit neutron-rich) matter that overlies the liquid core composed of free neutrons, protons, and electrons. The crust’s composition varies with depth in a rather abrupt manner. As an accreted nucleus gets buried under an increasingly thick layer of more recently accreted material, it undergoes a series of $e^-$ captures, neutron
emissions, and pycnonuclear reactions [16–18], resulting in layered composition. In Fig. 2a, we show schematically two such compositional layers (light and dark shading) sandwiched between the liquid core and the ocean. Since an appreciable fraction of the pressure is supplied by degenerate electrons, e\textsuperscript{−} captures induce abrupt density increases. In the outer crust, these density jumps are as large as \( \approx 10\% \), while in the inner crust the density contrast is \( \lesssim 1\% \). At \( T = 0 \), the e\textsuperscript{−} captures occur when the electron Fermi energy \( E_F \) is greater than the mass difference between the e\textsuperscript{−} capturer and the product of the reaction. In the absence of other effects, this depth is the same everywhere, and such an axisymmetric capture boundary (the dashed line in Fig. 2a) does not create a mass quadrupole moment.

However, in accreting NSs the crustal temperatures are high enough (in excess of \( 2 \times 10^8 \) K) that e\textsuperscript{−} capture rates become temperature-sensitive [19]. Bildsten [11] pointed out that if there is a lateral temperature gradient in the crust (the arrow in Fig. 2a), then regions of the crust that are hotter undergo captures at a lower density than the colder regions. The capture boundary becomes “wavy” (the solid line in Fig. 2a), with captures proceeding a height \( \Delta z_d \) higher on the hot side of the star, and \( \Delta z_d \) lower on the cold side. Such a temperature gradient, if misaligned from the spin axis, will give rise to a nonaxisymmetric density variation and a nonzero quadrupole moment \( Q_{22} \) [11].

The required quadrupole moment \( Q_{eq} \) such that GW emission is in equilibrium with the accretion torque is

\[
Q_{eq} = 3.5 \times 10^{37} \text{g cm}^2 \frac{M_{1.4}}{P_6^{1/4}} \left( \frac{\dot{M}}{10^{-9} M_\odot \text{ yr}^{-1}} \right)^{1/2} \left( \frac{300 \text{ Hz}}{\nu_s} \right)^{5/2},
\]

The range of \( \dot{M}'s \) in LMXBs is \( \approx 10^{-11} - 2 \times 10^{-8} M_\odot \text{ yr}^{-1} \), requiring \( Q_{22} \approx 10^{37} - 10^{38} \text{ g cm}^2 \) for \( \nu_s = 300 \text{ Hz} \) [11]. Can temperature-sensitive e\textsuperscript{−} captures sustain a quadrupole moment this large? The quadrupole moment generated by a temperature-sensitive capture boundary is \( Q_{22} \sim Q_{hid} \equiv \Delta \rho \Delta z_d R^4 \), where \( \Delta \rho \) is the density jump at the electron capture interface. \( Q_{hid} \) is the quadrupole moment that would result if the crust did not elastically adjust (or just moved horizontally) in response to the lateral pressure gradient due to wavy e\textsuperscript{−} captures. Using this estimate, Bildsten [11] argued that a single wavy capture boundary in the thin outer crust could generate \( Q_{22} \) sufficient to buffer the spinup due to accretion (Eq. [3]), provided that temperature variations of \( \approx 20\% \) are present in the crust.

However an important piece of physics is missing from this estimate: the shear modulus \( \mu \). If \( \mu = 0 \), the crust becomes a liquid and cannot support a non-zero \( Q_{22} \). Ushomirsky, Cutler, & Bildsten [20] recently calculated of the elastic response of the crust to the wavy e\textsuperscript{−} captures. They found that the predominant response of the crust to a lateral density perturbation is to sink, rather than move sideways. For this reason, \( Q_{22} \) generated in the outer crust
[11] is much too small to buffer the accretion torque. However, a single $e^-$ capture boundary in the deep inner crust can easily generate an adequate $Q_{22}$. Because of the much larger mass involved by captures in the inner crust, the temperature contrasts required are $\lesssim 5\%$, or only $\approx 10^6 - 10^7 \text{ K}$, not $\approx 10^8 \text{ K}$ as originally postulated [11].

