Equilibration of the Gluon-Minijet Plasma at RHIC and LHC

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Abstract

We study the production and equilibration of the gluon-minijet plasma expected to be formed in the central region of ultrarelativistic heavy-ion collisions at the BNL-RHIC and the CERN-LHC by solving a self-consistent relativistic transport equation. We compute the minijet production within perturbative QCD. Subsequent collisions among the semi-hard partons are treated by considering the elastic $gg \to gg$ processes with screening of the long wavelength modes taken into account. We find rather similar equilibration times at RHIC and LHC energies. However, the number densities, energy densities and temperatures of the minijet plasma are very different. The equilibration times are found to be $\sim 5 \text{ fm}$ with temperatures of 360 and 700 MeV at RHIC and LHC, respectively.

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I. INTRODUCTION

In the next few years, the BNL-RHIC (Au-Au collisions at $\sqrt{s}=200$ GeV per incident nucleon pair) and the CERN-LHC (Pb-Pb collisions at $\sqrt{s}=5.5A$ TeV) accelerators will provide the opportunity to study a new phase of matter, namely the so-called Quark-Gluon Plasma (QGP). Such a phase probably existed in the early universe, a few microseconds after the Big-Bang [1]. It is interesting and challenging if this state of matter can be recreated in the laboratory. Various signatures proposed to detect this very dense state of matter experimentally are $J/\Psi$ suppression, dilepton and direct photon production and strangeness enhancement. For a review on both theoretical and experimental status of the QGP search see ref. [2].

Lattice quantum chromodynamics predicts that such a phase of deconfined matter of quarks and gluons should be obtained from ordinary hadronic matter at very high temperature ($\sim 200$ MeV) and pressure [3]. The energy density needed to create such a state of matter is about $\sim 3$ GeV/fm$^3$. There is little doubt that such a high energy density state will be created in central collisions of heavy nuclei at RHIC and LHC [4], but it is not clear whether an equilibrated QGP will be formed in these collisions. This is because as the system expands in these ultra relativistic heavy-ion collisions, the temperature may drop below the transition temperature before equilibrium can actually set in. Note that unlike in the cosmological QCD phase transition the expansion rate in high-energy heavy-ion collisions is not many orders of magnitude smaller than the interaction rate [5].

In any case, it is very important and interesting to study whether the QGP actually does thermalize in those reactions, and if so, what is the actual energy density, number density and temperature at which it thermalized. For this purpose, and also for the calculation of all the signatures, it is necessary to study the space-time evolution of partons just after the nuclear collision. For example, the equilibration time is crucial for a quantitative understanding of $J/\Psi$ suppression [6], and it is a challenging task to determine this quantity accurately. Similarly, understanding the equilibration time is important for all the other signatures mentioned above, for example for dilepton emission and strangeness production. Once equilibrium is reached, the further space time evolution of partons can be described by the well known equations of hydrodynamics.

The evolution of the QGP towards (local) equilibrium can be studied by solving transport equations for quarks and gluons with all the dynamical effects taken into account. Obviously, the first problem one always encounters is the correct computation of the initial conditions needed to solve the transport equation. This is because one can not calculate the parton production in all range of momentum from perturbative QCD (pQCD). There are also coherence effects [7,8] that play an important role in the early stage of the nuclear collision at very high energy. For small $x$ and large nuclei, the QCD based calculation performed in [9] predicts the existence of a coherent field in a certain kinematical range. That field may play an important role in the equilibration of the plasma. In the present paper, however, we restrict our calculation to the initial incoherent parton production, which is computed within the framework of pQCD. We study the subsequent evolution of that minijet “plasma” by solving a relativistic transport equation, thus taking into account collisions between the produced partons. In the future, we intend to generalize our approach to include both coherent field and incoherent partons in the transport equation.
The paper is organized as follows. In section II we briefly review minijet production in high-energy nuclear collisions within pQCD. In section III we discuss the in-medium screening of long wavelength gluons which is relevant to our study. We present the relativistic transport equation in section IV, briefly describing the numerical strategy for its solution in section V. We discuss our main results in section VI and conclude in section VII. Throughout the manuscript we employ natural units, ħ = c = k = 1.

