Quantum (in)stability of dilatonic AdS backgrounds and holographic renormalization group with gravity

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Abstract

Stability of dilatonic AdS spaces due to quantum effects of dilaton coupled conformal matter is considered. When such spaces do not exist on classical level, their dynamical generation occurs. Explicit examples corresponding to quantum creation of d4 dilatonic AdS Universe and of d2 dilatonic AdS Black Hole (BH) are presented. Motivated by holographic RG, in the similar approach the complete d5 effective action is discussed. The intermediate region where it is the sum of two parts: bulk (classical gravity) and boundary quantum action is investigated. The effective equations solution representing d5 AdS Universe with warp scale factor is found. Four-dimensional de Sitter or AdS world is generated on the boundary of such Universe as a result of quantum effects.

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1 Introduction

The interest in the study of questions related with AdS backgrounds is caused by several reasons. From one side, in AdS/CFT set-up [1] the investigation of classical solutions of IIB SG with AdS sections may provide the information about boundary QFT in less dimensions. Second, AdS space has maximum number of Killing vectors and it is well-known candidate for vacuum state in various SGs. For strings the AdS-like backgrounds are often suspected to be exact vacuum state. Third, some cosmological data indicate to the presence of spatial sections with negative curvature in the inflationary Universe. The recent numerous studies of warped compactifications and brane cosmology have been initiated by refs.[2](the corresponding literature is vast, see [3] and refs. therein). That suggests the possibility of AdS-like stage which could be important for various reasons (hierarchy problem, etc) in the early Universe.

Reserving the possibility of dynamical generation of AdS regions at the early (multidimensional) Universe the question could arise: what are the mechanisms responsible for such process? In particular, if AdS BHs exist they should be presumably produced as the primordial BHs at the very early Universe where quantum effects play the dominant role. Hence, the natural mechanism for creation-annihilation of AdS backgrounds could be defined by quantum effects. Indeed, it has been shown recently [9] that quantum annihilation of 4d AdS Universe typically occurs. However, the presence of dilaton may change the situation to quantum creation of dilatonic AdS Universe (subject to fine-tuning of theory parameters).

In the present work we investigate the role of quantum conformal matter effects (using large $N$, anomaly induced effective action) in the dynamical realization of dilatonic AdS backgrounds in various dimensions. In the next section the 4d dilatonic classical gravity with $N$ dilaton coupled quantum fermions is considered. From the solution of effective equations it follows that quantum corrected dilatonic AdS Universe may occur, even in situation where such classical solution was absent. However, the probability of generation of AdS Universe is significantly less than of quantum creation of de Sitter Universe. It is interesting that some features of quantum creation of dilatonic AdS Universe should be also typical for the same process for primordial AdS BHs. To demonstrate this, in section three we present two-dimensional exactly solvable model. It is the same dilatonic gravity with dilaton coupled massless quantum fermions in two dimensions. The complete anomaly in-
duced effective action is found. The quantum 2d dilatonic AdS BH which did not existed on classical level is constructed as the solution of the effective equations. In other words, the creation of quantum 2d AdS BH occurs.

From another side, in AdS/CFT set-up where AdS background is introduced from the very beginning the question on realization of gravity in boundary world appears. Some progress in this direction is made within Randall-Sundrum scenario which may suggest the solution of hierarchy problem. In fourth section we discuss the holographic RG action leading to warped compactification (RS-like Universe) in the region where both sides of AdS/CFT correspondence (i.e. bulk and boundary) are still relevant. Then, using the same anomaly induced effective action of previous sections (boundary) we suggest the way it should appear in the dynamics of five-dimensional world on equal footing with 5d gravity action. As a result the dynamical effective equations could be solved realizing 5d AdS Universe with warp factor. On the boundary of such Universe the de Sitter (inflationary) world occurs. It is actually induced by quantum effects (not four-dimensional cosmological constant introduced on brane by hands). In the final section we present short resume and mention some open problems.

2 Quantum instability of 4d AdS dilaton universe

Let us consider the 4-dimensional dilaton gravity theory with the following action:

\[ S = \int d^4x \sqrt{-g} \left[ -\frac{1}{16\pi G} (R + 6\Lambda) + \alpha (\nabla_\mu \phi)(\nabla^\mu \phi) \right], \tag{1} \]

where \( \Lambda \) is the cosmological constant and \( \alpha \) some suitable parameter. For constant dilaton \( \phi \) and negative cosmological constant the classical background solution corresponds to the 4d AdS space. Even for non-constant dilaton, there are solutions interpolating between asymptotically AdS and flat space with singular dilaton [4]. Our primary interest will be in (asymptotically) AdS background for the theory defined by (1).