What causes the lateral temperature asymmetry, and can it persist despite the strong thermal contact with the almost perfectly conducting core? In LMXBs, the crusts are composed of the compressed products of nuclear burning of the accreted material. The exact composition depends on the local accretion rate, which could have a significant non-axisymmetric piece due to, e.g., the presence of a weak $B$ field. Moreover, except in the highest accretion rate LMXBs, nearly all of the nuclear burning occurs in type I X-ray bursts. Burst QPOs (see Sec. I) provide conclusive evidence that bursts themselves are not axisymmetric. Until the origin of this symmetry breaking is clearly understood, it is plausible to postulate that these burst asymmetries get imprinted into the crustal composition.

Ushomirsky et al. [20] showed that such a non-uniform composition leads directly to lateral temperature variations $\delta T$. Horizontal variations in the charge-to-mass ratio $Z^2/A$ (which determines the crustal conductivity) and/or nuclear energy release modulate the radial heat flux in the crust and set up a nonaxisymmetric $\delta T$. The $\delta T$’s required to induce a $Q_{22} \approx Q_{eq}$ can easily be maintained if there is a $\approx 10\%$ asymmetry in the nuclear heating or $Z^2/A$. So long as accretion continues, these $\delta T$’s persist despite the strong thermal contact with the isothermal NS core.

The $e^-$ capture $Q_{22}$ calculations [20] are summarized in Fig. 2b. If the size of the heating or $Z^2/A$ asymmetry is a constant fixed fraction, then for $\dot{M} \lesssim 0.5 \dot{M}_{\text{Edd}}$ the scaling of $Q_{22}(\dot{M})$ is just that needed for all of these NSs to have the same spin frequency (the normalization is proportional to the magnitude of the asymmetry, but the scaling is fixed by the microphysics). For $\dot{M} \gtrsim 0.5 \dot{M}_{\text{Edd}}$, in order to explain the spin clustering at exactly 300 Hz, this mechanism requires that the crustal asymmetry correlate with $\dot{M}$. Alternatively, if the asymmetry is the same as in the low $\dot{M}$ systems, then one would expect the bright LMXBs to have higher spins, a possibility that cannot be ruled out by current observations (Sec. I).

So long as crustal deformations are due to shear forces only, the crustal $Q_{22}$ is limited by the yield strain $\sigma_{\text{max}}$ to be less than [20]

$$Q_{\text{max}} \approx 10^{38} \text{ g cm}^{-2} \left( \frac{\sigma_{\text{max}}}{10^{-2}} \right) \frac{R_{\odot}^{0.26}}{M_{\odot}^{1.2}}.$$  \hspace{1cm} (4)

$Q_{22}$’s needed to buffer the accretion torque require strains $\sigma \approx 10^{-3} - 10^{-2}$ at $\approx 300 \text{ Hz}$, with $\sigma \gtrsim 10^{-2}$ in near-Eddington accretors. Estimates for the yield strain of the neutron star crust range anywhere from $10^{-1}$ for perfect one-component crystals to $10^{-5}$. Hence $\sigma \gtrsim 10^{-2}$ is probably higher than
yield strain, though this conclusion is based on extrapolating experimental results for terrestrial materials by > 10 orders of magnitude. Such high strains are perhaps the biggest problem with the crustal $Q_{22}$ mechanism. At high pressures ($\gg$ shear modulus) terrestrial materials tend to deform plastically rather than crack, and so the crusts of accreting NSs may be in a state of continual plastic flow. If accretion continually drives the crust to $\sigma_{\text{max}}$, this leads to a natural explanation for spin similarities near $\dot{M}_{\text{Edd}}$.

However, many fundamental issues remain unanswered. First, the calculation [20] is only good up to an overall prefactor set by the density of capture layers in the deep crust. We thus need an exploratory calculation of both the composition of the products of nuclear burning in the upper atmosphere over the entire range of $\dot{M}$ in LMXBs, and their detailed nuclear evolution under compression in the crust. Knowledge of the composition is also necessary for a robust calculation of the shear modulus, which is clearly the crucial number to know when computing the elastic response of the crust. Recent results [21,22] indicate that inner crusts of NSs are composed of highly nonspherical nuclei and may be more like liquid crystals (solids that provide no elastic restoring force for certain kinds of distortions) rather than simple Coulomb solids. Such improved calculations have implications far beyond the problem of the crustal quadrupole moment. The shear modulus of the crust affects the maximum elastic energy that can be stored in the crust, and hence the energetics of pulsar glitches and starquakes, as well as the models of magnetic field evolution that depend on crustal “plate tectonics” (see [23]). It even has bearing on the stability of r-modes in neutron stars (Sec. III). In addition, much work needs to be done on understanding what sets the shear strength $\sigma_{\text{max}}$ of multicomponent crystals, likely with defects and highly nonspherical nuclei, or what happens when $\sigma_{\text{max}}$ is exceeded and viscoelastic flow ensues.

