High-resolution timing of MIPs with RPCs – a model

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Abstract

Resistive Plate Chambers made with glass and metal electrodes forming accurately spaced gas gaps of a few hundred micrometers have reached timing accuracies below 50 ps σ with efficiencies above 95%. This type of detector, operating at atmospheric pressure with non-flammable gases, seems well suited for large area TOF systems, providing performances comparable to the existing scintillator-based TOF technology but with significantly lower price per channel.

In this work we discuss the principles of operation and describe the performance of several chamber configurations.

1. - Introduction

The recent development, in the framework of the ALICE experiment [1], of timing Resistive Plate Chambers (RPCs) opened the possibility to build large high-resolution TOF arrays at a low cost per channel. Beam tests have shown timing accuracies below 50 ps σ at 99% efficiency for single chamber configurations [2], and a 32 channel prototype has shown an average timing accuracy of 88 ps σ and an average efficiency of 97% [3].

Detailed studies of single gap RPCs [4], aimed to clarify experimentally the origin of the good detection efficiency observed for MIPs in four-gap chambers, have shown that the statistical distribution of the total induced charge per event ($Q_{T(ind)}$) strongly departs from the theoretically expected [5] $1/Q_{T(ind)}$ shape, being compatible with the distributions observed in four-gap chambers.

It was also shown that the avalanche develops under the influence of a strong space charge effect, and that the large observed detection efficiency (up to 45% for a single 0.1 mm gas gap) may not be fully explained by gas ionization alone.

In this paper we will describe the principle and some results of a Monte Carlo model of avalanche development in timing RPCs, including space charge effects and providing a computational framework for the interpretation of the experimental data as a function of the basic detector parameters.

2. – Monte-Carlo model

Efficient (macroscopic) Monte Carlo simulations of RPCs have been already described in the literature [5, 6], based on an analytic model of the avalanche development as a function of the appropriate geometric (Figure 1) and gas swarm

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[†] Part of this work was done at CERN, Geneva, Switzerland.

parameters¹. The stochastic nature of several intervening processes (ionization statistics, avalanche statistics) is taken into account by “randomizing” (see for instance [8]) the model input variables.

Figure 1 – Definition of some main geometric and electric quantities.

In Figure 1 are shown some of the main geometric and electric quantities: g is the width of the gas gap; d the thickness of the resistive plate; ϵ_r the relative dielectric constant of the resistive plate; N_0^j the initial number of electrons in cluster j ; x_0^j the distance from the cathode to the cluster j ; v the electrons drift velocity; x the avalanche development length; $q_e(t)$ the number of electrons in the avalanche as a function of the avalanche development time t ; $i(t)$ the current observed in the external circuit.

2.1 - Analytic model

In order to introduce the space charge effect in a reasonably realistic way we have chosen to include it in the underlying analytic avalanche development model. In fact, the avalanche growth equation for a point-like electron density distribution

$$\frac{d}{dt} q_e(t) = \alpha v q_e(t) \quad (1)$$

modified by the charge dependent first Townsend coefficient defined in Figure 2 has an exact solution given in Table 1.

A more accurate, but considerably heavier, approach would involve the computation of the avalanche development in a non-uniform electric field, influenced by the avalanche’s own space charge [9].

Figure 2 – The variation of the first Townsend coefficient with the avalanche charge ($\alpha(q_e)$) was modeled by specifying (inset) a plausible functional dependency. It remains essentially undisturbed (α_0) for $q_e \ll q_{sat}$ and tends asymptotically to 0 for $q_e \gg q_{sat}$.

Quantity	Exact solution	Small charge limit ($N_0 G \ll q_{sat}$)
Avalanche charge ($0 < t < (g - x_0)/v$)	$q_e(t) = N_0 \frac{W(M e^{\alpha_0 v t + M})}{M}$	$q_e(t) = N_0 e^{\alpha_0 v t}$
Induced electronic charge	$Q_{e(ind)} = \frac{N_0}{\alpha_0(g+d')} \frac{(W(u)+M+2)}{2} \left(\frac{W(u)}{M} - 1 \right)$	$Q_{e(ind)} = \frac{N_0}{\alpha_0(g+d')} (e^{\alpha_0(g-x_0)} - 1)$
Total induced charge	$Q_{(ind)} = N_0 \frac{g}{g+d'} \left(\frac{W(u)}{M} - \frac{x_0}{g} \right)$	$Q_{(ind)} = N_0 \frac{g}{g+d'} \left(G - \frac{x_0}{g} \right)$
Electronic/total induced charges (when $G \gg 1$)	$\frac{Q_{e(ind)}}{Q_{(ind)}} = \frac{W(u)+M+2}{2\alpha_0 g}$	$\frac{Q_{e(ind)}}{Q_{(ind)}} = \frac{1}{\alpha_0 g}$
Auxiliary quantities	$M = N_0 / q_{sat}$; $G_0 = e^{\alpha_0 g}$; $d^* = d / \epsilon_r$	$u = G M e^{-M}$; $G = G_0 e^{-\alpha_0 x_0}$; $W(x) e^{W(x)} = x$

Table 1 - Solution $q_e(t)$ of eq.1 and several important quantities derived from it by well known relations [10]. In the small avalanche limit the general expressions reduce to standard textbook formulas.

The expressions given in Table 1 apply to the amplification of a single cluster (being omitted the cluster index j) and use the Lambert function $W(x)$ [11], defined by $W e^W = x$ [†].