Few remarks are in order. There are different interpretations of action (1). First of all, this action may be considered as the compactification of the II B supergravity bosonic sector (say, a Freund-Rubin ansatz and background like \( S_5 \times S_1 \times AdS_4 \)).
Second, making a conformal transformation of the metric and a suitable redefinition of the dilaton, one arrives at the Brans-Dicke theory (for a review, see [5]), with non-trivial dilatonic potential. Hence, the action (1) may be considered as a Brans-Dicke theory in the Einstein frame, which has been argued to be the physical one (for a review, see [6]).

We will be interested in the investigation of the stability of the (classical) AdS background in the theory (1), under the quantum fluctuations of the conformal matter. As matter Lagrangian, we take the one which corresponds to $N$ massless dilaton coupled Dirac spinors

$$L_M = \exp(A\phi) \sum_{i=1}^{N} \bar{\psi}_i \gamma^\mu \nabla_\mu \psi^i .$$

(2)

Here $A$ is some constant parameter. The above strong matter-dilaton coupling is typical for any matter- Brans-Dicke theory in the Einstein frame. Transforming the variables in the theory (1) to the Brans-Dicke gravity in the Jordan frame, one can easily show that $L_M$ is transformed into the usual spinor Lagrangian with no dilaton coupling. That is why $L_M$ is just how the matter looks in the Brans-Dicke gravity within the Einstein frame. Additional motivations to consider the dilaton coupled matter Lagrangian as in (2) come from the supergravity side, where such coupling is a common feature when the dilaton is present in the supergravity multiplet. The number of spinors $N$ is to be considered very large, in order to justify why one can neglect the proper quantum gravity contributions.

The quantum effective action for dilaton coupled spinor has been found in ref. [7] by integrating the conformal anomaly. This quantum effective action should be added to the classical one $S$ (there is only the dilaton-gravitational background under consideration).

Let us now define the space-time we are going to work with. We consider the 4d AdS background with the static metric

$$ds^2 = e^{-2\lambda x_3} \left[ dt^2 - (dx_1)^2 - (dx_2)^2 \right] - (d\tilde{x}_3)^2 .$$

(3)

It has a negative cosmological constant $\Lambda = -\lambda^2$. Making the coordinates transformations

$$y = \frac{e^{\lambda x_3}}{\lambda} .$$

(4)
one can present (3) in the conformally flat form

$$ds^2 = a(y)^2 \eta_{\mu\nu} dx^\mu dx^\nu,$$

where

$$a = e^{-\lambda \bar{x}_3} = \frac{1}{\lambda y}.$$  \hspace{1cm} (6)

This form of the metric is very useful in the AdS/CFT correspondence. It is also clear that the classical AdS Universe may be trivially realized as solution of the theory (1), with negative cosmological constant and constant dilaton.

Here our purpose will be the investigation of the role of the quantum effects to the dilaton AdS universe. To this aim, we shall consider the metric (5) with an arbitrary scale factor to be determined dynamically. The anomaly induced effective action of ref. [7] on such background may be written in the following form [8]:

$$W = V_3 \int dy \left\{ 2b_1 \sigma_1 \sigma_1''' - 2(b + b_1) \left( \sigma_1'' - \sigma_1'^2 \right)^2 \right\}.$$  \hspace{1cm} (7)

Here, $V_3$ is the (infinite) volume of 3-dimensional flat space-time (time is included), $\sigma = \ln a(y)$, $\sigma_1 = \sigma + \frac{A\phi}{3}$, $' \equiv d/dy$, and $b = \frac{3N}{60(4\pi)^2}$, $b_1 = -\frac{11N}{360(4\pi)^2}$. It should be noted that (7) may be regarded as the complete one-loop effective action. The classical gravitational action on the background (3) with non-trivial dilaton may be obtained from (1) and reads

$$S = V_3 \int dy \left\{ \frac{1}{\chi} \left[ 6(\sigma'' + \sigma'^2)e^{2\sigma} - 6\Lambda e^{4\sigma} \right] + \alpha e^{2\sigma} \right\}.$$  \hspace{1cm} (8)

with $\chi = 16\pi G$. The quantum corrections can be taken into account starting from the effective equations obtained from the effective action $S + W$. These equations look similarly to the ones associated with the De Sitter quantum corrected universe [8], with the addition of the cosmological constant contribution and an opposite sign in the Einstein term:

$$\mathcal{C}_{e^{(A\phi/3)}} - \frac{12}{\chi} a'' - \frac{24\Lambda a^3}{\chi} + 2\alpha a \phi'^2 = 0$$

$$\frac{A}{3} \mathcal{C}_{ae^{(A\phi/3)}} - 2\alpha (a^2 \phi')' = 0,$$  \hspace{1cm} (9)
where
\[ \tilde{C} = -\frac{4b}{a} \left[ \frac{a^m}{a} - \frac{4\tilde{a}^m}{\tilde{a}^2} - 3\tilde{a}^{nm} \right] - \frac{24}{\tilde{a}^4} \left[ (b - b_1)\tilde{a}^m\tilde{a}^2 + b_1\tilde{a}^4 \right], \tag{10} \]

and
\[ \tilde{a} = ae^{(A\phi/3)}. \tag{11} \]

