In the MSSM, cosmological scalar field condensates formed along flat directions of the scalar potential (Affleck-Dine condensates) are typically unstable with respect to formation of Q-balls, a type of non-topological soliton. I discuss the creation and growth of the quantum seed fluctuations which catalyse the collapse of the condensate. In D-term inflation models, the fluctuations of squark fields in the flat directions also give rise to isocurvature density fluctuations stored in the Affleck-Dine condensate. After the condensate breaks up, these can be perturbations in the baryon number, or, in the case where the present neutralino density comes directly from B-ball decay, perturbations in the number of dark matter neutralinos. The latter case results in a large enhancement of the isocurvature perturbation, which should be observable by PLANCK.

1 AD condensate lumps and their fragmentation

The quantum fluctuations of the inflaton field give rise to fluctuations of the energy density which are adiabatic. However, in the minimal supersymmetric standard model (MSSM), or its extensions, the inflaton is not the only fluctuating field. It is well known that the MSSM scalar field potential has many flat directions, along which a non-zero expectation value can form during inflation, leading to a condensate after inflation, the so-called Affleck-Dine (AD) condensate.

An F- and D-flat direction of the MSSM with gravity-mediated SUSY breaking has a scalar potential of the form

\[
U(\Phi) \approx (m^2 - cH^2) \left( 1 + K \log \left( \frac{|\Phi|^2}{M^2} \right) \right) |\Phi|^2 + \frac{\lambda^2 |\Phi|^2 (d-1)}{M^{2(d-3)}} + \frac{A_\lambda \lambda \Phi^d}{dM^{d-3}} + h.c.,
\]

(1)
where \( m \) is the conventional gravity-mediated soft SUSY breaking scalar mass term \( (m \approx 100 \text{ GeV}) \), \( d \) is the dimension of the non-renormalizable term in the superpotential which lifts the flat direction, \( cH^2 \) gives the order \( H^2 \) correction to the scalar mass (with \( c \) positive typically of the order of 1 for AD scalars \(^2\)) and we assume that the natural scale of the non-renormalizable terms is \( M_s \), where \( M_s = M_{Pl}/8\pi \) is the supergravity mass scale. The A-term also receives order \( H \) corrections, \( A_\lambda = A_{\lambda o} + a_\lambda H \), where \( A_{\lambda o} \) is the gravity-mediated soft SUSY breaking term and \( a_\lambda \) depends on the nature of the inflation model; for F-term inflation \( |a_\lambda| \) is typically of the order of \(^2 1\) whilst for minimal D-term inflation models it is zero \(^6\).

The logarithmic correction to the scalar mass term, which occurs along flat directions with Yukawa and gauge interactions, is crucial for the growth of perturbations of the AD field. This growth occurs if \( K < 0 \), which is usually the case for AD scalars with gauge interactions, since \( K \) is dominated by gaugino corrections \(^4\). Typically \( K \approx -(0.1 - 0.01) \).

When the Hubble rate becomes becomes of the order of the curvature of the potential, given by the susy breaking mass \( m_S \), the condensate starts to oscillate. At this stage B-violating terms are comparable to the mass term so that the condensate achieves a net baryonic charge. An important point is that the AD condensate is not stable but typically breaks up into non-topological solitons \(^7\)\(^4\) which carry baryon (and/or lepton) number \(^8\)\(^9\) and are therefore called B-balls (L-balls).

The properties of the B-balls depend on SUSY breaking and on the flat direction along which the AD condensate forms. We will consider SUSY breaking mediated to the observable sector by gravity. In this case the B-balls are unstable but long-lived, decaying well after the electroweak phase transition has taken place \(^4\), with a natural order of magnitude for decay temperature \( T_d \sim \mathcal{O}(1) \text{ GeV} \). This assumes a reheating temperature after inflation, \( T_R \), which is less than about \( 10^4 \) GeV. Such a low value of \( T_R \) can easily be realized in D-term inflation models because these need to have discrete symmetries in order to ensure the flatness of the inflaton potential which can simultaneously lead to a suppression of the reheating temperature \(^10\).