II. MINIJET PRODUCTION IN NUCLEAR COLLISIONS AT RHIC AND LHC

In this section we review the computation of the single-inclusive semi-hard cross section in lowest order pQCD, cf. also [4,10]. The 2 → 2 minijet cross section per nucleon in AA collision is given by

\[
\sigma_{jet} = \int dp_1 dp_2 dy_1 dy_2 \frac{2\pi p_t}{s} \sum_{ijkl} x_1 f_{i/A}(x_1, p_{1t}^2) x_2 f_{j/A}(x_2, p_{2t}^2) \hat{\sigma}_{ij-kl}(s, \hat{t}, \hat{u}). \tag{1}
\]

Here \(x_1\) and \(x_2\) are the light-cone momentum fractions carried by the partons \(i\) and \(j\) from the projectile and the target, respectively. \(f_{j/A}\) are the distribution functions of the parton species \(j\) within a nucleon bound in a nucleus of mass number \(A\). \(y_1\) and \(y_2\) denote the rapidities of the scattered partons. The symbols with carets refer to the parton-parton c.m. system. \(\hat{\sigma}_{ij-kl}\) is the elementary pQCD parton cross section.

\[
\hat{s} = x_1 x_2 s = 4p_t^2 \cosh^2\left(\frac{y_1 - y_2}{2}\right),
\]

gives the total c.m.-energy of the parton-parton scattering. The rapidities \(y_1\), \(y_2\) and the momentum fractions \(x_1\), \(x_2\) are related by,

\[
x_1 = p_t (e^{y_1} + e^{-y_2})/\sqrt{s}, \quad x_2 = p_t (e^{-y_1} + e^{-y_2})/\sqrt{s}.
\]

The limits of integrations of rapidities \(y_1\) and \(y_2\) are given by \(|y_1| \leq \ln(\sqrt{s}/2p_t + \sqrt{s/4p_t^2 - 1})\) and \(-\ln(\sqrt{s}/p_t - e^{-y_2}) \leq y_2 \leq \ln(\sqrt{s}/p_t - e^{-y_1})\), respectively. We multiply the above minijet cross sections by the phenomenological factor \(K = 2\) to account for higher-order contributions.

The minijet cross section, Eq. (1), can be related to the total number of produced partons via

\[
N = T(0) \int dp_1 dp_2 dy_1 dy_2 \frac{2\pi p_t}{s} \sum_{ijkl} x_1 f_{i/A}(x_1, p_{1t}^2) x_2 f_{j/A}(x_2, p_{2t}^2) \hat{\sigma}_{ij-kl}(s, \hat{t}, \hat{u}),
\]

where \(T(0) = 9A^2/8\pi R_A^2\) is the nuclear geometrical factor for head-on AA collisions (for a nucleus with a sharp surface). \(R_A = 1.1A^{1/3}\) fm is the nuclear radius. Similarly, the total transverse energy \(E_t\) of minijets is given by

\[
E_t = T(0) \int dp_1 dp_2 dy_1 dy_2 \frac{2\pi p_t}{s} \sum_{ijkl} x_1 f_{i/A}(x_1, p_{1t}^2) x_2 f_{j/A}(x_2, p_{2t}^2) \hat{\sigma}_{ij-kl}(s, \hat{t}, \hat{u}).
\]
We employ the “EKS98” set of parton distribution functions in a bound nucleon from ref. [11], based on the GRV98 set of parton distributions for a free nucleon [12]. We choose the transverse momentum cutoff for minijet production, $p_0$, to be 1 and 2 GeV at RHIC and LHC, respectively. These values are obtained from the McLerran-Venugopalan model using their initial conditions [8]. These scales provide the initial cutoff for the (semi-)hard scatterings. Below those scales the incoherent parton picture is not valid and coherence effects have to be taken into account.

