FeynHiggsFast is a Fortran code for the calculation of the masses and the mixing angles of the neutral CP-even Higgs bosons in the MSSM up to two-loop order. It is based on a compact analytical approximation formula of the complete diagrammatic result in the MSSM up to two-loop order. It is available for arbitrary choices of the parameters in the MSSM sector.

Abstract

in the Higgs sector of the MSSM

FeynHiggsFast: a program for a fast calculation

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1 Introduction

One of the most striking phenomenological implications of Supersymmetry (SUSY) is the prediction of a relatively light Higgs boson, which is common to all supersymmetric models whose couplings remain in the perturbative regime up to a very high energy scale \([1]\). The search for the lightest Higgs boson thus allows a crucial test of SUSY and especially the Minimal Supersymmetric Standard Model (MSSM) \([2]\). Therefore it is one of the main goals at the present and the next generation of colliders. A precise knowledge of the dependence of the masses and mixing angles of the Higgs sector of the MSSM on the relevant SUSY parameters is necessary for a detailed analysis of SUSY phenomenology at LEP2, the upgraded Tevatron, and also for the LHC and a future linear \(e^+e^-\) collider, where high-precision measurements in the Higgs sector might become possible.

The mass of the lightest \(CP\)-even MSSM Higgs boson, \(m_h\), is bounded from above by about \(m_h \lesssim 135\) GeV, including radiative corrections up to the two-loop level \([3, 7]\). The mixing angle \(\alpha_{\text{eff}}\) that diagonalizes the neutral \(CP\)-even Higgs sector receives the same kind of corrections. By incorporating \(\alpha_{\text{eff}}\) into the Higgs decay widths, the leading electroweak corrections to the decay of the neutral \(CP\)-even Higgs bosons are taken into account.

In this note we present the Fortran code \textit{FeynHiggsFast}. Using low energy MSSM parameters as input, it evaluates the masses of the neutral \(CP\)-even Higgs bosons, \(m_h\) and \(m_H\), as well as the corresponding mixing angle, \(\alpha_{\text{eff}}\), at the two-loop level. In addition the mass of the charged Higgs boson, \(m_{H^\pm}\), is evaluated at the one-loop level. The \(\rho\)-parameter, leading to constraints in the scalar fermion sector of the MSSM, is evaluated up to \(O(\alpha_s)\), taking into account the gluon exchange contribution at the two-loop level \([8]\). \textit{FeynHiggsFast} is based on a compact analytical approximation formula, containing at the two-loop level the dominant corrections in \(O(\alpha_s)\) obtained in the Feynman-diagrammatic (FD) approach \([4]\) and subdominant corrections of \(O(G_F^2m_t^6)\) obtained with renormalization group (RG) methods \([5, 7]\). In comparison with the FD result, consisting of the complete one-loop and the two-loop contributions as given in Ref. \([4]\) (incorporated into the Fortran code \textit{FeynHiggs} \([9]\)), the approximation formula is much shorter. Thus \textit{FeynHiggsFast} is about \(3 \times 10^4\) times faster than \textit{FeynHiggs}. Agreement between these two codes of better than 2 GeV is found for most parts of the MSSM parameter space.

The following sections are organized as follows: In Sect. 2 we shortly summarize the analytical approximation formula and perform a comparison with the full FD result. A description of how to use \textit{FeynHiggsFast} is given in Sect. 3. The conclusions can be found in Sect. 4.

2 The analytical approximation

In order to fix our notations, we first list the conventions for the MSSM scalar top sector: the mass matrix in the basis of the current eigenstates \(\tilde{t}_L\) and \(\tilde{t}_R\) is given by (neglecting
contributions $\sim M_t^2$):

$$M_i^2 = \begin{pmatrix} M_{i_L}^2 + m_i^2 & m_i X_i \\ m_i X_i & M_{i_R}^2 + m_i^2 \end{pmatrix},$$

where

$$m_i X_i = m_i (A_t - \mu \cot \beta).$$

Furtheron the following simplification is used:

$$M_{i_L} = M_{i_R} := m_i.$$

In this simplified case we define:

$$M_2^2 := m_{\tilde{g}}^2 + m_t^2.$$

For further details see Ref. [10].