## 2 Fluctuations of the AD field

The AD field \( \Phi = \phi e^{i\theta}/\sqrt{2} \equiv (\phi_1 + i\phi_2)/\sqrt{2} \) is a complex field and, in the D-term inflation models \(^11\), is effectively massless during inflation. Therefore both its modulus and phase are subject to fluctuations with

\[
\delta \phi_\lambda(\vec{x}) = \sqrt{V} \int \frac{d^3k}{(2\pi)^3} e^{-ik \cdot \vec{x}} \delta \phi_\lambda(k),
\]

where

\[
\delta \phi_\lambda(k) = \sqrt{\frac{2}{k^3}} \frac{d^3k}{(2\pi)^3} e^{i\vec{k} \cdot \vec{x}} \delta \phi_\lambda(k).
\]
where $V$ is a normalizing volume and where the power spectrum is the same as for the inflaton field,

$$\frac{k^3|\delta_k|^2}{2\pi^2} = \left(\frac{H_I}{2\pi}\right)^2,$$

where $H_I$ is the value of the Hubble parameter during inflation. One can then find the solution to the linear perturbation equations and use that as a starting point for the non-linear evolution.

Let us consider the evolution of a single spherical condensate lump. Such lumps are described in general by

$$\phi_1(r, t) = A\cos(mt)(1 + \cos(\pi r/r_0))$$

and

$$\phi_2(r, t) = B\sin(mt)(1 + \cos(\pi r/r_0)),$$

for $r \leq r_0$ and by $\phi_{1,2} = 0$ otherwise. The initial radius of the lump is $2r_0$, where $r_0 = \pi/\sqrt{2|K|^{1/2}m}$, and the condensate field equations of motion are given by

$$\ddot{\phi}_i + 3H\dot{\phi}_i - \nabla^2\phi_i = -m^2(1 + K)\phi_i - Km^2\phi_i\log\left(\frac{\phi_1^2 + \phi_2^2}{\phi_0^2}\right).$$

where $\phi_i$ ($i=1,2$) and we may take $H = 0$.

In general, the condensate lump pulsates while charge is flowing out until the lump reaches a (quasi-)equilibrium pseudo-breather configuration, with the lump pulsating with only a small difference between the maximum and minimum field amplitudes. We refer to this state as a Q-axiton. Compared with the natural time scale $t = 1/m$, the time taken to reach the Q-axiton state is long, of the order of $1600/m$ for the case $K = -0.05$ and the condensate is maximally charged, $Q = Q_{\text{max}}$, which is about five expansion times ($H_i^{-1} \approx 300/m$). This is shown in Fig. 1, where we display the oscillation of the field amplitude at the origin as a function of time, as well as the spatial profile of the whole lump as it pulsates.

In Fig. 2 we show the time evolution of the charge and the energy of the whole configuration, integrated out to the distance $8r_0$. At first charge and energy flows out of the volume, but the slow approach to equilibrium can also readily be seen, with the axiton lump starting with initial charge $Q = 1.18\;\text{GeV}^{-2}$ and stabilizing to $Q \approx 1.09\;\text{GeV}^{-2}$.

We have also calculated $f_B$, the ratio of the charge in the equilibrium Q-axiton state to the charge of the initial condensate lump. $f_B$ is found to vary between 0.3 and 0.95, depending on the initial condensate charge.

AD condensate lump evolution has also other interesting features, and Q-ball scattering resembles the scattering of topological solitons. AD condensate formation has also recently been simulated in the lattice.
Figure 1: (a) The time evolution of the field amplitude at the origin, $\phi(0)$; (b) a detail of the early time evolution, showing the pulsation cycles modulating the coherent oscillations (time is in units of $1/m$); (c) the first pulsation of the condensate lump: a) initial lump; b) maximal lump; c) minimal lump; (d) the final equilibrium $Q$-axiton compared with the initial lump ($r$ is in units of $r_0$). In all cases $K = -0.05$ and $Q = Q_{\text{max}}$. 
Figure 2: (a) The time evolution (in units of $1/m$) of the $Q$-axiton energy $E$ and (b) charge $Q$ (in units of $\text{GeV}^{-2}$) for $K = -0.05$ and initial $Q = Q_{\text{max}}$.

3 Density fluctuations

The fluctuations of the condensate phase correspond to fluctuations in the local baryon number density, or isocurvature fluctuations, while the fluctuations of the modulus give rise to adiabatic density fluctuations. For given background values $\theta$ and $\phi$, (with $\theta$ naturally of the order of 1) one finds \(^{(7)}\)

$$\left( \frac{\delta \theta}{\tan(\theta)} \right)_k = \frac{H_I}{\tan(\theta)\phi} = \frac{H_I k^{-3/2}}{\sqrt{2}\tan(\theta)\phi I},$$

where $\phi_I$ is the value of $\phi$ when the perturbation leaves the horizon. The magnitude of the AD field $\Phi$ remains at the non-zero minimum of its potential until $H \simeq m_S$, after which the baryon asymmetry $n_B \propto \sin(\theta)$ forms. Thus the isocurvature fluctuation reads

$$\left( \frac{\delta n_B}{n_B} \right)_k \equiv \delta_B^{(i)} = \left( \frac{\delta \theta}{\tan(\theta)} \right)_k$$