### III R-MODES IN ACCRETING NS CORES

Bildsten [11] and Andersson, Kokkotas, & Stergioulas [15] pointed out that the r-mode instability (see the review by B. Owen in this volume for the introduction and notation) may also explain the spins of NSs in LMXBs, and, if so, produce GW signal detectable by LIGO-II. An accreting NS is spun up (along a line in $(\nu_s, T)$ plane marked with an arrow in Fig. 3) until it reaches the r-mode instability line (the solid line in Fig. 3). At that point (marked by a thick dot in Fig. 3) the r-mode amplitude needed to balance the accretion torque is rather small. The NS can then hover at the instability line, with $1/\tau_G + 1/\tau_V = 0$, and the r-mode amplitude such that it balances the accretion torque. However, at $T = \text{few} \times 10^8$ K, the r-mode–accretion equilibrium spin frequency would be $\approx 150$ Hz, rather than $\approx 300$ Hz, resulting in an apparent disagreement with the observed spins of LMXBs. Bildsten [11] and Andersson et al. [15] speculated that including other sources of viscosity, e.g., superfluid
mutual friction, is likely to raise the instability curve, resulting in equilibrium frequencies close to the canonical 300 Hz. Finally, the narrow range of the observed spin frequencies would presumably arise because of the similar core temperatures of the accreting NSs (shown by the shaded box in Fig. 3).

Recent theoretical work brought up several challenges to this scenario. Levin [24] and Spruit [25] showed that steady-state equilibrium between accretion and r-modes is thermally unstable for normal fluid cores. In a normal fluid (i.e., not superfluid), the shear viscosity scales as $T^{-2}$, so the increase in the core temperature due to viscous heating decreases the shear viscosity. The smaller shear viscosity increases the growth rate of the r-mode, leading to an unstable runaway. Using a phenomenological model of nonlinear r-mode evolution [26], Levin [24] showed that in this case, instead of just hovering near the instability line, the r-mode grows rapidly until saturation, heats up the star, and spins it down and out of the instability region in less than 1 yr. Therefore, if NSs in LMXBs have normal fluid cores, we would not expect to see any of them with $\approx 300$ Hz spins.

The unstable regime for r-modes in normal fluid NSs (above the solid line in Fig. 3) encompasses much of the parameter space occupied by NSs in LMXBs and newborn NSs. Because of the large torques exerted by the unstable r-modes, we would not expect to see any NSs in this region. In addition, the existence of two 1.6 ms radio pulsars (the spins and upper limits on core temperatures of which are shown by arrows in Fig. 3) means that rapidly rotating NSs are formed in spite of the r-mode instability [15]. While it is not clear whether their current core temperatures place these pulsars within the r-mode instability region, normal-fluid r-mode theory says that they were certainly unstable during spinup.

Superfluid r-mode calculations have been eagerly awaited, as they could resolve these conflicts. However, Lindblom & Mendell [27] showed that, for most values of the neutron-proton entrainment parameter, the superfluid dissipation is not competitive with gravitational radiation. Only over about 3% of the possible entrainment parameter values is mutual friction strong enough to compete with gravitational radiation. The r-mode instability line in this case is an approximately horizontal line (see Fig. 8 of [27]) separating the unstable spin frequencies ($\nu_s > \nu_{\text{crit}}$) from the stable ones ($\nu_s < \nu_{\text{crit}}$). If the superfluid entrainment parameter has a value such that $\nu_{\text{crit}} \approx 300$ Hz, then the LMXB spin frequencies could still be understood in terms of the r-mode instability and the special nature of the NS superfluid.