At the tree level the mass matrix of the neutral $CP$-even Higgs bosons in the $\phi_1, \phi_2$ basis can be expressed as follows:

$$M_{\text{Higgs}}^{2,\text{tree}} = \begin{pmatrix} m_{\phi_1}^2 & m_{\phi_1 \phi_2} \\ m_{\phi_1 \phi_2} & m_{\phi_2}^2 \end{pmatrix} = \begin{pmatrix} M_A^2 \sin^2 \beta + M_Z^2 \cos^2 \beta & -(M_A^2 + M_Z^2) \sin \beta \cos \beta \\ -(M_A^2 + M_Z^2) \sin \beta \cos \beta & M_A^2 \cos^2 \beta + M_Z^2 \sin^2 \beta \end{pmatrix}.$$  

$M_A$ denotes the mass of the $CP$-odd Higgs boson, $M_Z$ is the mass of the Z boson, and $\tan \beta = v_2/v_1$ is the ratio of the two vacuum expectation values of the two Higgs doublets in the MSSM, see Ref. [11].

At higher orders, the Higgs mass matrix (5) is supplemented by the renormalized self-energies $\Sigma_i(q^2), \ s = \phi_1, \phi_2, \phi_1 \phi_2$. Here we use the approximation of neglected external momentum: $\Sigma_i \equiv \Sigma_i(0)$. The masses $m_h$ and $m_H$ are then obtained by diagonalizing the higher order corrected mass matrix with the mixing angle $\alpha_{\text{eff}}$:

$$\begin{pmatrix} m_{\phi_1}^2 - \Sigma_{\phi_1} & m_{\phi_1 \phi_2} - \Sigma_{\phi_1 \phi_2} \\ m_{\phi_1 \phi_2} - \Sigma_{\phi_1 \phi_2} & m_{\phi_2}^2 - \Sigma_{\phi_2} \end{pmatrix} \xrightarrow{\alpha_{\text{eff}}} \begin{pmatrix} m_H^2 & 0 \\ 0 & m_h^2 \end{pmatrix},$$

$$\alpha_{\text{eff}} = \arctan \left[ \frac{-(M_A^2 + M_Z^2) \sin \beta \cos \beta - \Sigma_{\phi_1 \phi_2}}{M_Z^2 \cos^2 \beta + M_A^2 \sin^2 \beta - \Sigma_{\phi_1} - m_h^2} \right], \quad -\frac{\pi}{2} < \alpha_{\text{eff}} < \frac{\pi}{2}.$$  

Here we only give a very brief outline of the calculation of the renormalized Higgs boson self-energies. A detailed description can be found in Ref. [10].

The mass of the lightest Higgs boson receives contributions from all sectors of the MSSM, but not all are numerically of equal relevance. The dominant corrections arise from the $t - \tilde{t}$ sector of the MSSM: The results for the renormalized self-energies $\Sigma_i$ have been derived analytically in the FD approach in Refs. [12,13]. These results, however, are rather lengthy. In order to derive a compact analytical expression, several approximations have been made.

1 The more general case with $M_{i_L} \neq M_{i_R}$ can be found in eq. (10).
as described in Ref. [10]. The main step of the approximation consists of a Taylor expansion in

\[ \Delta_i = \frac{|m_t X_i|}{M_S^2} \quad (8) \]

of the \( \hat{\Sigma}_i(0) \). For the one-loop correction the expansion has been performed up to \( \mathcal{O}(\Delta_i^2) \); all three renormalized one-loop Higgs-boson self-energies give a contribution. Concerning the two-loop self-energies, the expansion has been carried out up to \( \mathcal{O}(\Delta_i^3) \); in the approximation considered here only \( \hat{\Sigma}_{i(2)}^{(2)} \) gives a non-zero contribution.

Leading contributions beyond \( \mathcal{O}(\alpha\alpha_s) \) have been taken into account by incorporating the leading two-loop Yukawa correction of \( \mathcal{O}(G_F m_t^6) \) [11]. Furthermore the result has been expressed in terms of the \( \overline{\text{MS}} \) top-quark mass

\[ \overline{m}_t = m_t(m_t) \approx \frac{m_t}{1 + \frac{4}{3\pi \alpha_s(m_t)}} \quad (9) \]

instead of the pole mass \( m_t \). This leads to an additional contribution in \( \mathcal{O}(\alpha\alpha_s^2) \).