An important model parameter is the quantity G_0 , that corresponds to the gas gain from cathode to anode in the absence of space charge effects.

¹ These parameters can be computed from the microscopic cross-sections by existing programs [7].

[†] Note that for small x , $W(x) \approx x$. The small avalanche limit can be obtained by noting that in most practical cases $M \ll 1$ and if $N_0 G \ll q_{sat}$ then $u \approx G M \ll 1$, so that $W(u)/M \approx G$.

2.2 – Stochastic processes

There are 3 main stochastic processes to be taken into account: a) the variable number k and position x_0^j of the clusters created by the ionizing particles; b) the number N_0^j of electrons released per ionizing collision; c) the variable gain of the avalanche amplification process.

The process a) is taken into account by generating k clusters at random positions in the gap. The number of clusters (k) follows a Poisson distribution with parameter $\langle k \rangle = \lambda g$, being λ the mean number of ionizing collisions per unit length.

For process b) we generate values of N_0^j following the distribution given in [12], but the exact shape of this distribution has little influence in the final results.

For process c) we assume that avalanches initiated by single electrons released at a fixed position in the gap will result in an exponentially distributed final avalanche charge². Since the avalanche gain fluctuations occur mainly in the first few amplification steps we replace the initial number of electrons in the cluster N_0^j by an apparent value

$$N_0^{j*} = \sum_{i=1}^{N_0^j} h, \quad (2)$$

where h is an exponentially distributed random variable with unitary mean value.

Quantities resulting from the development of the avalanches are calculated by summing the appropriate expressions taken from Table 1 over all clusters in the gap. For instance, the total induced charge per event will be given by

$$Q_{T(ind)} = \sum_{j=1}^k Q_{(ind)}(N_0 = N_0^{j*}, x_0 = x_0^j) \quad (3)$$

² Other distributions, like the Polya distribution, were also considered, but no significant influence on the final results was found. The exponential distribution, having a larger relative variance, was taken as the “worst case”.

3. – Results

Two sets of parameters (“scenarios”) that reproduce the observed detection efficiency and charge distributions for single 0.3 mm gap chambers were identified.

The “gas only” scenario corresponds to a rather large cluster density $\lambda = 10/\text{mm}$ (compare with data in [13]) and an extremely large $G_0 = 10^{11}$, being doubtful whether a real avalanche can progress under such extreme conditions without developing a forward streamer [10] and subsequent discharge. Smaller G_0 values could be considered at the expense of a larger λ , which also seems unlikely [6, 13]. To reproduce the efficiency levels observed in single 0.1 mm gap chambers even more extreme values of λ and G_0 would need to be considered.

The “gas + surface” scenario allows for more reasonable values, $\lambda = 5/\text{mm}$ and $G_0 = 10^9$, but the emission of one electron from the cathode with a probability of 60% must be considered. This possibility has also been discussed in [6].

For both scenarios we used $q_{sat} = 1 \text{ pC}$.

Figure 3 – Comparison between the experimental charge distributions measured in a single 0.3 mm gap RPC filled with isobutane and the model-generated distributions of $Q_{T(ind)}$, showing a good agreement in both “scenarios”. The overlaid straight line represents the theoretically expected [5] $1/Q_{T(ind)}$ distribution.

In Figure 3 we compare an experimental charge distribution with the model-generated ones, showing a good agreement for both “scenarios”. It is clear that the smaller avalanches (with $q_e < q_{sat}$) that

populate the distribution close to the origin are not affected by the space charge effect, resulting in a $I/Q_{T(ind)}$ distribution shape there (solid line). However the unaffected spectrum would extend much beyond the Raether limit (around 5 pC) generating an unacceptably large rate of discharges. The space charge effect suppresses the larger avalanches allowing for safe operation at a gain level that would not be otherwise possible.

Additionally, the statistical combination of four identical but independent single gaps³ results in a peaked distribution [4], as observed in four-gap chambers. This would not be the case if the single gap distributions were of the $I/Q_{T(ind)}$ type.

4. – Conclusions

Timing RPCs made with glass and metal electrodes, forming four of accurately spaced gas gaps of 0.3 mm, have reached time resolutions below 50 ps σ with a detection efficiency of 99% for MIPs [2]. A 32-channel prototype equipped with such chambers has shown an average resolution of 88 ps σ with a spread of 9 ps and an average efficiency of 98 % with a spread of 0.5 %. The crosstalk between neighboring channels generally did not exceed 1% [3].

It was observed in single gap chambers that the signal charge distribution departs strongly from the theoretically expected shape and that the gas amplification process seems to be strongly influenced by a space charge effect [4].

A Monte Carlo model of avalanche growth under the influence of a space charge effect was developed and reproduces well the observed charge distributions. It can be concluded that the peaked distributions observed in four-gap chambers (allowing for a very high detection efficiency) is ultimately related to the modification (compression) of the right tail of the single-gap charge distributions by the space charge effect.

However, to reproduce the observed efficiencies either one has to assume unrealistic gas ionization densities or consider the emission of an electron from the cathode with a probability of 60%. It is not

clear at this stage which possibility is to be preferred (see also [6]). Eventually a third mechanism, not yet considered, could be also involved.

5. – Acknowledgements

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6. - References

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³ Quadruple self-convolution of the single gap distributions.

Table captions

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Figure captions

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Figure 2

Figure 3