Relativistic Symmetry Suppresses Quark Spin-Orbit Splitting

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Abstract

Experimental data indicate small spin-orbit splittings in hadrons. For heavy-light mesons we identify a relativistic symmetry that suppresses these splittings. We suggest an experimental test in electron-positron annihilation. Furthermore, we argue that the dynamics necessary for this symmetry are possible in QCD.

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I. INTRODUCTION

Recently, Isgur [1] has re-emphasized the experimental fact that spin-orbit splittings in meson and baryon systems, which might be expected to originate from one-gluon-exchange (OGE) effects between quarks, are absent from the observed spectrum. He conjectures that this is due to a fairly precise, but accidental, cancellation between OGE and Thomas precession effects. Taking the point of view that precise cancellations reflect symmetries rather than accidents, we have examined what dynamical requirements would lead to such a result. One of us recently observed [2] that a relativistic symmetry is the origin of pseudospin degeneracies first observed in nuclei more than thirty years ago [3,4]. We find that a close relative of that dynamics can account for the spin degeneracies observed in hadrons composed of one light quark (antiquark) and one heavy antiquark (quark).

Below, we first elucidate the experimental evidence for small spin-orbit splittings. Then we identify the symmetry involved in terms of potentials in the Dirac equation for heavy-light quark systems, and note the relation to the symmetry for pseudospin. We show that the former symmetry predicts that the Dirac spatial wavefunctions in momentum space will be identical for the two states in the doublet, leading to a proposed experimental test. Finally, we argue that the required relation between the potentials may be plausible from known features of QCD.

II. EXPERIMENTAL AND LATTICE QCD SPECTRUM

In the limit where some of the (anti)quarks are infinitely heavy, the angular momentum of the light degrees of freedom, \( j \), is separately conserved [5]. The states can be labelled by \( l_j \), where \( l \) is the orbital angular momentum of the light degrees of freedom. In non-relativistic models of conventional mesons and baryons the splitting between \( l_{+\frac{1}{2}} \) and \( l_{-\frac{1}{2}} \) levels, e.g. the \( p_{\frac{3}{2}} \) and \( p_{\frac{1}{2}} \) or \( d_{\frac{5}{2}} \) and \( d_{\frac{3}{2}} \) levels, can only arise
from spin-orbit interactions \[1\]. The \(p_{\frac{1}{2}}\) level corresponds to two degenerate broad states with different total angular momenta \(J = j \pm s_Q\), where \(s_Q\) is the spin of the heavy (anti)quarks \[5\]. For example in the case of \(D\)-mesons \(s_Q = \frac{1}{2}\) and the two states are called \(D_0^*\) and \(D_1^*\). There are also two degenerate narrow \(p_{\frac{3}{2}}\) states \(D_1\) and \(D_2^*\) \[5\]. The degenerate states separate as one moves slightly away from the heavy quark limit, and their spin-averaged mass remains approximately equal to the mass before separation.

For the \(D\)-mesons, the CLEO collaboration claims a broad \(J^P = 1^+\) state at \(2461_{-34}^{+41} \pm 10 \pm 32\) MeV \[6\], belonging to the \(p_{\frac{1}{2}}\) level, in close vicinity to the \(D_2^*\) at \(2459 \pm 2\) MeV \[7\], belonging to the \(p_{\frac{3}{2}}\) level, indicating a remarkable \(p_{\frac{3}{2}}-p_{\frac{1}{2}}\) spin-orbit degeneracy of \(-2 \pm 50\) MeV. It is appropriate to extract the spin-orbit splitting this way since: Firstly, the charm quark behaves like a heavy quark. Secondly, the difference between the \(D_1^*\) and \(D_2^*\) levels is the best indicator \[8\] of the \(p_{\frac{3}{2}}-p_{\frac{1}{2}}\) splitting in the absence of experimental data on the \(D_0^*\), as opposed to the difference between the \(D_1^*\) and spin-averaged \(p_{\frac{1}{2}}\) level at \(2446 \pm 2\) MeV. Spin-averaged masses are determined from experiment \[7\].

For the \(K\)-mesons, the \(p_{\frac{1}{2}}\) level is at \(1409 \pm 5\) MeV, with \(p_{\frac{3}{2}}\) nearby at \(1371 \pm 3\) MeV, corresponding to a \(p_{\frac{3}{2}}-p_{\frac{1}{2}}\) splitting of \(-38 \pm 6\) MeV. The splitting between the higher-lying \(d_{\frac{1}{2}}\) and \(d_{\frac{3}{2}}\) levels is \(-4 \pm 14\) MeV or \(41 \pm 13\) MeV, depending on how the states are paired into doublets. These results indicate a near spin-orbit degeneracy if the strange quark can be treated as heavy.