Before learning about these results, Brown & Ushomirsky [28] ruled out such a simple superfluid equilibrium observationally for a subset of LMXBs. In steady state, the shear in the r-mode deposits $\approx 10$ MeV of heat per accreted baryon into the NS core. When the core is superfluid, Urca neutrino emission from it is suppressed, and this heat must flow to the NS surface and be radiated thermally. In steadily accreting systems (such as Sco X-1) this thermal emission is dwarfed by the accretion luminosity of $GMM/R \approx$
200 MeV per accreted baryon. However, in transiently accreting systems, such as Aql X-1, when accretion ceases, the r-mode heating should be directly detectable as enhanced X-ray luminosity from the NS surface. For Aql X-1 and other NS transients, Brown & Ushomirsky [28] showed that, if the superfluid r-mode equilibrium prevails, then the quiescent luminosity should be about 5–10 times greater than is actually observed.

A possible resolution of this conundrum has been recently proposed by Bildsten & Ushomirsky [29]. All but the hottest (\( \gtrsim 10^{10} \) K) NSs have solid crusts. The r-mode motions are mostly transverse, and reach their maximum amplitude near the crust-core boundary. The fluid therefore rubs against the crust, which creates a thin (few cm) boundary layer. Because of the short length scale, the dissipation in this boundary layer is very large. The damping time due to rubbing is [29]

\[
\tau_{\text{rub}} \approx 100 \text{ s } T_{8}^{1/4} \frac{M_{1.4}}{R_{6}^{2}} \left( \frac{1 \text{ kHz}}{\nu_{s}} \right)^{1/2},
\]

substantially shorter than the viscous damping times due to the shear and bulk viscosities in the stellar interior, as well as the mutual friction damping time for most values of the superfluid entrainment parameter.

The critical frequency for the r-mode instability in NSs with crusts is shown by the dashed line in Fig. 3 for the case where all nucleons are normal, and the dark shading around it represents the range of frequencies when either neutrons or all nucleons are superfluid. The crust-core rubbing raises the minimum frequency for the r-mode instability in NSs with crusts to \( \gtrsim 500 \) Hz for \( T \approx 10^{10} \) K, nearly a factor of five higher than previous estimates. This substantially reduces the parameter space for the instability to operate, especially for older, colder NSs, such as those accreting in binaries and millisecond pulsars. In particular, the smallest unstable frequency for the temperatures characteristic of LMXBs is \( \gtrsim 700 \) Hz, safely above all measured spin frequencies. This work resolves the discrepancy between the theoretical understanding of the r-mode instability and the observations of millisecond pulsars and LMXBs, and, along with observational inferences [28], likely rules out r-modes as the explanation for the clustering of spin frequencies of neutron stars in LMXBs around 300 Hz.

To summarize, a significant role of steady-state r-modes in LMXBs has probably been ruled out, both on theoretical grounds [29] (unless crust-core coupling is much stronger than was estimated), and observationally [28] (for Aql X-1 in particular). However, stochastically excited r-modes that decay rapidly may still play a significant role in accreting systems, as even a very small amplitude (\( \alpha \lesssim 10^{-5} \)) can balance the accretion torque at \( \dot{M}_{\text{Edd}} \). In addition, the issues of crustal shear modulus and the structure of the crust-core boundary, highlighted in Sec. II, are of paramount importance for r-modes as well. Crustal quadrupoles [11,20] can explain the spins of LMXBs.
and remain a viable source of continuous GWs, but the strains at $M_{\text{Edd}}$ are rather high. The crustal breaking strain $\sigma_{\text{max}}$ is not likely to be understood theoretically any time soon, and detection of GWs from LMXBs with LIGO-II type instruments will surely teach us many new things about NSs.

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REFERENCES

**FIGURE 2.** Left (a) A cartoon description of how a transverse temperature gradient leads to a varying altitude for the $e^-$ captures. Right (b) The quadrupole $Q_{22}$ due to a single capture layer in the inner crust as a function of $\dot{M}$. The solid and dashed lines denote the results for several NS models for a 10% composition asymmetry, while the dotted line is the relation given by Eq. (3), i.e., the quadrupole necessary for spin equilibrium at $\nu_s = 300$ Hz as a function of $\dot{M}$. 
**FIGURE 3.** Critical spin frequencies of the r-mode instability. The solid line is the critical frequency set only by shear and bulk viscosities in the core (same as Fig. 1 of B. Owen’s review in this volume). The dashed line also includes viscous boundary layer damping, while the shading around it displays the effect of core superfluidity.