The initial number density $n_0 = N/V_0$ and energy density $\varepsilon_0 = E_t/V_0$ of the minijet plasma are obtained from Eqs. (4) and (5). The initial volume of the cylindrical system is given by $V_0 = \pi R^2_A \tau_0$, with the initial time $\tau_0 = 1/p_0$ obtained from the uncertainty principle. We will use these initial conditions to study the evolution of the parton “plasma” at RHIC and LHC by solving a self-consistent relativistic transport equation (see section IV). As most of the produced minijets are gluons, we will simplify our considerations by considering gluons only.

III. SCREENING IN NON-EQUILIBRIUM

In this section we describe the screening of long wavelength transverse fields in the parton medium. It will play an important role in defining the finite collision term of our transport equation.

The gluon screening mass is given by the infrared limit of the real part of the gluon self-energy, calculated in the given background that is described by the distribution function $f$ (not to be confused with the parton distribution function $f_{j/A}$ introduced in section II). In ref. [13] the following expression has been derived (in Coulomb gauge) for a medium of gluonic excitations:

$$m^2 = -\frac{3\alpha_s}{\pi^2} \lim_{|\vec{q}| \to 0} \int d^3p \frac{|\vec{p}|}{\vec{q} \cdot \vec{p}} \vec{q} \cdot \nabla_p f(p)$$

(6)

In the above equation $\vec{q}$ is the momentum of the test particle, $f(p)$ is the non-equilibrium distribution function of the gluons and $\alpha_s$ is the strong coupling constant. We will consider the transverse screening mass in the following, which will be introduced below as a cutoff in parton-parton elastic scattering to obtain a finite transport cross-section. Performing an integration by parts we obtain the transverse screening mass

$$m_t^2 = \frac{3\alpha_s}{\pi^2} \int \frac{d^3p}{p^t} f(p).$$

(7)

Following Bjorken hypothesis [14], we express all the quantities in terms of longitudinal-boost invariant parameters $\tau$, $\xi$ and $p_t$. Here $\tau = \sqrt{t^2 - z^2}$, $\xi = \eta - y$ with $\eta = \text{Artanh} (z/t)$ being the space-time rapidity and $y = \text{Artanh} (p_z/p_0)$ the momentum-space rapidity. We assume that the above equation is also valid for a space-time dependent distribution function $f(x, p)$ and hence use

$$m_t^2(\tau) = \frac{6\alpha_s(\tau)}{\pi} \int dp_t \int d\xi f(\tau, p_t, \xi).$$

(8)
Improving earlier approaches [15], we do not assume factorization of the distribution function in the form $f(\tau, p_t, \xi) = g(\xi)h(\tau, p_t)$. Also, in our case the screening mass enters the collision kernel of the transport equation and thus determines the rate of equilibration.

In the above equation the QCD running coupling constant $\alpha_s(\tau)$ becomes time dependent. To obtain the time-dependent coupling constant we use

$$\alpha_s(\tau) = \alpha_s(\langle p_t^2(\tau) \rangle),$$

with the average transverse momentum square of the excitations of the medium defined as

$$\langle p_t^2(\tau) \rangle = \frac{\int d\Gamma p_\mu u^\mu p_t^2 f(\tau, p_t, \xi)}{\int d\Gamma p_\mu u^\mu f(\tau, p_t, \xi)}.$$  \hspace{1cm} (10)

Here $d\Gamma = d^3p/(2\pi)^3 p^0 = dp.dp.d\xi/(2\pi)^2$ is the invariant momentum-space measure. Throughout our calculation we use the GRV98 calculation of $\alpha_s(\langle p_t^2(\tau) \rangle)$ [12] with $\langle p_t^2(\tau) \rangle$ calculated from Eq. (10).

A. Initial Screening Mass from Minijets

The initial screening mass can be obtained directly from the number and energy of the minijets, as discussed in section II.