The analytical expressions arising from the \( t - \bar{t} \) sector are given at the one-loop level as follows:

\[ \hat{\Sigma}_{\phi_1}^{(1)}(0) = \frac{G_F \sqrt{2}}{\pi^2} \frac{\Lambda}{M^2} \cos^2 \beta \log \left( \frac{\overline{m}_t^2}{M_S^2} \right), \quad (10) \]

\[ \hat{\Sigma}_{\phi_1 \phi_2}^{(1)}(0) = -\frac{G_F \sqrt{2}}{\pi^2} \frac{X_6}{M_S^2} \cot \beta \left[ -\frac{3}{8} \frac{M^2_t}{m_t^2} + \frac{5}{8} \Lambda \sin^2 \beta \right] \log \left( \frac{\overline{m}_t^2}{M_S^2} \right), \quad (11) \]

\[ \hat{\Sigma}_{\phi_2}^{(1)}(0) = \frac{G_F \sqrt{2}}{\pi^2} \frac{X_6}{8 \sin^2 \beta} \left\{ -2 \frac{M^2_S}{m_t^2} + \frac{11}{10} \frac{M^4_S}{m_t^4} \right. \]

\[ + \left[ 12 - \frac{6}{m_t^2} \frac{M^2_S}{m_t^2} \sin^2 \beta + \frac{8}{m_t^2} \frac{M^4_S}{m_t^4} \Lambda \sin^4 \beta \right] \log \left( \frac{\overline{m}_t^2}{M_S^2} \right) \]

\[ + \left[ X_6 \overline{m}_t^2 \left[ -12 + \frac{12}{m_t^2} \frac{M^2_S}{m_t^2} + \frac{6}{m_t^2} \right] + \frac{X_6}{M_S^2} \left[ -4 \frac{m_t^2}{M_S^2} + 3 \overline{m}_t^2 \right] \right. \]

\[ + \left. \frac{X_6}{M_S^2} \left[ 3 \frac{m_t^4}{M_S^4} - 12 \frac{m_t^6}{M_S^4} + \frac{2}{M_S^4} \right] \right\}, \quad (12) \]

with

\[ \Lambda = \left( \frac{1}{8} - \frac{1}{3}s_w^2 + \frac{4}{9}s_w^4 \right), \quad s_w^2 = 1 - \frac{M^2_W}{M^2_Z}. \quad (13) \]

The two-loop contributions in \( \mathcal{O}(\alpha\alpha_s) \) read:

\[ \hat{\Sigma}_{\phi_1}^{(2)}(0) = 0, \]

\[ \hat{\Sigma}_{\phi_1 \phi_2}^{(2)}(0) = 0, \]

3
\[\hat{\Sigma}_{\phi_2}^{(2)}(0) = \frac{G_F \sqrt{2} \alpha_s}{\pi^2} \frac{m_i^4}{\pi \sin^2 \beta} \left[ 4 + 3 \log^2 \left( \frac{m_i}{M_S^2} \right) + 2 \log \left( \frac{m_i^2}{M_S^2} \right) - 6 \frac{X_i}{M_S} - \frac{X_i^2}{M_S^2} \right], \tag{14}\]

The two-loop Yukawa correction in this approximation reads:
\[\hat{\Sigma}_{\phi_2}^{(2)}(0) = -\frac{9}{16 \pi^4} \frac{G_F^2 m_i^6}{\sin^2 \beta} \left[ \log^2 \left( \frac{m_i}{M_S^2} \right) - \frac{2 X_i^2}{M_S^2} \log \left( \frac{m_i^2}{M_S^2} \right) + \frac{1}{6} X_i^4 \log \left( \frac{m_i^2}{M_S^2} \right) \right]. \tag{15}\]

\(M_S\) has to be chosen according to
\[M_S = \begin{cases} \sqrt{m_i^2 + m_i^4} & : M_{i_L} = M_{i_R} = m_i \\ \left[ M_{i_L}^2 M_{i_R}^2 + m_i^2 (M_{i_L}^2 + M_{i_R}^2) + m_i^4 \right]^{1/2} & : M_{i_L} \neq M_{i_R} \end{cases} \tag{16}\]

For the one-loop corrections from the other sectors of the MSSM the logarithmic approximation given in Ref. [21] has been used for \textit{FeynHiggsFast}, see also Ref. [10]. In order to obtain the radiatively corrected Higgs boson masses and the mixing angle, the renormalized Higgs boson self-energies have to be inserted into eq. (12), and the corresponding diagonalization has to be performed.

