$|V_{ub}|$ and $|V_{cb}|$, Charm Counting and Lifetime Differences in Inclusive Bottom Hadron Decays

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Abstract

Inclusive bottom hadron decays are analyzed based on the heavy quark effective field theory (HQEFT). Special attentions in this paper are paid to the $b \to u$ transitions and nonspectator effects. As a consequence, the CKM quark mixing matrix elements $|V_{ub}|$ and $|V_{cb}|$ are reliably extracted from the inclusive semileptonic decays $B \to X_u e \nu$ and $B \to X_c e \nu$. Various observables, such as the semileptonic branch ratio $B_{SL}$, the lifetime differences among $B^-, B^0, B_s$ and $\Lambda_b$ hadrons, the charm counting $n_c$, are predicted and found to be consistent with the present experimental data.

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I. INTRODUCTION

Recently, the heavy quark effective field theory (HQEFT) with keeping both quark and antiquark fields [1] have been investigated and applied to both exclusive [2] and inclusive decays [3] of heavy hadrons. It has been seen that the contributions and effects from the antiquark field can play a significant role for certain physical observables and are also necessary to be considered from the point of view of quantum field theory. Consequently, in this new framework of HQEFT, one can arrive at a consistent description on both exclusive and inclusive decays of heavy hadrons. For instance, at zero recoil, 1/$m_Q$ corrections in both exclusive and inclusive decays are automatically absent when the physical observables are presented in terms of heavy hadron masses\(^1\) $m_H$ [2,3]. Our basic point of the considerations is based on the fact that a single heavy quark within hadron is off-mass shell by amount of binding energy $\bar{\Lambda}$. Thus a more reliable heavy quark expansion should be carried out in terms of the so-called "dressed heavy quark" mass

$$\hat{m}_Q \equiv \lim_{m_Q \to \infty} m_H = m_Q + \bar{\Lambda}$$

with $m_H$ the heavy hadron mass. The new framework of HQEFT developed in [1-3] enables us to describe a slightly off-mass shell heavy quark within hadrons. Thus such an HQEFT is expected to provide a reliable way to determine the CKM matrix elements $|V_{cb}|$ and $|V_{ub}|$ as well as to explain the lifetime differences of heavy hadrons.

In this paper we are going to investigate the inclusive bottom hadron decays and will mainly pay attention to the analysis on the nonspectator effects and $b \to u$ transitions within the framework of HQEFT. One can extract $|V_{cb}|$ either from the end point of the

\(^1\)Note that in the usual heavy quark expansion or in the expansion based on the usual heavy quark effective theory (HQET), 1/$m_Q$ corrections in the inclusive decays are absent only when the inclusive decay rate is presented in terms of heavy quark mass ($m_Q$) rather than the heavy hadron mass ($m_H$), the situation seems to be conflict with the case in the exclusive decays where the normalization is given in term of heavy hadron mass. Such an inconsistency may be the reason that leads to the difficulty for understanding the lifetime differences among the bottom hadrons.
differential decay rate of exclusive $B \rightarrow D(D^*)$ decays or from the total decay rate of inclusive semileptonic decays. By including nonspectator effects considered in this paper, we will present more reliable results for most of the interesting quantities, such as, $|V_{ub}|$ and $|V_{cb}|$, charm counting $n_c$ and lifetime differences among $B^−, B^0, B_s, \Lambda_b$ hadrons.