First, we note that in absence of dilaton and quantum corrections (\(a = b = b_1 = 0\)), there exists a special solution
\[ a(y) = \frac{1}{\sqrt{-A y}}, \tag{12} \]
corresponding to the AdS Universe, namely such universe exists at classical level.

Furthermore, in the absence of the dilaton and vanishing cosmological constant, only the first of (9) survives. It may be solved via the special ansatz \( a = c/y \) with the constraint \( c^2 = b_1\chi \). Since \( b_1 < 0 \), one gets an imaginary scale factor \( a \), namely a quantum annihilation of the AdS universe, as it was shown in detail in ref. [9]. Hence, classical AdS Universe is not stable under the action of quantum fluctuations. Note that dilaton is constant there.

Now, one can ask what happens in the general situation within dilatonic gravity with non-constant dilaton with regard to the AdS universe. In other words, there is the question about the stability of the AdS universe in II B Supergravity under the action of the quantum matter fluctuations.

The general solution of the system of differential equations (9) is very difficult to find. Apart from numerical studies, the best one can do is to search for special solutions. Motivated by the similar dilatonic De Sitter universe solution [8], one may try
\[ a(y) = \frac{1}{H y}, \quad \phi'(y) = \frac{1}{H_1 y}, \tag{13} \]
where \( H \) and \( H_1 \) are some constants to be determined. From Eqs. (9) one obtains
\[ \frac{aA}{3} \left[ \frac{12\alpha}{\chi} + \frac{24\Lambda a^3}{\chi} - 2\alpha a\phi'^2 \right] - 2\alpha (a^2\phi')' = 0. \tag{14} \]
Substituting Eqs. (13) in the second of (9) and (14), we obtain

\[
\frac{12\Lambda}{\chi H^2} = \frac{\alpha}{H_1^2} - \frac{9\alpha}{AH_1} - \frac{12}{\chi}.
\]

(15)

\[
\frac{81\alpha}{2AbH^2} = -(A - 3H_1) \left( \frac{18H_1^2 - 2\tilde{b}(A - 3H_1)^2}{H_1^2} \right),
\]

(16)

with \( \tilde{b} = 1 + \frac{4}{5} \). As a result, one may eliminate \( H \) from the first equation and obtain a third order complete algebraic equation for the quantity \( H_1 \), which in principle can be solved. However, for \( \Lambda = 0 \), there is the complete decoupling of the two equations and one easily arrives at

\[
H_1 = \frac{1}{24} \left[ -\frac{9\alpha\chi}{A} \pm \sqrt{\frac{81\alpha^2\chi^2}{A^2} + 48\alpha\chi} \right],
\]

(17)

and the other quantity \( H \) may be obtained from equation (16):

\[
\frac{1}{H^2} = \frac{2b}{81\alpha} \left[ (-18 + 81\tilde{b}) A^2 + \frac{24\tilde{b}A^4}{\alpha\chi} \pm \frac{\alpha\chi}{3bA} \sqrt{\frac{81\alpha^2\chi^2}{A^2} + 48\alpha\chi} \right].
\]

(18)

Since \( \tilde{b} = \frac{7}{11} > 0 \) and \(-18 + 81\tilde{b} > 0\), there is always a real solution for \( H \) at least for positive \( \alpha \) if we choose the sign \( \pm \) in front of the square root properly.

Hence, we demonstrated that in presence of non-constant dilaton the quantum AdS Universe solution in dilatonic gravity is less unstable. At least, it may be realized while it didnt exist on classical level! However, it is less stable than corresponding de Sitter Universe [8] as it may be easily seen from the explicit solution. In fact, the mechanism presented in this section may serve as the one corresponding to quantum creation of primordial AdS Black Holes in early Universe. However, for 4d AdS BH one should calculate the extra piece of effective action which is non-local and very complicated. The complete calculation of it is not known, presumably it could be found only as expansion on theory parameters. That is the reason we prefer to present such analysis only in two dimensions.
3 Quantum creation of 2d AdS black hole