We have also implemented the result of the MSSM contributions to \(\Delta \rho\) [3, 13]. Here the corrections arising from \(t\bar{b}\)-loops up to \(O(\alpha_s)\) have been taken into account. The result is valid for arbitrary parameters in the \(t\)- and \(b\)-sector, also taking into account the mixing in the \(b\)-sector which can have a non-negligible effect in the large \(\tan \beta\) scenario [13].

The two-loop result can be separated into the pure gluon-exchange contribution, which can be expressed by a very compact formula, allowing a very fast evaluation, and the pure gluino-exchange contribution, which is given by a rather lengthy expression. The latter correction goes to zero with increasing gluino mass and can thus be discarded for a heavy gluino. Concerning the implementation of the \(\rho\)-parameter into \textit{FeynHiggsFast}, we have neglected the gluino exchange contribution. The \(\rho\)-parameter can be used as an additional constraint (besides the experimental bounds) on the squark masses. A value of \(\Delta \rho\) outside the experimentally preferred region of \(\Delta \rho_{\text{SUSY}} \approx 10^{-3}\) [14] indicates experimentally disfavored \(t\)- and \(b\)-masses.

For illustration of the quality of the compact approximation, we compare \(m_h\) and \(\alpha_{\text{eff}}\) with the results from the complete Feynman-diagrammatic calculation. The FD calculation contains the full diagrammatic one-loop contribution [13], the complete leading two-loop corrections in \(O(\alpha_s)\) [13, 16], and the two contributions beyond \(O(\alpha_s)\) (see eq. (2) and eq. (13)) without approximation.

For the Standard Model parameters we use \(M_Z = 91.187\) GeV, \(M_W = 80.39\) GeV, \(G_F = 1.16639 \times 10^{-5}\) GeV\(^{-2}\), \(\alpha_s(m_t) = 0.1095\), and \(m_t = 175\) GeV. In the numerical evaluation we have furthermore chosen the trilinear couplings in the scalar top and bottom sector to be \(A_b = A_t\). This fixes together with the choice of \(\mu\) (the Higgs mixing parameter) the mixing
Figure 1: $m_h$ as a function of $m_{\tilde{q}}$, calculated from the full formula and from the approximation formula for $M_A = 500$ GeV, $m_{\tilde{g}} = 500$ GeV and $\tan \beta = 1.6$ or 40.

Figure 2: $m_h$ as a function of $M_A$, calculated from the full formula and from the approximation formula for $m_{\tilde{q}} = 1000$ GeV, $m_{\tilde{g}} = 500$ GeV and $\tan \beta = 1.6$ or 40.

Figure 3: The relative difference $(\sin \alpha_{\text{eff}} - \sin \alpha_{\text{eff}}(\text{approx}))/\sin \alpha_{\text{eff}}$ is shown as a function of $M_A$ for three values of $\tan \beta$ in the no mixing and the maximal mixing scenario. The other parameters are $\mu = -100$ GeV, $M = m_{\tilde{q}}$, $m_{\tilde{g}} = 500$ GeV and $A_t = A_i$. 
in the scalar bottom sector. The parameter $M$ appearing in the plots is the $SU(2)$ gaugino mass parameter, it enters as an independent parameter in the full result only, see Ref. [9].

In Refs. [1], [2] it has been shown that the lightest Higgs boson mass $m_h$ as a function of $X_t$ reaches a maximum at around $|X_t/m_t| \approx 2$. This case we refer to as 'maximal mixing'. A minimum of $m_h$ is reached for $X_t \approx 0$, which we refer to as 'no mixing'.

Fig. 1 displays the dependence of $m_h$ on $m_t$ for the cases of no mixing and maximal mixing, and we have set $M_A = 500$ GeV. For tan $\beta$ we have chosen two typical values: tan $\beta = 1.6$ as a low and tan $\beta = 40$ as a typical high value. Very good agreement is found in the no-mixing scenario as well as in the maximal-mixing scenario, the deviations lie below 2 GeV.

The dependence on $M_A$ is shown in Fig. 2. The quality of the approximation is typically better than 1 GeV for the no-mixing case and better than 2 GeV for the maximal-mixing case. Only for very small (and experimentally already excluded) values of $M_A$ a deviation of 5 GeV occurs. The peaks in the plot for tan $\beta = 1.6$ in the full result are due to the threshold $M_A = 2 m_t$ in the one-loop contribution, originating from the top-loop diagram in the $A$ self-energy. This peak does not occur in the approximation formula (where the momentum dependence of the $A$ self-energy has been neglected) and can thus lead to a larger deviation around the threshold.