In general, it is not so favorable to extract, in comparison with the extraction of $|V_{cb}|$, the CKM matrix element $|V_{ub}|$ due to either experimental or theoretical reasons. In experimental side, there exists an overwhelming background of $b \rightarrow c\ell\bar{\nu}$ decays, its magnitude could be as large as about two orders. Even a small leakage of the measurement for $b \rightarrow c\ell\bar{\nu}$ decays could affect the identification of $b \rightarrow u\ell\bar{\nu}$ decays to a large extent. Moreover, there still have other background sources such as nonleptonic $b$ decays to charmed hadrons which can undergo a semileptonic decay and lead to a leptonic misidentification. Thus it may result in a large experimental error when extracting $|V_{ub}|$ directly from the total decay rate. Another method of determining $|V_{ub}|$ is from the lepton spectrum at the slice where the energy of the electron $E_e$ is higher than $(M_B^2 - M_D^2)/2M_B \approx 2.31$ GeV, since this region can only arise from $b \rightarrow u\ell\bar{\nu}$ decay. However, only 10% of the total identified events of $b \rightarrow u\ell\bar{\nu}$ decays is contained in such a region. Moreover, in theoretical side, as such a region lies at the high end point, the bound state effect as well as hadronization cannot be neglected in that region. Luckily, recent new developments enable one to extract the events of $b \rightarrow u$ transitions from the dominant $b \rightarrow c$ background. The basic point is to use the invariant hadronic mass spectrum in the final state, i.e., $s_H = (P_b - q)^2$ with $q$ being the momentum of the lepton pair [5,6]. In $b \rightarrow u$ semileptonic inclusive decays, more than 90% events lie in the arrange $s_H < m_c^2$. ALEPH collaboration has used this method to identify the $b \rightarrow u$ events and reported interesting results for the $b \rightarrow u\nu_e$ decay [7].

Another important issue is the nonspectator effects in the bottom hadron decays. In the usual heavy quark effective theory (HQET), those are the main effects which could result in lifetime differences among different bottom hadrons $B^0, B^0_s$ and $\Lambda_b$. Nevertheless, the effects were found to be only less than 5% of the total decay rates and seem to be too small to explain the experimental data.

Our paper is organized as follows: In section 2 we derive the general formalism for the
total decay widths of $b \to u(c)$ transitions in the framework of HQEFT and also the one from nonspectator effects. In section 3 we provide the numerical results of $|V_{ub}|$ and $|V_{cb}|$ ($|V_{ub}|/|V_{cb}|$) from $b \to u\ell\bar{\nu}$ and $b \to c\ell\bar{\nu}$ semileptonic decays, the charm counting $n_c$ with including the nonspectator effects and the results of the ratios

$$r_{uu} = \frac{\mathcal{B}(b \to u\bar{u}d')}{\mathcal{B}(b \to u\tau\bar{\nu})}, \quad r_{uu} = \frac{\mathcal{B}(b \to u\bar{u}d')}{\mathcal{B}(b \to u\tau\bar{\nu})}, \quad r_{cu} = \frac{\mathcal{B}(b \to u\bar{c}s')}{\mathcal{B}(b \to u\ell\bar{\nu})},$$

(1.1)

with $s' = sV_{cs} + dV_{cd}$ and $d' = sV_{us} + dV_{ud}$. We also present, to a good approximation, a simplified analytic expression for both $|V_{ub}|$ and $|V_{cb}|$ as functions of fundamental parameters $\alpha_s(\mu)$ and $m_c$ as well as $\kappa_1$, which may be useful for phenomenological analyses. Our conclusions and remarks are presented in the last section.

II. $B \to U(C)$ TRANSITIONS AND NONSPECTATOR EFFECTS

The decays of bottom hadrons are mediated by the following effective weak Lagrangian renormalized at the scale $\mu = m_b$

$$\mathcal{L}_{\text{eff}} = -\frac{4G_F}{\sqrt{2}} \sum_{q=u,c} V_{qb} \left\{ c_1(m_b) \left[ \bar{d}_L^q \gamma_\mu u_L \bar{q}_L \gamma^\mu b_L + \bar{s}_L^q \gamma_\mu c_L \bar{q}_L \gamma^\mu b_L \right] \\
+ c_2(m_b) \left[ \bar{q}_L \gamma_\mu u_L \bar{d}_L^q \gamma^\mu b_L + \bar{q}_L \gamma_\mu c_L \bar{s}_L^q \gamma^\mu b_L \right] \\
+ \sum_{\ell=e,\mu,\tau} \bar{\ell}_L \gamma_\mu \nu_\ell \bar{q}_L \gamma^\mu b_L \right\} + \text{h.c.},$$