In this section we investigate the possibility for quantum creation of 2d AdS BHs using methods developed in previous section. Motivated by the 4-dimensional case, we may assume that the classical action for the 2-dimensional dilaton gravity theory reads

$$S = \int d^2x \sqrt{-g(x)} \left[ -\frac{R + 6\Lambda}{\chi} + \frac{1}{2}(\nabla_\mu \phi)(\nabla^\mu \phi) + \exp(A\phi)L_M \right], \quad (19)$$

where $A$ is a constant parameter and the matter Lagrangian is the one of two-dimensional Majorana spinors:

$$L_M = \sum_{i=1}^{N} \bar{\psi}_i \gamma^\mu \nabla_\mu \psi^i. \quad (20)$$

Let us neglect the classical matter contribution since we are interested only in the one-loop EA induced by the conformal anomaly of the quantum matter.

In two dimensions, a general static metric may be written in the form

$$ds^2 = V(r)dt^2 - \frac{1}{W(r)}dr^2. \quad (21)$$

It is well know that introducing the new radial coordinate $r^*$, defined by

$$r^* = \int \frac{dr}{\sqrt{V(r)W(r)}}, \quad (22)$$

one gets a conformally flat space-time,

$$g_{\mu\nu} = V(r(r^*))\eta_{\mu\nu} = e^{2\sigma(r^*)}\eta_{\mu\nu}. \quad (23)$$

We also assume that the field $\phi$ depends only on $r^*$.

Since the conformal anomaly for the dilaton coupled spinor is (see [7])

$$T = \frac{c}{2}\tilde{R}, \quad (24)$$

where $c = N/(12\pi)$ and $\tilde{R}$ is calculated on the metric $\tilde{g}_{\mu\nu} = e^{2A\phi}g_{\mu\nu}$, the anomaly induced EA in the local, non-covariant form reads

$$W = \frac{c}{2} \int d^2x \tilde{\sigma}\tilde{\sigma}''. \quad (25)$$
Here $\dot{\sigma} = \sigma + A\phi$ and $' = \frac{d}{dr^*}$. Note that this is, up to a non-essential constant, an exact one-loop expression. The total one-loop effective action is $S + W$, i.e.

$$S + W = V_1 \int dr^* \left\{ \frac{k_G}{2} (\sigma'' - 6Ae^{2\sigma}) + \frac{1}{2}(\phi')^2 + \frac{c}{2} (\sigma + A\phi) (\sigma'' + A\phi'') \right\},$$

(26)

where $k_G = \frac{1}{8\pi G}$, and $V_1$ the (infinite) temporal volume. Since 2d Einstein theory is trivial, the whole dynamics appears as a result of quantum effects.

The equations of motion given by the variations of $\phi$ and $\sigma$ are

$$0 = - (1 - cA^2) \phi'' + cA\sigma''$$

(27)

$$0 = c (\sigma'' + A\phi'') - 6k_G Ae^{2\sigma}.$$  

(28)

From (27) and (28), we obtain

$$0 = -6k_G Ae^{2\sigma} + \frac{c}{1 - cA^2} \sigma'' ,$$

(29)

which gives a constant $E$ of motion

$$E = -3k_G Ae^{2\sigma} + \frac{c}{2 (1 - cA^2)} (\sigma')^2 .$$

(30)

We now change the coordinate by

$$r = \int e^{2\sigma} dr^* ,$$

(31)

which gives

$$W(r) = V(r) = e^{2\sigma}.$$  

(32)

Then Equation (30) can be integrated to give

$$e^{2\sigma} = \frac{(r - r_0)^2}{b} - a$$

$$b \equiv \frac{6(1 - cA^2)k_G}{c}, \quad a \equiv \frac{E}{3k_G\Lambda} .$$

(33)

Here $r_0$ is a constant of the integration. If we further redefine,

$$\frac{1}{l^2} \equiv b , \quad cM \equiv \frac{2r_0}{b} , \quad k \equiv -a + \frac{r_0^2}{b} ,$$

(34)
we obtain a generic 2d AdS black hole solution,

\[ W(r) = V(r) = e^{2\sigma} = k - cMr + \frac{r^2}{l^2} \]  

(35)

where \( M \) may be interpreted as the mass of the BH. We may take \( k = \pm 1 \), or \( k = 0 \). In general, we have a simple positive root, interpreted as horizon radius. In the case \( k = 1 \) one must have \( cM > 2 \). It is easy to show that the above metric has a negative constant scalar curvature and for large \( r \), \( V(r) \approx \frac{r^2}{l^2} \), namely one gets the AdS asymptotic behavior. For the sake of simplicity, let us consider the case \( k = 0 \). In this case the horizon radius and the Hawking temperature read

\[ r_H = cMl^2, \beta_H = \frac{4\pi}{cM} = \frac{4\pi l^2}{r_H}. \]  