For \(B\)-mesons, L3 has performed an analysis, using input from theoretical models and heavy quark effective theory, to determine that the \(p_{\frac{3}{2}}-p_{\frac{1}{2}}\) splitting is \(97 \pm 11\) MeV \[8\]. In the same analysis the mass difference between the \(B_2^*\) and \(B_0^*\), an approximate indicator of the \(p_{\frac{3}{2}}-p_{\frac{1}{2}}\) splitting, is \(110 \pm 11\) MeV. The latter agrees with lattice QCD estimates of \(155_{-13}^{+9} \pm 32\) MeV \[9\] and \(183 \pm 34\) MeV \[10\]. However, according to another estimate \[11\], the splitting is less than \(100\) MeV, and consistent with zero.
In summary, the $p_u^2 - p_d^2$ splitting appears to increase from a negative to a positive value as one increases the mass of the heavy quark. However, results for the bottom quark are not based on a model-independent experiment.

There is also evidence in light quark mesons and baryonic systems that the spin-orbit interaction is small [1]. In non-relativistic models, meson and “two-body” baryon spin-orbit interactions are related and, for a specific class of baryons, the spin-orbit interaction is small for exactly the same reasons that it is small in mesons [1].

III. A DYNAMICAL SYMMETRY FOR THE DIRAC EQUATION

If we consider a system of a (sufficiently) heavy antiquark (quark) and light quark (antiquark), the dynamics may well be represented by the motion of the light quark (antiquark) in a fixed potential provided by the heavy antiquark (quark). Let us assume that both vector and scalar potentials are present. Then the Dirac equation describing the motion of the light quark is

$$H = \tilde{\alpha} \cdot \tilde{p} + \beta(m + V_S) + V_V + M,$$

where we have set $\hbar = c = 1$, $\tilde{\alpha}$, $\beta$ are the usual Dirac matrices, $\tilde{p}$ is the three-momentum, $m$ is the mass of the light quark and $M$ is the mass of the heavy quark.

This one quark Dirac equation follows from the two-body Bethe-Salpeter equation in the equal time approximation, the spectator (Gross) equation with a simple kernel, and a two quark Dirac equation, in the limit that $M$ is large [12–14]. If the vector potential, $V_V(\tilde{r})$, is equal to the scalar potential plus a constant potential, $U$, which is independent of the spatial location of the light quark relative to the heavy one, $V_V(\tilde{r}) = V_S(\tilde{r}) + U$, then the Dirac Hamiltonian is invariant under a spin symmetry [15,16], $[H, \hat{S}_i] = 0$, where the generators of that symmetry are given by,

$$\hat{S}_i = \begin{pmatrix} \hat{s}_i & 0 \\ 0 & \hat{s}_i \end{pmatrix}. $$

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where \( \hat{s}_i = \sigma_i/2 \) are the usual spin generators, \( \sigma_i \) the Pauli matrices, and \( \hat{\tilde{s}}_i = U_p \hat{s}_i U_p \) with \( U_p = \frac{\sigma \cdot \vec{p}}{p} \). Thus Dirac eigenstates can be labeled by the orientation of the spin, even though the system may be highly relativistic, and the eigenstates with different spin orientation will be degenerate.

For spherically symmetric potentials, \( V_V(\vec{r}) = V_V(r) \), \( V_S(\vec{r}) = V_S(r) \), the Dirac Hamiltonian has an additional invariant algebra; namely, the orbital angular momentum,

\[
\hat{L}_i = \begin{pmatrix} \hat{\ell}_i & 0 \\ 0 & \hat{\ell}_i \end{pmatrix},
\]

where \( \hat{\ell}_i = U_p \hat{\ell}_i U_p \) and \( \hat{\ell}_i = \vec{r} \times \vec{p} \). This means that the Dirac eigenstates can be labeled with orbital angular momentum as well as spin, and the states with the same orbital angular momentum projection will be degenerate. Thus, for example, the \( n_r p_{1/2} \) and \( n_r d_{3/2} \) states will be degenerate, where \( n_r \) is the radial quantum number.

Thus, we have identified a symmetry in the heavy-light quark system which produces spin-orbit degeneracies independent of the details of the potential. If this potential is strong, the heavy-light quark system will be very relativistic; that is, the lower component for the light quark will be comparable in magnitude to the upper component of the light quark. It is remarkable that non-relativistic behaviour of energy levels can arise for such fully relativistic systems.