(2.1)

where $q_L = \frac{1}{2}(1 - \gamma_5)q$ denotes a left-handed quark field, and $d' = dV_{ud} + sV_{us}$ and $s' = sV_{cs} + dV_{cd}$. The Wilson coefficients $c_1$ and $c_2$ at leading-order are

$$c_1 = \frac{1}{2}(c_+ + c_-), \quad c_2 = \frac{1}{2}(c_+ - c_-),$$

(2.2a)

$$c_\pm(m_b) = \left( \frac{\alpha_s(m_W)}{\alpha_s(m_b)} \right)^{a_\pm}, \quad a_- = -2a_+ = -\frac{12}{33 - 2n_f}. \quad (2.2b)$$

Due to optical theorem, the inclusive decay width of bottom hadrons may be expressed as the absorptive part of the forward scattering amplitude of bottom hadron $H_b$

$$\Gamma(H_b \to X) = \frac{1}{2m_{H_b}} \text{Im} \left( i \int d^4x < H_b | T \left\{ \mathcal{L}_{\text{eff}}^{(X)}(x), \mathcal{L}_{\text{eff}}^{(X)}(0) \right\} | H_b > \right),$$

(2.3)

where $\mathcal{L}_{\text{eff}}^{(X)}(x)$ is the part of the complete $\Delta B = 1$ effective Lagrangian which contributes to the particular inclusive final state $X$ under consideration. Because the energy released
in the process is rather large, one may calculate the inclusive decay width with operator product expansion. For non-perturbative part, one can use heavy quark expansion based on the HQEFT, which has been shown to be reliable for $b \to c$ transitions [3] and is expected to be applicable for $b \to u$ transitions.

**A. $b \to u(c)$ Inclusive Decays**

It is interesting to note that up to the $1/m_b^2$ order the inclusive decay width of bottom hadrons only depends on two matrix elements

\[
A = \frac{1}{6m_{H_b}} < H_b | \bar{b} e^{-im_b \not{v} \cdot x} (iD_{\perp})^2 e^{im_b \not{v} \cdot x} b | H_b >,
\]

\[
N_b = \frac{1}{12m_{H_b}} < H_b | \bar{b} g \sigma_{\mu \nu} G^{\mu \nu} b | H_b >,
\]

with $g$ the QCD coupling constant. The decay width for $b \to c$ transitions can be written in the following general form

\[
\Gamma(H_b \to c + X) = \hat{\Gamma}_0^c \eta_{cX}\{I_0(\rho, \rho_X, \hat{\rho}) + I_1(\rho, \rho_X, \hat{\rho}) A + I_2(\rho, \rho_X, \hat{\rho}) N_b\},
\]

where the functions $\eta_{cX}$ arise from QCD radiative corrections. $\hat{\Gamma}_0^c$ is given by

\[
\hat{\Gamma}_0^q \equiv \frac{G_F^2 \hat{m}_b^5 |V_{qb}|^2}{192\pi^3}, \quad q = c, u.
\]

with $\hat{m}_b = m_b + \bar{\Lambda}$ the “dressed bottom quark” mass. The functions $I_0$, $I_1$ and $I_2$ are phase space factors [3] with

\[
\rho \equiv m_c^2/\hat{m}_b^2; \quad \hat{\rho} \equiv \hat{m}_c^2/\hat{m}_b^2,
\]