(36)

Since

\[ r^* = l^2 \frac{r - r_H}{r_H} \ln \frac{r}{r_H}, \]  

(37)

the old radial coordinate as a function of the new one is

\[ r = \frac{r_H}{1 - e^{\frac{4\pi r_H}{r^*}}}. \]  

(38)

Note that the horizon \( r = r_H \) corresponds to \( r^* \) going to \(-\infty\). The \( \sigma \) function is given by

\[ \sigma(r^*) = \ln \frac{r_H}{l} - \ln(1 - e^{4\pi T_H r^*}) + 2\pi T_H r^*. \]  

(39)

From (27), one obtains the following first integral of motion

\[ -\left(1 - cA^2\right) \phi'(r^*) + cA\sigma'(r^*) = c_1, \]  

(40)

where \( c_1 \) is an integration constant. Thus the solution for the dilaton field is

\[ \phi(r^*) = \phi_0 + c_1 r^* + \frac{cA}{1 - cA^2} \sigma(r^*), \]  

(41)

where \( c_1 \) is an integration constant. As a function of the old radial coordinate one has

\[ \phi(r) = \phi_0 + c_1 \ln \frac{r - r_H}{r} + \frac{cA}{1 - cA^2} \ln \frac{\sqrt{r(r - r_H)}}{l}. \]  

(42)
Note that the quantum correction to the dilaton diverges on the horizon.

Thus, using anomaly induced effective action for dilaton coupled spinor we proved the possibility of quantum realization of 2d AdS BH which did not exist on classical level. It is interesting that unlike to 4d case where EA for AdS BH is not completely known, 2d case is exactly solvable. Our solution may be interpreted as quantum creation of dilatonic AdS BH.

The finite Hawking temperature in (36) tells that the Hawking radiation (for a recent review, see [12]) should occur for our BH solution. If we consider the time dependent perturbation, one could trace the fate of the black hole. This will be discussed elsewhere.

4 Holographic Renormalization Group and Dynamical Gravity

In the standard AdS/CFT correspondence [1] one can think about the simultaneous incorporation of string compactification with exponential warp factor (Randall-Sundrum compactification [2, 3]) and the holographic map between 5d supergravity and 4d boundary (gauge) theory. Moreover, it could be extremely interesting to do it in such a way that dynamical gravity would appear on the boundary side. One possibility to realize such a mechanism is presumably related with holographic renormalization group (RG), see [10, 11] for an introduction. Probably the most important element of such RG (as well as in holographic correspondence between 5d SG and 4d dual gauge theory) is the identification of fifth AdS coordinate with the RG parameter of 4d boundary theory. In particular, this gives the way to study RG flows in 4d gauge theory via the investigation of classical AdS-like solutions of 5d gauged SGs.

There was very interesting suggestion in this respect in ref.[11] to consider low-energy effective action (EA) in the region where field theoretical quantities and analogous supergravity quantities could be considered on equal foot. In other words, this is the way to match two dual descriptions into the global picture of some, more universal RG flow. Immediate consequence of such point of view is the possible explanation of smallness of cosmological constant, the stability of flat spacetime along the RG flow and possible understanding of 4d gravity appearence in standard AdS/CFT set-up.
Our purpose will be to find the explicit realization of ideas of ref. [11] via the construction of the corresponding phenomenological model. First of all, it will be necessary to repeat two starting points of the consideration in ref. [11].

In the calculation of complete low-energy EA in AdS/CFT set-up one can divide it into a high energy and low energy pieces, separated by some given RG scale (fixed value of radial coordinate):

\[ S = S_{UV} + S_{IR}. \]  

(43)

Here \( S_{UV} \) is obtained from the original stringy action as a result of specific compactification. \( S_{IR} \) may be identified with the quantum effective action of (gauge and matter) low energy theory.

Let us start now the explicit construction of the model. Consider 5d warped AdS metric:

\[ ds^2 = a_1^2(r) \tilde{g}_{\alpha\beta} dx^\alpha dx^\beta - dr^2 \]  