In Fig. 3 we display the quality of the short analytical approximation formula for the effective mixing angle. The relative difference between $\alpha_{\text{eff}}$ obtained from the full calculation (i.e. calculating the renormalized Higgs self-energies $\Sigma_A(0)$ without any approximation) and the angle obtained with the help of the approximated Higgs self-energies, denoted as $\alpha_{\text{eff(approx)}}$ is shown as a function of $M_A$. We use three values of tan $\beta$, tan $\beta = 3, 20, 40$. Sizable deviations occur only in the region $100$ GeV $\leq M_A \leq 150$ GeV, especially for large tan $\beta$. In this region of parameter space the values of $m_h$ and $m_H$ are very close to each other. This results in a high sensitivity to small deviations in the Higgs boson self-energies entering the Higgs-boson mass matrix (8), (7). Otherwise the relative difference stays below 3%.

3 The Fortran program **FeynHiggsFast**

The complete program *FeynHiggsFast* consists of about 1300 lines Fortran code. The executable file fills about 65 KB disk space. The calculation for one set of parameters, including the $\Delta \rho$ constraint, takes about $2 \times 10^{-5}$ seconds on a Sigma station (Alpha processor, 600 MHz processing speed, 512 MB RAM). The program can be obtained from the *FeynHiggs* home page:

http://www-itp.physik.uni-karlsruhe.de/feynhiggs

Here the code is available, together with a short instruction, information about bug fixes etc.

*FeynHiggsFast* consists of a front-end (program *FeynHiggsFast*) and the main part where the evaluation is performed (starting with subroutine *feynhiggsfastsub*). The front-end can be manipulated by the user at will, whereas the main part should not be changed. In this way *FeynHiggsFast* can be accommodated as a subroutine to existing
programs\textsuperscript{2}, thus providing an extremely fast and for many purposes sufficiently accurate evaluation for the masses and mixing angles in the MSSM Higgs sector.

The input of \textit{FeynHiggsFast} are the low energy SUSY parameters, listed in Tab. \ref{tab:input}. Concerning the $t$-sector, the user has the option to enter either the physical parameters, i.e. the masses and the mixing angle ($m_{\tilde{t}_1}$, $m_{\tilde{t}_2}$ and $\sin \theta_t$) or the unphysical parameters $M_{\tilde{t}_L}$, $M_{\tilde{t}_R}$ and $X_t$. From these input parameters \textit{FeynHiggsFast} calculates the masses and the mixing angle of the neutral $\mathcal{C}\mathcal{P}$-even Higgs sector, as well as the mass of the charged Higgs boson and the $\rho$-parameter.

<table>
<thead>
<tr>
<th>input for \textit{FeynHiggsFast}</th>
<th>MSSM expr.</th>
<th>internal expr. in \textit{FeynHiggsFast}</th>
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<tr>
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</table>

Table 1: The meaning of the different MSSM variables which have to be entered into \textit{FeynHiggsFast}.

4 Conclusions

\textit{FeynHiggsFast} is a Fortran code for the calculation of masses and mixing angles in the Higgs sector of the MSSM. In addition it evaluates the SUSY corrections to the $\rho$-parameter, arising from the scalar top and bottom sector. Concerning the evaluation in the neutral $\mathcal{C}\mathcal{P}$-even Higgs sector, \textit{FeynHiggsFast} is based on a simple analytic formula, derived from a more complete result obtained in the Feynman-diagrammatic approach. Beyond the one-loop level, \textit{FeynHiggsFast} contains the dominant corrections in $\mathcal{O}(\alpha_s)$ and further subdominant contributions. The accuracy of the approximation compared to the full result is better than 2 GeV for most parts of the MSSM parameter space.

\textsuperscript{2} This has been carried out, for example, for the program HDECAY \textsuperscript{16,17}, into which \textit{FeynHiggsFast} has been incorporated as a subroutine.
The program is available via the WWW page

http://www-physik.uni-karlsruhe.de/~feynhiggs.

The code consists of a front-end and a subroutine. The front-end can be manipulated at the user’s will. By accommodating the front-end, *FeynHiggsFast* can serve as a subroutine to existing programs, thus providing an extremely fast and reasonably accurate evaluation of the masses and mixing angles in the MSSM Higgs sector. These can then be used as inputs for further computations.

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**References**