here $\hat{m}_c = m_c + \bar{\Lambda}$ is the “dressed charm quark” mass. For $b \to c + \ell \bar{\nu}$ semileptonic decays, $\rho_X \equiv m_c^2/\hat{m}_b^2$, and for $b \to c + \bar{u}d'$ decays, $\rho_X \approx 0$ since the light quarks mass is much smaller than the bottom quark mass. For $b \to c + \bar{c} s'$ decays, we take $\rho_X \simeq \hat{\rho}$ which means that the emitted anti-charm quark $\bar{c}$ is also treated as a “dressed heavy quark”. This is slightly different from the consideration in ref. [3] where $\rho_X \approx \rho$. As a consequence, the charm counting $n_c$ has less dependence on the charm quark mass $m_c$. It is noticed that the parameter $\rho$ arises from the propagator of charm quark and the parameters $\hat{\rho}$ and $\rho_X$
arise from the phase space integral of the differential decay rate \( d\Gamma/dy \) with \( y \equiv 2E_\ell/\hat{m}_b \) for semileptonic decays and \( y \equiv 2E_{e(\mu)}/\hat{m}_b \) for nonleptonic decays. The integral region of \( y \) is

\[
2\sqrt{\rho x} \leq y \leq 1 + \rho x - \hat{\rho}.
\]

We now extend the above considerations for \( b \to c \) transitions to \( b \to u \) transitions. To do that, one only needs to notice the differences between \( b \to c \) and \( b \to u \) transitions. One important difference is that for \( b \to c \) decays the final charm quark \( c \) remains to be regarded as a “dressed heavy quark”. While for \( b \to u \) decays, as the masses of \( u \) quark and lightest hadrons (i.e., pions) are very small, their effects are highly suppressed by \( 1/m_b \) and will be neglected in a good approximation \( m_u \ll m_c \ll \hat{m}_b \). With this consideration, the differential decay width of \( b \to u\ell\bar{\nu} \) transition is simplified

\[
\frac{1}{\Gamma_0^u} \frac{d\Gamma}{dy} = 2(3 - 2y)y^2 - \frac{6N_b}{\hat{m}_b^2} \{3y^2 - \delta(y - 1)\} + \frac{2A}{m_b^2} \{3y^2 + 2\delta(y - 1) + 2\delta'(y - 1)\} \tag{2.6}
\]

with \( y \equiv 2E_{e(\mu)}/\hat{m}_b \). The integral region of the phase space is

\[
0 \leq y \leq 1 - m_c^2/\hat{m}_b^2 + \rho_e,
\]

which indicates that the \( \delta \)-functions cannot contribute to the total decay width as \( y < 1 \).

The \( b \to u\ell\bar{\nu} \) decay width is simply given by

\[
\frac{\Gamma}{\Gamma_0^u} = 1 + \frac{2A}{\hat{m}_b^2} - \frac{6N_b}{\hat{m}_b^2} + O\left(1/\hat{m}_b^2\right), \tag{2.7}
\]

where we have neglected the terms of \( O(m_u^2/\hat{m}_b^2) \).

When including the one-loop QCD corrections, we have the following general forms for \( b \to u \) transitions

\[
\frac{\Gamma(b \to u\ell\bar{\nu})}{\Gamma_0^u} = \left\{1 - \frac{2}{3\pi}\left(\pi^2 - \frac{25}{4}\right)\alpha_s(\mu)\right\}(1 + \frac{2A}{\hat{m}_b^2} - \frac{6N_b}{\hat{m}_b^2}); \quad \ell = e, \quad \mu, \tag{2.8a}
\]

\[
\frac{\Gamma(b \to u\tau\bar{\nu})}{\Gamma_0^u} = \left\{1 - 0.665\alpha_s(\mu)\right\}(f(\rho_\tau) + \frac{2A}{\hat{m}_b^2}(1 - \rho_\tau)^4 - \frac{6N_b}{\hat{m}_b^2}(1 + \rho_\tau)^3), \tag{2.8b}
\]