(44)

where \( r \) is radius of d5 AdS, or RS Universe, i.e., \( a_1 = a_1(r) \) is scale factor of d5 AdS and \( a_1(r) \) usually depends exponentially on the radial coordinate. \( \tilde{g}_{\alpha\beta} \) is 4d metric of boundary, time dependent FRW Universe. We assume that \( \tilde{g}_{\alpha\beta} = a^2(\eta) \eta_{\alpha\beta} \) where \( \eta \) is conformal time and \( \eta_{\alpha\beta} \) is 4d Minkowski tensor. As \( \tilde{g}_{\mu\nu} \) corresponds to conformally flat space, it is defined by conformal time dependent scale factor \( a \). Hence we have two scale factors. One can discuss now the structure of low-energy effective action. Truncation of \( S_{UV} \) gives basically the bosonic sector of 5d gauged supergravity (for simplicity, we consider the situation with only one scalar (dilaton) which is quite typical):

\[ S_{UV} = \int d^5x \sqrt{-g(5)} \left\{ V(\phi) - \frac{1}{H(\phi)} R(5) + V_1(\phi) \nabla_\mu \phi \nabla^\mu \phi \right\}. \]  

(45)

At first step, we limit ourselves to even simpler situation of constant dilaton. (Note that holographic RG implies that \( \phi \) corresponds to the coupling constant of dual QFT). The reason is that we are searching for 4d dynamic gravity, at least qualitatively. Then,

\[ S_{UV} = \int d^5x \sqrt{-g(5)} \left\{ -\frac{1}{H} R(5) - \frac{6\Lambda}{H} \right\}. \]  

(46)
where $V(\phi = \text{const}) \equiv \frac{6A}{H^2}$. We consider 5d AdS background with some 4d time-dependent conformally-flat boundary in this theory as vacuum state. The question is: can boundary quantum effects (instead of 4d cosmological constant) stabilize such space?

The 4d quantum effective action of low-energy theory on the conformally-flat space $g_{\mu\nu} = a^2(\eta)\eta_{\mu\nu}$ looks as (see, for example, [9])

$$W = V_3 \int d\eta \left\{ 2b_1 \sigma''' - 2(b + b_1) \left( \sigma'' - \sigma^2 \right) \right\}$$  \hspace{1cm} (47)

where $\sigma = \ln a(\eta)$, $V_3$ is space volume, $\sigma' = \frac{da}{d\eta}$, for $N = 4$ $SU(N)$ SYM theory $b = \frac{N^2 - 1}{4(4\pi)^2}$, $b_1 = -b$. In general, $b > 0$, $b_1 < 0$ and $b \neq b_1$.

One has to relate $W$ with $S_{\text{IR}}$. We consider d5 AdS background with the metric of the form:

$$ds_5^2 = -dr^2 + a_1^2(r) a^2(\eta) \eta_{\mu\nu} dx^\mu dx^\nu .$$  \hspace{1cm} (48)

One knows that $S_{\text{IR}} = W$ at $r = r_0$, cut-off scale. On the other side in AdS limit the description is completely from supergravity side. So, at $r \to r_A$, $S_{\text{IR}} \to 0$. Then one can adopt the phenomenological approach where

$$S_{\text{IR}} = \int f(a_1(r)) W dr$$  \hspace{1cm} (49)

so that $f(a_1(r))$ satisfies above relations connecting $S_{\text{IR}}$ and $W$. Some choice for it is explicitly given below. Then, one can solve Eqs. of motion from $S_{\text{UV}} + S_{\text{IR}}$ on the background (48).

Under the assumption of the metric in (48), the sum of the actions (46) and (49) has the following form

$$S_{\text{UV}} + S_{\text{IR}}$$
$$= V_3 \int dr d\eta \left\{ - \left\{ e^{2\varphi + 2\sigma} \left( 6\sigma_{,\eta\eta} + 6\sigma_{,\eta}^2 \right) ight\} ight. $$
$$+ e^{4\varphi + 4\sigma} \left( -8\varphi_{,rr} - 20\varphi_{,r}^2 - \frac{16}{T} \varphi_{,r} \right) \right\} \frac{1}{H} $$
$$- \frac{6A}{H} e^{4\varphi + 4\sigma} + f(\varphi) \left\{ 2b_1 \sigma_{,\eta}^2 - 2(b + b_1) \left( \sigma_{,\eta} - \sigma_{,\eta}^2 \right) \right\} ^2 $$
$$+ \frac{A}{4H} \left( e^{4\varphi + 4\sigma} \right)_{,r} + \frac{B}{H} \left( e^{2\varphi + 2\sigma} \sigma_{,\eta} \right)_{,\eta} .$$  \hspace{1cm} (50)