This symmetry is similar to the relativistic symmetry [2] identified as being responsible for pseudospin degeneracies observed in nuclei [3,4]. In contrast to spin symmetry, pseudospin symmetry has the pairs of states \( ((n_r - 1)s_{1/2}, n_r d_{3/2}) \), \( ((n_r - 1)p_{3/2}, n_r f_{5/2}) \), etc. degenerate, making the origin of this symmetry less transparent. The pseudospin generators are

\[
\hat{\tilde{S}}_i = \begin{pmatrix} \hat{\tilde{s}}_i & 0 \\ 0 & \hat{\tilde{s}}_i \end{pmatrix}.
\]

For pseudospin symmetry, the nuclear mean scalar and vector potential must be equal
in magnitude and opposite in sign, up to a constant, $V_V = -V_S + U$. Relativistic mean field representations of the nuclear potential do have this property; that is, $V_S \approx -V_V$ [17,18]. We will return later to the question of whether the relation $V_V = V_S + U$ arises in QCD.

It has previously been observed that pseudospin symmetry improves with increasing energy of the states, for various potentials [2]. A similar behaviour may be expected for spin symmetry, consistent with the experimental observations that spin–orbit splittings decrease for higher mass states [1,7].

The terms in the Dirac equation (1) include the possibility of the vector OGE and vector and scalar Thomas precession spin-dependent terms customarily included in non-relativistic models [1].

**IV. EXPERIMENTAL TEST**

In the spin symmetry limit, the radial wavefunctions of the upper components of the Dirac wavefunction of the two states in the spin doublet will be identical, behaving “non-relativistically”, whereas the lower components will have different radial wavefunctions. This follows from the form of the spin generators given in Equation (2). The $(1,1)$ entry of the operator matrix is simply the non-relativistic spin operator which relates the upper component of the Dirac wavefunction of one state in the doublet to the upper component of the other state in the doublet. Since this operator does not affect the radial wavefunction, the two radial wavefunctions must be the same. By contrast, the lower component wavefunction is operated on by $U_p$ which does operate on the radial wavefunction because of the momentum operator.

As an example, we show in Figure 1 the upper and lower components for Dirac wavefunctions of the $p_{1/2} - p_{3/2}$ doublet in which the scalar and vector potentials were determined by fitting the spectrum of the $K$-mesons. In this realistic case, $V_V \approx V_S + U$, so the radial wavefunctions for the upper components are not exactly
identical but are very close, whereas the radial wavefunctions for the lower components are very different.

Likewise the wavefunctions in momentum space for the upper components will be very similar, as seen in Figure 2, again because the spin operator does not affect the wavefunction. However, since $U_p$ depends only on the angular part of the momentum, $\hat{p} = \frac{\vec{p}}{p}$, it does not affect the radial wavefunction in momentum space. In Figure 2 we see that the radial wavefunctions in momentum space are almost identical for the lower components as well. This prediction of the symmetry can be tested in the following experiment.

The annihilation $e^+e^- \rightarrow D_0^*D_0^*$, $D_0^*D_2^*$ and $D_2^*D_2^*$ allows for the extraction of the $D_0^*$ and $D_2^*$ electromagnetic static form factors and the $D_0^*$ to $D_2^*$ electromagnetic transition form factor. The photon interaction $\gamma_{\mu}$ ensures that all radial wavefunctions of the light quark are accessed, because it acts both diagonally and off-diagonally, and because it does not act on the (infinitely) heavy quark. When spin symmetry is realised, there are only two independent radial momentum space wavefunctions, which should enable the prediction of one of the three form factors in terms of the other two. This should enable the verification of the predictions of spin symmetry. On the other hand, non-relativistic models, with no lower components for the wavefunctions, have only one independent radial wavefunction, which will lead to the prediction of two of the form factors in terms the remaining one. This might be too restrictive. The proposed experiment can be carried out at the Beijing Electron Positron Collider at an energy of approximately 1 GeV above the $\psi(4040)$ peak in the final state $DD\pi\pi$.

An equivalent experiment for K-mesons would involve detection of the $KK\pi\pi$ final state, which has already been measured [19].