First of all the initial distribution function of gluons, $f_0$, can be obtained from the initial number density of the minijets as

$$f_0 = \frac{(2\pi)^3}{g_G} \frac{dN}{V_0 d^3p},$$

with $N$ from Eq. (4). Here $d^3p = d^2p_\perp dp_z = p_t^2 dp_t \cosh y_1 dy_1$. $g_G = 16$ is the product of the spin and color degeneracy factor for gluons. The initial screening mass $m_{i0}$ can be obtained from Eq. (8) by using the above expression for $f_0$. We find

$$m_{i0}^2 = \frac{27\alpha_s A^2}{8\pi_0 R_A^2} \int dp dp_1 dp_2 \frac{1}{s} \sum_{ijkl} x_1 f_{i/A}(x_1, p_t^2) x_2 f_{j/A}(x_2, p_t^2) \hat{\sigma}_{ijkl}(\hat{s}, \hat{t}, \hat{u}).$$

In the above equation we use $\alpha_s(\langle p_t^2 \rangle)$ with $\langle p_t^2 \rangle$ defined by

$$\langle p_t^2 \rangle = \frac{1}{\sigma_{jet}} \int dp dp_t dp_1 dp_2 \frac{2\pi p_t}{s} \sum_{ijkl} x_1 f_{i/A}(x_1, p_t^2) x_2 f_{j/A}(x_2, p_t^2) \hat{\sigma}_{ijkl}(\hat{s}, \hat{t}, \hat{u}).$$

IV. SOLUTION OF THE TRANSPORT EQUATION WITH SCREENING

In the absence of any coherent color field the space-time evolution of the produced partons at RHIC and LHC can be studied by solving the Boltzmann transport equation

$$p^\mu \partial_\mu f(x, p) = C(x, p),$$

(14)
where $f(x, p)$ is the distribution function of gluons and $C_i(x, p)$ is the collision term. To solve the above transport equation with the initial value of $f_0$ given by Eq. (11), we employ the relaxation time approximation for the collision term [16,17]:

$$C(\tau, \xi, p_t) = -p^\mu u_\mu \left[ f(\tau, \xi, p_t) - f^{eq}(\tau, \xi, p_t) \right] / \tau_c(\tau). \quad (15)$$

$u^\mu$ is the four-velocity of the local rest-frame of the medium, $f^{eq}$ is the equilibrium distribution function, and $\tau_c(\tau)$ is the time dependent relaxation time of the plasma. With this collision term the formal solution of the transport equation becomes

$$f(\tau, \xi, p_t) = \int_0^\tau d\tau' \exp \left[ \int_0^{\tau'} \frac{d\tau''}{\tau_c(\tau'')} \right] \frac{f^{eq}(\tau', \xi', p_t)}{\tau_c(\tau')} + f_0(\xi) \exp \left( -\int_0^\tau \frac{d\tau''}{\tau_c(\tau'')} \right), \quad (16)$$

where $\xi'$ is the solution of

$$\sinh \xi' = \frac{\tau}{\tau'} \sinh \xi. \quad (17)$$

We write the relaxation time for collisions, $\tau_c(\tau)$, as

$$\tau_c(\tau) = \frac{1}{\sigma_t(\tau) n(\tau)}, \quad (18)$$

where

$$n(\tau) = g_G \int d\Gamma p_\mu u^\mu f(\tau, p_t, \xi) \quad (19)$$

is the number density of the gluon-minijet plasma, and

$$\sigma_t(\tau) = \int d\Omega \frac{d\sigma}{d\Omega} \sin^2 \theta_{c.m.}$$

(20) denotes the time dependent transport cross-section for the collision processes [17,18]. We assume that small-angle scattering gives the dominant contribution to the transport cross-section [18], such that $\sin^2 \theta_{CM} = 4i\hat{u}/\hat{s}$. We mention again that, in our study, all the quantities such as $m_t$, $\sigma_t$, $n$, $\tau_c$ are time dependent and have been obtained from the non-equilibrium distribution function of the gluon minijets.