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Here \( \varphi = \ln a_1(r) \), \( \frac{2}{r^2} = \Lambda \) and \( \cdot = \frac{d}{dr}, \cdot' = \frac{d}{d\eta} \). We add the total derivative terms proportional to constant parameters \( A \) and \( B \) and we also rewrite the term proportional to \( b_1 \) by using the total derivative. We should note, however, that the total derivative terms are not irrelevant to the equations of motion as we show it explicitly later, which might be caused by the fact that the 5d general covariance is not always kept in (49). The parameters \( A \) and \( B \) can be later determined by the consistency conditions. The equations of motion given by the variation over \( \sigma \) and \( \varphi \) have the following forms:

\[
0 = -\frac{1}{H} e^{2\varphi + 2\sigma} \left( 18\sigma,_{\eta\eta} + 24\sigma^2,_{\eta} \right)
+ \frac{4}{H} e^{4\varphi + 4\sigma} \left( -8\varphi,_{rr} - 20\varphi^2,_{r} - \frac{16}{l} \varphi,_{r} + \frac{A}{l} \varphi,_{r} - 6\Lambda \right)
+ 4f(\varphi) \left\{ 4b_1\sigma,_{\eta\eta\eta} - 2(b + b_1) (2\sigma,_{\eta\eta\eta} - 2\sigma,_{\eta}\sigma,_{\eta\eta})
+ 2\sigma^2,_{\eta} - 4\sigma^2,_{\eta}\sigma,_{\eta\eta} \right\}
\]

\[
0 = -\frac{1}{H} e^{2\varphi + 2\sigma} \left\{ 6\sigma,_{\eta\eta} + 6\sigma^2,_{\eta} - B (\sigma,_{\eta\eta} + 2\sigma^2,_{\eta}) \right\}
+ \frac{1}{H} e^{4\varphi + 4\sigma} \left( -24\varphi,_{rr} - 48\varphi^2,_{r} - 24\Lambda \right)
+ f'(\varphi) \left\{ 2b_1\sigma^2,_{\eta\eta} - 2(b + b_1) (\sigma,_{\eta\eta} - \sigma^2,_{\eta}\right\} \right\} .
\]  

(51)

Multiplying \( e^{-4\sigma} \) and differentiating it with respect to \( \eta \), one gets

\[
0 = -\frac{1}{H} e^{2\varphi} \frac{d}{d\eta} \left\{ \left( 18\sigma,_{\eta\eta} + 24\sigma^2,_{\eta} \right) \right\}
+ 4f(\varphi) \frac{d}{d\eta} \left\{ \left( 4b_1\sigma,_{\eta\eta\eta} - 2(b + b_1) (2\sigma,_{\eta\eta\eta} - 2\sigma,_{\eta}\sigma,_{\eta\eta})
+ 2\sigma^2,_{\eta} - 4\sigma^2,_{\eta}\sigma,_{\eta\eta} \right) \right\} .
\]

(53)

In order that Eq.(53) has some reasonable solution, we choose that \( f(\varphi) \) should be proportional to \( e^{2\varphi} \):

\[
f(\varphi) = e^{2(\varphi - \varphi_0)} .
\]

(54)

Then UV limit, where \( f = 0 \), corresponds to \( \varphi = -\infty \) and IR limit, where \( f = 1 \), corresponds to \( \varphi = \varphi_0 \). The value of \( \varphi \) is determined later from the equations of motion. Assuming

\[
\sigma = -\ln \eta , \quad \varphi = \frac{a}{l} r , \quad (a \text{ is a constant})
\]

(55)
one obtains the following from the equations of motion (51) and (52):

\[
0 = \left[ -\frac{42}{H} + 4e^{-2\varphi_0} \left\{ 24b_1 - 28(b + b_1) \right\} \right] \frac{1}{\eta^4} \\
+ \frac{4}{l^2 H} e^\frac{2\varphi}{l} \left( -20a^2 - 16a + Aa + 72 \right)
\]

\[
0 = \left[ -\frac{24 + 6B}{H} + 4e^{-2\varphi_0 b_1} \right] \frac{1}{\eta^4} \\
+ \frac{48}{l^2 H} e^\frac{2\varphi}{l} \left( -a^2 + 1 \right).
\]

Then the solution is

\[
e^{2\varphi_0} = H \left\{ \frac{16}{7} b_1 - \frac{8}{3} (b + b_1) \right\}, \quad B = \frac{89b_1 - 112(b + b_1)}{2b_1 - 28(b + b_1)}
\]

\[
(a, A) = (1, -32) \text{ or } (-1, 68).
\]

Then the metric is given by

\[
ds^2 = -dr^2 + \frac{e^{\pm 2r_{0}l}}{\eta^2} \left( d\eta^2 - \sum_{i=1}^{3} (dx^i)^2 \right).
\]

Here \( \pm \frac{r_{0}l}{l} = \varphi_0 \). The metric of the wall of the brane is

\[
ds_{\text{wall}}^2 = \frac{1}{\eta^2} \left( d\eta^2 - \sum_{i=1}^{3} (dx^i)^2 \right).
\]

If one changes the time variable \( \eta \) by \( \eta = e^{-t} \), we obtain

\[
ds_{\text{wall}}^2 = dt^2 - e^{2t} \sum_{i=1}^{3} (dx^i)^2.
\]

It is nothing but that of de Sitter space, which can be regarded as inflationary universe. It is interesting that Hubble parameter is depending from radial coordinate of 5d AdS Universe. Therefore we have obtained the time dependent solution in the form of warped compactification, which is caused by the quantum correction coming from the boundary QFT. In the above treatment, we have assumed that the wall lies at \( r = r_0 \) since \( f = 1 \) there. We need, however, to check the dynamics of the wall by solving junction
equation coming from the surface counterterm, which should include $W$ in (47) as a quantum correction.