If B-mesons also exhibit spin symmetry, one can do equivalent experiments around 1 GeV above the $\Upsilon$ peak at the SLAC, KEK or CESR B-factories.
If such a dynamical symmetry can explain the suppression of spin-orbit splitting in the hadron spectrum, the question remains as to why it might be expected to appear in QCD. To address this, we first recall the ongoing argument as to whether confinement corresponds to a vector or scalar potential [20]. The first natural expectation was that confinement reflected the infrared growth of the QCD coupling constant, enhancing the color-Coulomb interaction at large distances, see e.g. Ref. [21]). An involved two- (or multi-) gluon effect has been proposed [22] to account for the origin of a scalar confining potential.

The existence of one or the other of these vector and scalar potentials is not necessarily exclusionary – they may both be realised. The arguments in Ref. [23]) suggest further that they are related, with the scalar exceeding the vector by an amount which may be approximately constant as one saturates into the linear confining region at large separations. We very briefly reiterate the basic argument of Ref. [23]) here.

The starting point is to accept the standard approach [21] that renormalization-group-improved single-gluon-exchange produces a linearly increasing vector potential between a quark and an antiquark. One then considers what to expect for multiple gluon exchange, starting with two gluons. Since two gluons are attracted to each other in a color singlet channel, and also have a zero mass threshold (as for massless quark-antiquark pairs), it is reasonable to conclude that a (Lorentz and color) scalar gluonic condensate develops, along with a mass gap for a glueball state. These developments are indeed observed in lattice QCD calculations.

Ref. [23]) goes on to argue that renormalization-group-improved single-glueball-exchange involves the square of the QCD coupling and so, despite the massiveness of the object exchanged, also leads to a (now scalar) confining potential between quarks and antiquarks. This further implies that the ratio of the slopes of the two potentials in their common linear (confining) region is given by the square of the ratio
of the QCD scale for growth of the coupling constant to the value of the mass gap of the condensate formation. This ratio may be expected to be of order one as both quantities are determined by the underlying QCD scale.

If the two potentials do indeed have similar slopes in some region, they would necessarily differ only by an approximately constant value, in that region. Thus, the origin of the dynamical symmetry may not be unreasonable, and may indeed be a natural outcome of non-perturbative QCD.

On the other hand, \textit{identically} equal vector and scalar potentials, except for a constant difference, would appear to be coincidental. An ameliorating effect is that to produce an approximation to the spin symmetry of Eq. (2) this condition need only hold in regions where the wavefunctions are substantial.

The determination of QCD potentials, from models like the minimal area law, stochastic vacuum model, or dual QCD, and from lattice QCD, is hampered by the problem of rigorously defining the concept of a potential from QCD when one quark is light. It suffices to say that there is no agreement on the mixed Lorentz character of the potential even between two heavy quarks \cite{24}, where the potential can be rigorously defined, although lattice QCD results are consistent with simply a vector Coulomb and scalar linear potential \cite{25}.

\textbf{VI. SUMMARY}

The observation of “accidental” spin-orbit degeneracies observed in heavy-light quark mesons can be explained by a relativistic symmetry of the Dirac Hamiltonian which occurs when the vector and scalar potentials exerted on the light quark by the heavy antiquark differ approximately by a constant, $V_V \approx V_S + U$. Conversely, spin-orbit degeneracies imply a specific Dirac structure for confinement. The approximate symmetry predicts that the spatial Dirac wavefunction for the spin doublets will be approximately equal in momentum space, a feature which can be tested in
electron-positron annihilation. We have argued that $V_V \approx V_S + U$ may occur in QCD, particularly for regions of space dominated by the light quark wavefunction. The extension of the above to purely light quark systems is not trivial.

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REFERENCES


a) The square of the Dirac radial wavefunction of the upper component times $r^2$. 
b) The square of the Dirac radial wavefunction of the lower component times $r^2$. 

$p_{3/2}$ is the solid red line and $p_{1/2}$ is the dashed blue line. Note that lower component is comparable in magnitude to the upper component.
a) The square of the Dirac momentum spatial wavefunction of the upper component time.

b) The square of the Dirac momentum spatial wavefunction of the lower component times $p_{3/2}$ is the solid red line and $p_{1/2}$ is the dashed blue line.
a) The square of the Dirac momentum spatial wavefunction of the upper component times $q^2$.
b) The square of the Dirac momentum spatial wavefunction of the lower component times $q^2$.
$p_{3/2}$ is the solid red line and $p_{1/2}$ is the dashed blue line.
a) The square of the Dirac radial wavefunction of the upper component times \( r^2 \).
b) The square of the Dirac radial wavefunction of the lower component times \( r^2 \).
\( p_{3/2} \) is the solid red line and \( p_{1/2} \) is the dashed blue line. Note that lower component is comparable in magnitude to the upper component.

Figure 1