We shall consider the leading order elastic scattering processes $gg \rightarrow gg$. The differential cross section for this process is given by

$$\frac{d\sigma}{dt} = \frac{9\pi \alpha_s^2}{2s^2} \left| 3 - \frac{\hat{u}\hat{t}}{s^2} - \frac{\hat{u}\hat{s}}{t^2} - \frac{\hat{s}\hat{t}}{u^2} \right|. \quad (21)$$

In the limit of small-angle scattering (of identical particles) it simplifies to

$$\frac{d\sigma}{dt} = \frac{9\pi \alpha_s^2}{2t^2} \frac{1}{t^2} \quad (22)$$

and the transport cross-section $\sigma_t$ diverges logarithmically due to exchange of long-wavelength gluons. However, as discussed in section III, long-wavelength fields will be
screened by the dense medium. For our studies we therefore employ the medium-modified elastic cross-section [19]

\[
d\sigma/d\hat{t} = \frac{9\pi\alpha_s^2}{2} \left( \frac{m_t^2}{s} + 1 \right) \frac{1}{(\hat{t} - m_t^2)^2}. \tag{23}
\]

Using Eq. (23) in Eq. (20) we obtain the medium modified finite transport cross-section

\[
\sigma_t = \frac{9\pi\alpha_s^2}{2} \left( \frac{m_t^2}{s} + 1 \right) \left[ (\hat{s} + 2m_t^2) \log \left( \frac{\hat{s}}{m_t^2} + 1 \right) - 2\hat{s} \right]. \tag{24}
\]

To simplify the considerations we replace \( \hat{s} \) by its average value

\[
\hat{s}(\tau) = 4\langle E(\tau) \rangle^2. \tag{25}
\]

\( \langle E(\tau) \rangle \) is the time dependent average energy per particle given by

\[
\langle E(\tau) \rangle = \frac{\epsilon(\tau)}{n(\tau)},
\]

where

\[
\epsilon(\tau) = g_G \int d\Gamma(p_{\mu}u^{\mu})^2 f(\tau, p_t, \xi), \tag{27}
\]

is the energy density of the minijet "plasma" and \( n(\tau) \) is the local number density of the plasma as defined in Eq. (19).

To obtain the collision term (15) in each time step we also have to determine the equilibrium distribution towards which the evolution is supposed to converge. In other words, we have to determine the "equivalent" plasma temperature from (27). To this end we employ the relation between energy density and temperature in an ideal gluon gas in thermodynamical equilibrium,

\[
\epsilon = g_G \frac{\pi^2}{30} T^4. \tag{28}
\]

This should be a reasonable approximation as long as the system is not too close to the hadronization phase transition.

**V. NUMERICAL SOLUTION**

The expression for the distribution function \( f(\tau, \xi, p_t) \), Eq. (16), involves \( T(\tau) \) and \( \tau_c(\tau) \) which are again defined through the distribution function \( f(\tau, \xi, p_t) \). We solve these coupled set of equations self-consistently. At any time \( \tau \) we start with the old trial values \( T_0, n_O, \alpha_{sO}, m_{tO} \) and \( \hat{s}_O \) from which we get \( f_O \) via Eq. (16). This \( f_O \) is used in Eq. (27) to calculate \( \epsilon(\tau) \) which gives a new temperature \( T_N \) through Eq. (28). This new temperature, but the old values of \( n_O, \alpha_{sO}, m_{tO} \) and \( \hat{s}_O \) are again used in Eq. (16) to get a new \( f_1 \). This \( f_1 \) is used in Eq. (19) to calculate a new \( n_N \) which also gives a new value \( \hat{s}_N \) via Eq. (25). These
new values $T_N, n_N$ and $s_N$ and the old values $\alpha_{sO}$ and $m_{tO}$ are again used in Eq. (16) to get a new $f_2$. Using this $f_2$ in Eq. (10) we obtain $\alpha_{sN}$ from Eq. (9). These new values $T_N, n_N,$ $s_N$, $\alpha_{sN}$ and old value $m_{tO}$ are again used in Eq. (16) to obtain a new $f_3$. Using this $f_3$ in Eq. (8) we obtain a new $m_{tN}$. Thus, starting with the old set of values $T_O$, $n_O$, $\alpha_{sO}$, $m_{tO}$ and $s_O$ we obtain a new set of values $T_N$, $n_N$, $\alpha_{sN}$, $m_{tN}$ and $s_N$. This process is iterated until convergence is attained to the required accuracy. This gives us the self-consistent values of $\epsilon(\tau)$, $T(\tau)$, $n(\tau)$, $\alpha_{s}(\tau)$, $m_{t}(\tau)$ and $s(\tau)$ at any time $\tau$.