If we assume that the metric has the following form instead of (48),

$$ds^2_5 = -dr^2 + a_1^2(r) a^2(y) \left( dt^2 - dy^2 - \left( dx^1 \right)^2 - \left( dx^2 \right)^2 \right), \tag{62}$$

one finds $e^{2\varphi_0}$ in (54) is

$$e^{2\varphi_0} = -H \left\{ \frac{16}{7} b_1 - \frac{8}{3} (b + b_1) \right\}, \tag{63}$$

instead of (58). Since $\frac{16}{7} b_1 - \frac{8}{3} (b + b_1) > 0$ in most of cases, Eq.(63) seems to be inconsistent. In case of $\frac{16}{7} b_1 - \frac{8}{3} (b + b_1) < 0$, however, one obtains the following metric

$$ds^2 = -dr^2 + \frac{e^{\pm 2(r-r_0)}}{y^2} \left( dt^2 - dy^2 - \left( dx^1 \right)^2 - \left( dx^2 \right)^2 \right). \tag{64}$$

Then the metric of the wall of the brane is given by

$$ds^2_{\text{wall}} = \frac{1}{y^2} \left( dt^2 - dy^2 - \left( dx^1 \right)^2 - \left( dx^2 \right)^2 \right). \tag{65}$$

The metric in (65) is nothing but that of 4d AdS. Hence, one can get 5d AdS Universe with warp scale factor a la Randall-Sundrum where 4d AdS world is generated on the wall. Again, as in section 2 the probability of realization of 4d AdS is less than the one for de Sitter Universe.

Hence, we presented the model where warped RS type scenario may be realized simultaneously with generation of inflationary Universe (or less stable AdS) on the wall. Dynamical 4d gravity is induced from background gravitational field on the boundary. The source for such mechanism is quantum effects due to boundary QFT. It is not quite clear how one can estimate exactly these quantum effects. That is the reason we adopted the phenomenological approach introducing some cut-off, interpolating, fifth coordinate dependent function in such a way that near AdS the theory is described by 5d SG. Far away of AdS, at some fixed radius it is described by anomaly induced effective action of dual 4d QFT. There is, of course, some ambiguity in the choice of this function. However, that may be considered as kind of usual regularization dependence in frames of holographic RG.
5 Discussion

In the present work the dynamical generation of AdS backgrounds in dilatonic gravity with quantum dilaton coupled matter is discussed. The dynamical generation is caused by quantum effects which we incorporate via using the anomaly induced effective action. It is shown that in such a way the 2d dilatonic AdS BH as well as 4d AdS Universe may be created. Hence, via effective action approach the account of quantum corrections gives rise the possibility to create the primordial AdS BHs or primordial regions with negative curvature in the early Universe. Of course, the probability to induce such spaces is normally less than the corresponding one for de Sitter regions or de Sitter-like BHs.

Holographic RG is also considered. The holographic effective action is taken in the intermediate region where it consists of two parts: UV (bulk, i.e. 5d classical gravity) and IR (boundary QFT contribution derived via anomaly induced effective action). These two terms are of the same order. The solution of effective equations suggests that one can realize dynamically (i.e. thanks to quantum effects) 5d AdS Universe with warp scale factor where 4d boundary is inflationary Universe. Less stable boundary 4d AdS world could also occur.

There are different ways to generalize the results of this work. First of all, one can study in detail the properties of dynamically generated AdS backgrounds, say, Hawking radiation in AdS BH, realization of 4d AdS BH, etc. More general backgrounds which are asymptotically AdS ones may be found also but only numerically. Second, it would be interesting to generalize the results of section four to the case of non-constant dilaton. However in that case the role of interpolating, cut-off function should be better understood. Or, another way to introduce the 5d IR effective action due to boundary QFT should be presented. In any case, the introduction of background gravity via anomaly induced effective action on the boundary leads to appearance of dynamical gravity in self-consistent way, via solution of effective equations. Clearly, that such mechanism may be realized also for another dimensions. These questions will be discussed elsewhere.

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