\[
\frac{\Gamma(b \to u\bar{d}'d')}{\Gamma_0^u} = \left\{1 - \eta_{uu}\alpha_s(\mu)\right\}\left\{1 + \frac{2A}{\hat{m}_b^2} - \frac{6N_b}{\hat{m}_b^2}\right\} + (c_+^2(\mu) - c_-^2(\mu))\frac{6N_b}{\hat{m}_b^2}, \tag{2.8c}
\]

\[
\frac{\Gamma(b \to u\bar{c}'s')}{\Gamma_0^u} = \left\{1 - \eta_{uc}\alpha_s(\mu)\right\}\left\{f(\hat{\rho}) + \frac{2A}{\hat{m}_b^2}(1 - \hat{\rho})^4 - \frac{6N_b}{\hat{m}_b^2}(1 - \hat{\rho})^3\right\}
\]

\[+(c_+^2(\mu) - c_-^2(\mu))\frac{6(1 - \hat{\rho})^3N_b}{\hat{m}_b^2}, \tag{2.8d}
\]

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with
\[ \rho_x \equiv m_x^2/m_b^2; \quad \hat{\rho} \equiv \hat{m}_x^2/\hat{m}_b^2; \quad d' = dV_{ud} + sV_{us}, \]
and
\[ f(x) = 1 - 8x + 8x^3 - x^4 - 12x^2 \ln x. \]

The two loop QCD corrections for \( b \to u \ell \nu \) decays have recently been carried out in ref. [12]. The one-loop QCD corrections for \( b \to u \bar{u} d \) decays can be obtained from the ones for \( b \to c \) transitions given in refs. [10,13,14] by simply taking the limit \( a \equiv m_c^2/m_b^2 \to 0 \). And the one loop QCD corrections for \( b \to u \bar{c} s \) decays have been calculated in ref. [17]. For consistent, we shall use the one loop results for \( b \to u \ell \nu \) decays to calculate the ratios \( r_{uu}, r_{u}, \) and \( r_{cu} \), and adopt the two loop QCD corrections for the semileptonic decays \( b \to q \ell \nu \) (\( q = c, u \)) to extract the CKM matrix elements \( |V_{ub}| \) and \( |V_{cb}| \) as well as the ratio \( |V_{ub}|/|V_{cb}| \).

B. Nonspectator Effects in Bottom Hadron Decays

To order of \( 1/m_b^3 \), the nonspectator effects due to Pauli interference and \( W \)-exchange, \( B_i, \varepsilon_i \) (\( i = 1, 2 \)), \( \tilde{B_i} \), and \( r \), may have sizeable contributions to the lifetime differences of bottom hadrons due to a phase-space enhancement by a factor of \( 16\pi^2 \). The four-quark operators relevant to inclusive nonleptonic \( B \) decays are

\[
O_{V-A}^q = \bar{b}_L \gamma_\mu q_L \bar{q}_L \gamma^\mu b_L, \tag{2.9a}
\]
\[
O_{S-P}^q = \bar{b}_R q_L b_R, \tag{2.9b}
\]
\[
T_{V-A}^q = \bar{b}_L \gamma_\mu t^a q_L \bar{q}_L \gamma^\mu t^a b_L, \tag{2.9c}
\]
\[
T_{S-P}^q = \bar{b}_R t^a q_L \bar{q}_L t^a b_R, \tag{2.9d}
\]

where \( q_{R,L} = \frac{1+\gamma_5}{2} q \) and \( t^a = \lambda^a/2 \) with \( \lambda^a \) being the Gell-Mann matrices. For the matrix elements of these four-quark operators between \( B \) hadron states, we follow the definitions given in ref. [11]