**VI. RESULTS AND DISCUSSIONS**

The purpose of this paper is to study various bulk properties of the gluon minijet plasma. In particular, we discuss the time evolution of the number density, the energy density, the temperature, as well as the collision-relaxation time and the transport cross section. Calculations of signatures from this equilibrating minijet plasma will be presented elsewhere.

We compare the time evolution of various physical quantities with that obtained in the equilibrium limit, i.e. the hydrodynamical evolution [14]. For purely longitudinal expansion one has

$$n(\tau) = n_0 \left( \frac{\tau}{\tau_{eq}} \right)^{-1}, \quad (29)$$

$$\epsilon(\tau) = \epsilon_0 \left( \frac{\tau}{\tau_{eq}} \right)^{-\frac{4}{3}}, \quad (30)$$

$$T(\tau) = T_0 \left( \frac{\tau}{\tau_{eq}} \right)^{-\frac{1}{3}}. \quad (31)$$

In the above equations $\epsilon_0$, $n_0$ and $T_0$ are the energy density, number density and temperature of the gluon plasma at $\tau = \tau_{eq}$, where equilibrium is reached. The latter two equations actually depend on the equation of state of the minijet plasma; we have assumed an ideal gas of gluons.

The evolutions of the number densities are shown in Fig. 1. The solid lines depict the results from our self-consistent transport calculations. Not surprisingly, the plasma is found to be much denser at LHC than at RHIC. The dashed lines are the number densities corresponding to Eq. (29) with $n_0$ obtained from the non-equilibrium studies at $\tau_{eq}= 5.4$ and 4.8 fm at RHIC and LHC, respectively. After $\tau = \tau_{eq}$ the number densities behave as $n(\tau) \propto \tau^{-\alpha}$ with $\alpha = 0.994$ and 1.003 at RHIC and LHC respectively. These values can be compared with $\alpha = 1$ for the case of 1+1 dimensional hydrodynamic evolution, suggesting that the equilibrium is indeed reached at these values of $\tau_{eq}$. As a cross-check we also examine the behavior of the energy densities and temperatures.

The time evolution of the energy densities at RHIC and LHC are shown in Fig. 2. The solid lines are obtained from the solution of our transport equation, while dashed lines are obtained from Eq. (30), $\epsilon_0$ being extracted from the non-equilibrium distribution function at $\tau_{eq}$. Again, we see that the energy densities follow the hydrodynamic evolution equation at $\tau_{eq} = 5.4$ and 4.8 fm at RHIC and LHC, respectively. After $\tau = \tau_{eq}$ the energy density
behaves as $\epsilon(\tau) \propto \tau^{-\alpha}$ with $\alpha = 1.329$ and 1.332 at RHIC and LHC respectively. These values can be compared with $\alpha = 4/3$ obtained for $1+1$ dimensional hydrodynamic evolution, Eq. (30).

Finally, the time evolution of the temperatures of the minijet plasma at RHIC and LHC are depicted in Fig. 3. After $\tau = \tau_{eq}$ the temperature behaves as $T(\tau) \propto \tau^{-\alpha}$ with $\alpha = 0.335$ and 0.34 at RHIC and LHC. These values can be compared with $\alpha = 1/3$ as appropriate for $1+1$ dimensional hydrodynamic evolution, Eq. (31).

The equilibration times and scaling exponents given above represent the optimal fit to the solution of the transport equation. However, to sketch when the evolution actually comes close to ideal hydrodynamics, we have refitted the scaling exponents, putting by hand $\tau_{eq}' = 2$ fm (RHIC) and 1.6 fm (LHC), respectively. The corresponding curves are shown by the dot-dashed lines in Figs. 1-3. The scaling exponents in this case are 0.9085, 0.9029 for the number density; 0.3165, 0.3194 for the temperature; and 1.208, 1.206 for the energy density at RHIC and LHC, respectively. These values have to be compared to $\alpha = 1, 1/3$ and 4/3 for the case of ideal 1+1 dimensional hydrodynamic flow.