The adiabatic fluctuations of the AD field may dominate over the inflaton fluctuations, with potentially adverse consequences for the scale invariance of the perturbation spectrum, thus imposing an upper bound on the amplitude of the AD field \(^{(16)}\). In the simplest D-term inflation model, the inflaton is coupled to the matter fields $\psi_-$ and $\psi_+$ carrying opposite Fayet-Iliopoulos
charges through a superpotential term \( W = \kappa S \psi_+ \psi^- \). At one loop level the inflaton potential reads

\[
V(S) = V_0 + \frac{g^4 \xi^4}{32 \pi^2} \ln \left( \frac{\kappa^2 S^2}{Q^2} \right); \quad V_0 = \frac{g^2 \xi^4}{2},
\]

where \( \xi \) is the Fayet-Iliopoulos term and \( g \) the gauge coupling associated with it. COBE normalization fixes \( \xi = 6.6 \times 10^{15} \) GeV. In addition, we must consider the contribution of the AD field to the adiabatic perturbation. During inflation, the potential of the \( d = 6 \) flat AD field is simply given by

\[
V(\phi) = \frac{\lambda^2}{32 M^6} \phi^{10}.
\]

Taking both \( S \) and \( \phi \) to be slow rolling fields one finds that the adiabatic part of the invariant perturbation is given by

\[
\zeta = \delta \rho / (\rho + p) = \frac{3}{4} \frac{\delta \rho^{(a)}}{\rho^{(a)}} \propto \frac{V'(\phi) + V''(S)}{V'(\phi)^2 + V''(S)^2} \delta \phi.
\]

Thus the field which dominates the spectral index of the perturbation will be that with the largest value of \( V' \) and \( V'' \).

The index of the power spectrum is given by \( n = 1 + 2 \eta - 6 \epsilon \), where \( \epsilon \) and \( \eta \) are defined as

\[
\epsilon = \frac{1}{2} M^2 \left( \frac{V'}{V} \right)^2, \quad \eta = M^2 \frac{V''}{V}.
\]

The present lower bounds imply \( |\Delta n| \lesssim 0.2 \). It is easy to find out that the condition that the spectral index is acceptably close to scale invariance essentially reduces to the condition that the spectral index is dominated by the inflaton, \( V'(\phi) < V'(S) \) and \( V''(\phi) < V''(S) \). The latter requirement turns out to be slightly more stringent and implies a lower bound on the AD condensate field \( \phi \) with \( \phi \lesssim 0.48 (g/\lambda)^{1/4} (M \xi)^{1/2} \).

As a consequence, there is a lower bound on the isocurvature fluctuation amplitude. Because the B-ball is essentially a squark condensate, in R-parity conserving models its decay produces both baryons and neutralinos \( \chi \), which we assume to be the lightest supersymmetric particles, with \( n_\chi \simeq 3 n_B \). This case is particularly interesting, as the simultaneous production of baryons and neutralinos may help to explain the remarkable similarity of the baryon and dark matter neutralino number densities. Therefore in this scenario the cold dark matter particles can have both isocurvature and adiabatic density fluctuations, resulting in an enhancement of the isocurvature contribution relative to the baryonic case.
One can show that the relative isocurvature contribution is

\[ \beta \equiv \left( \frac{\delta \rho_{\gamma}^{(i)}}{\delta \rho_{\gamma}^{(a)}} \right)^2 = \frac{1}{9} \omega^2 \left( \frac{M^2 V'(S)}{V(S) \tan(\theta) \phi} \right)^2 . \]  \tag{13}

It then follows that the lower limit on \( \beta \) is

\[ \beta \gtrsim 2.5 \times 10^{-2} g^{3/2} \lambda^{1/2} \omega^2 \tan(\tilde{\theta})^{-2} . \] \tag{14}

Thus significant isocurvature fluctuations are a definite prediction of the AD mechanism. When the polarization data is included, detecting isocurvature fluctuations at the level of \( \beta \sim 10^{-4} \) should be quite realistic at Planck. Thus the forthcoming CMB experiments offer a test not only of the inflationary Universe but also of AD baryogenesis.

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References