\[
\frac{1}{2m_{B_q}} \langle \bar{B}_q | O_{V-A}^q | \bar{B}_q \rangle \equiv \frac{f_{B_q}^2 m_{B_q}}{8} B_1, \tag{2.10a}
\]
\[
\frac{1}{2m_{B_q}} \langle \bar{B}_q | O_{S-P}^q | \bar{B}_q \rangle \equiv \frac{f_{B_q}^2 m_{B_q}}{8} B_2, \tag{2.10b}
\]
\[
\frac{1}{2m_{B_q}} \langle \bar{B}_q | T^q_{V-A} | B_q \rangle = \frac{f_{B_q}^2 m_{B_q}}{8} \varepsilon_1 , 
\]
(2.10c)

\[
\frac{1}{2m_{B_q}} \langle \bar{B}_q | T^q_{S-P} | B_q \rangle = \frac{f_{B_q}^2 m_{B_q}}{8} \varepsilon_2 , 
\]
(2.10d)

\[
\frac{1}{2m_{\Lambda_b}} \langle \Lambda_b | O^q_{V-A} | \Lambda_b \rangle = -\frac{f_{B_q}^2 m_{B_q}}{48} r , 
\]
(2.10e)

\[
\frac{1}{2m_{\Lambda_b}} \langle \Lambda_b | T^q_{V-A} | \Lambda_b \rangle = -\frac{1}{2} (\tilde{B} + \frac{1}{3}) \frac{1}{2m_{\Lambda_b}} \langle \Lambda_b | O^q_{V-A} | \Lambda_b \rangle , 
\]
(2.10f)

where \( f_{B_q} \) is the \( B_q \) meson decay constant defined via

\[
\langle 0 | \bar{q} \gamma_\mu \gamma_5 b | B_q (p) \rangle = i f_{B_q} p_\mu . 
\]
(2.11)

Under the factorization approximation, \( B_i = 1 \) and \( \varepsilon_i = 0 \), and under the valence quark approximation \( \tilde{B} = 1 \).

Applying the treatment in ref. [11], the decay widths due to nonspectator effects have the following form in our present considerations

\[
\frac{1}{2m_B} \langle B^- | \Gamma_{\text{spec}} | B^- \rangle = \tilde{\Gamma}_0^u \hat{\eta}_{\text{spec}} (1 - \hat{\rho})^2 \left\{ \left( 2c_+^2 - c_-^2 \right) B_1 + 3(c_+^2 + c_-^2) \varepsilon_1 \right\} , 
\]
(2.12a)

\[
\frac{1}{2m_B} \langle B_d | \Gamma_{\text{spec}} | B_d \rangle = -\tilde{\Gamma}_0^u \hat{\eta}_{\text{spec}} (1 - \hat{\rho})^2 |V_{ud}|^2 \left\{ \frac{1}{3} \left( 2c_+ - c_- \right)^2 \left[ (1 + \frac{2}{3}) B_1 - (1 + 2\hat{\rho}) B_2 \right] 
\right.
\]
\[+ \frac{1}{2} (c_+ + c_-)^2 \left[ (1 + \frac{2}{3}) \varepsilon_1 - (1 + 2\hat{\rho}) \varepsilon_2 \right] \left\} 
\]
\[- \tilde{\Gamma}_0^u \hat{\eta}_{\text{spec}} \sqrt{1 - 4\hat{\rho}} |V_{us}|^2 \left\{ \frac{1}{3} \left( 2c_+ - c_- \right)^2 \left[ (1 - \hat{\rho}) B_1 - (1 + 2\hat{\rho}) B_2 \right] 
\right.
\]
\[+ \frac{1}{2} (c_+ + c_-)^2 \left[ (1 - \hat{\rho}) \varepsilon_1 - (1 + 2\hat{\rho}) \varepsilon_2 \right] \left\} , 
\]
(2.12b)