We present the parametrizations of energy density, number density and temperature of the gluon minijet plasma after $\tau = \tau_{eq}$ at RHIC and LHC. They are given by

$$\epsilon(\tau) = 12.336\left(\frac{\tau}{5.4}\right)^{-1.329} [GeV/fm^3],$$

$$n(\tau) = 12.313\left(\frac{\tau}{5.4}\right)^{-0.994} [fm^{-3}],$$

$$T(\tau) = 366\left(\frac{\tau}{5.4}\right)^{-0.335} [MeV],$$

at RHIC for $\tau \geq 5.4$ fm and

$$\epsilon(\tau) = 161.246\left(\frac{\tau}{4.8}\right)^{-1.332} [GeV/fm^3],$$

$$n(\tau) = 84.563\left(\frac{\tau}{4.8}\right)^{-1.003} [fm^{-3}],$$

$$T(\tau) = 696\left(\frac{\tau}{4.8}\right)^{-0.34} [MeV]$$

at LHC for $\tau \geq 4.8$ fm.

The time evolutions of the transport cross sections at RHIC and LHC are shown in Fig. 4. In our calculation the minijet cutoff $p_0$ is taken to be 1 and 2 GeV at RHIC and LHC, respectively (see section II). Therefore, more energetic partons are produced at LHC and the transport cross section is much smaller. These transport cross sections play a crucial role in the equilibration of the plasma, cf. Eq. (18). In Fig. 5 we have displayed the time evolution of the relaxation time $\tau_c(\tau)$. One observes that the relaxation times at RHIC and LHC do not differ by much, despite the much higher density of partons obtained at LHC. As the relaxation time determines the equilibration rate $1/\tau_c$ of the plasma we obtain almost the same values for the equilibration time (5.4 and 4.8 fm) at RHIC and LHC.
VII. CONCLUSIONS

We have studied the production and equilibration of the gluon minijet plasma produced in the central region of high-energy nuclear collisions by solving the relativistic Boltzmann transport equation. The initial conditions are obtained from pQCD. We have solved the transport equation employing a collision term based on $2 \rightarrow 2$ elastic collisions. The collinear divergence in the perturbative cross section to lowest order is removed by incorporating the screening of very soft interactions by the medium. The screening mass is calculated from the non-equilibrium distribution function of the partons.

The present study indicates that the plasma is more dense and hot at LHC. At RHIC the minijet plasma equilibrates at 5.4 fm with a temperature of 360 MeV. The corresponding values at LHC are 4.8 fm and 700 MeV. We do not find significant differences in the equilibration time of the gluon-minijet plasma at RHIC and LHC since the product of comoving parton density and transport cross-section is similar. However, there are significant differences in other physical quantities such as number density, energy density and temperature. In our study the equilibration time is found to be somewhat larger than that obtained in the parton cascade model [20] and HIJING [21].

For simplicity, we have not incorporated any coherent field in the present study. Including a coherent field in the initial condition may further decrease the equilibration times and increase the energy density, the number density and the temperature of the plasma. We attempt to study the production and equilibration of a QGP at RHIC and LHC with both coherent field and incoherent partons taken into account in a forthcoming paper.

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REFERENCES

Figure captions

FIG. 1. Time evolution of the number densities of the gluon-minijets at RHIC and LHC. The solid lines are obtained from the self-consistent solution of the transport equation. Dashed and dot-dashed lines are obtained by a fit to \( n(\tau) \propto \tau^{-\alpha} \), starting at \( \tau = \tau_{eq} \) and \( \tau'_{eq} \), respectively (see text).

FIG. 2. Time evolution of the energy densities of the gluon-minijets at RHIC and LHC. The curves correspond to the various cases explained in Fig. 1.

FIG. 3. Time evolution of the temperatures of the gluon-minijets at RHIC and LHC. The curves correspond to the various cases explained in Fig. 1.

FIG. 4. Time evolution of the in-medium transport cross sections at RHIC and LHC.

FIG. 5. Time evolution of the relaxation times at RHIC and LHC.