\[
\frac{1}{2m_{\Lambda_b}} \langle \Lambda_b | \Gamma_{\text{spec}} | \Lambda_b \rangle = \tilde{\Gamma}_0^u \hat{\eta}_{\text{spec}} \frac{r}{16} \left\{ (1 - \hat{\rho})^2 \left[ (c_-^2 - c_+^2) + (c_+^2 + c_-^2) \right] \tilde{B} 
\right.
\]
\[- \left[ (1 - \hat{\rho})^2 (1 + \hat{\rho}) |V_{ud}|^2 + \sqrt{1 - 4\hat{\rho}} |V_{us}|^2 \right] 
\times \left[ (c_- - c_+) (5c_+ - c_-) + (c_- + c_+)^2 \tilde{B} \right] \left\} . 
\]
(2.12c)

where \( c_\pm = c_1 \pm c_2 \), and

\[
\hat{\eta}_{\text{spec}} = 16\pi^2 f_{B_q}^2 m_B \frac{m_B^3}{m_b^3} . 
\]
(2.13)

The spectator contribution to the width of \( B_s \) meson is simply obtained from that of the \( B_d \) meson by the replacement: \(|V_{ud}| \leftrightarrow |V_{us}|\), and \((f_B, m_B) \rightarrow (f_{B_s}, m_{B_s})\). Strictly
speaking, the values of the parameters $B_i$ and $\varepsilon_i$ for the $B_s$ meson should be different from those for the $B_d$ meson due to SU(3)-breaking effects.

III. NUMERICAL ANALYSIS

A. Basic Formulae

It has been shown from above section that the leading order nonperturbative corrections for $b \to u$ transitions only involve two matrix elements at the order of $1/m_b^2$. In the framework of new formulation of HQEFT, the mass formulae for the hadrons containing a single heavy quark are

$$ m_H = m_Q + \bar{\Lambda} - \frac{<H_v|\bar{Q}_v^{(+)}(i \not{\partial}_\perp)^2Q_v^{(+)}|H_v>}{2\bar{\Lambda} \cdot m_Q} + O\left(\frac{1}{m_Q^2}\right). \quad (3.1) $$

Define

$$ \kappa_1 \equiv - <H_v|\bar{Q}_v^{(+)}D_\perp^2Q_v^{(+)}|H_v> / (2\bar{\Lambda}), \quad (3.2a) $$

$$ \kappa_2 \equiv - <H_v|\bar{Q}_v^{(+)}\sigma_{\mu\nu}G^{\mu\nu}Q_v^{(+)}|H_v> / (4d_H\bar{\Lambda}). \quad (3.2b) $$

with $d_H = -3$ for pseudoscalar mesons, $d_H = 1$ for vector mesons and $d_H = 0$ for ground state heavy baryons, the mass formulae can be reexpressed as

$$ m_H = m_Q + \bar{\Lambda} - \frac{\kappa_1}{m_Q} - \frac{d_H\kappa_2}{m_Q} + O\left(\frac{1}{m_Q^2}\right). \quad (3.3) $$

Thus the “dressed bottom quark” mass $\hat{m}_b$ can be rewritten in terms of the hadron mass

$$ \hat{m}_b = m_{H_b} + \frac{\kappa_1 - d_H\kappa_2}{m_b} + O\left(\frac{1}{m_b^2}\right) $$

$$ = m_{H_b} + \frac{\kappa_1 - d_H\kappa_2}{m_{H_b}} + O\left(\frac{1}{m_{H_b}^2}\right). \quad (3.4) $$

Using eq.(3.3), the value of $\kappa_2$ can be directly extracted from the known $B - B^*$ mass splitting

$$ \kappa_2 \simeq \frac{1}{8}(m_{B^*}^2 - m_{B^0}^2) = 0.06 \text{ GeV}^2. \quad (3.5) $$
which has an accuracy up to the power correction of $\Lambdabar/2m_b \sim 5\%$. The value of $\kappa_1$ depends on bottom quark mass and binding energy, only a reliable range of $\kappa_1$ can be obtained. In the following analysis we shall take the range

$$-0.6 \text{ GeV}^2 \leq \kappa_1 \leq -0.1 \text{ GeV}^2.$$  

For the two matrix elements $A$ and $N_b$, we only need to consider the leading order terms in $1/m_b$ when the decay rates are evaluated up to $1/m_b^2$ order. It is then not difficult to yield

$$A = \frac{\kappa_1}{3}, \quad N_b = \frac{d_H\kappa_2}{3}. \quad (3.6)$$

The magnitudes of nonspectator effects depend on the values of $\varepsilon_1$ and $B_1$. From theoretical calculations [11,19–22], there remain large uncertainties. For a conservative consideration, we may take following values for those parameters

$$B_1(m_b) \simeq B_2(m_b) \simeq 1,$$

$$\varepsilon_1(m_b) \simeq \varepsilon_2(m_b) = -0.10 \pm 0.05,$$

$$r \simeq 0.3, \quad \tilde{B} \approx 1. \quad (3.7)$$

**B. $|V_{ub}|$, $|V_{cb}|$ and $|V_{ub}|/|V_{cb}|$ from Inclusive Semileptonic Decays**

With the above analyses, the two important CKM matrix elements $|V_{ub}|$ and $|V_{cb}|$ can be extracted from inclusive semileptonic decays. It is seen that up to $1/m_b^2$ order the $b \to c$ and $b \to u$ semileptonic decays mainly relate to the variables $\kappa_1$ and $\kappa_2$ as well as the energy scale $\mu$ and charm quark mass $m_c$. Fixing $\kappa_2 = 0.06$ and fitting to the current experimental data for total decay rates of the inclusive $b \to c$ and $b \to u$ semileptonic decays, we find that $|V_{ub}|$ and $|V_{cb}|$ may be given, to a good approximation, by the following form

$$|V_{ub}| = 3.45 \times 10^{-3} \left\{ 1 - 0.02\left(1 - \frac{\kappa_1}{-0.2 \text{ GeV}^2}\right) + 4 \times 10^{-4}\left(1 - \frac{\kappa_1}{-0.2 \text{ GeV}^2}\right)^2 \right\}$$

$$\times \left\{ 1 - 2.40(\alpha_s - 0.3) - 2.04(\alpha_s - 0.3)^2 \right\}^{\frac{1}{2}} \left( \frac{B(B^0 \to X_u e\nu)}{0.00173} \right)^{\frac{1}{2}} \left( \frac{1.56 \text{ ps}}{\tau(B^0)} \right)^{\frac{1}{2}}, \quad (3.8a)$$
\[ |V_{ub}| = 3.93 \times 10^{-2} \{1 - 4 \times 10^{-3}(1 - \frac{\kappa_1}{-0.2 \text{ GeV}^2})^2 - (1 - \frac{\kappa_1}{-0.2 \text{ GeV}^2}) \times [0.038 + 0.091(1 - \frac{m_c}{1.75 \text{ GeV}})] - 0.7(1 - \frac{m_c}{1.75 \text{ GeV}}) - 0.18(1 - \frac{m_c}{1.75 \text{ GeV}})^2 \} \times [1 - 2.05(\alpha_s - 0.3) - 2.17(\alpha_s - 0.3)^2]^{-\frac{1}{2}} \left( \frac{B(B^0 \rightarrow X_c e\bar{\nu})}{0.1048} \right)^{\frac{1}{2}} \left( \frac{1.56 \text{ ps}}{\tau(B^0)} \right)^{\frac{1}{2}}. \]

Let us now discuss possible theoretical uncertainties for the quantities \( |V_{ub}| \) and \( |V_{cb}| \) or \( |V_{ub}|/|V_{cb}| \).

(1). From discarding high order nonperturbative corrections. In the present analysis we have only expanded the matrix element to \( 1/m_b^2 \) order. It has been noted that there is no \( 1/m_b \) order corrections and the magnitude of \( 1/m_b^2 \) corrections is less than 5.5%. Since the higher order nonperturbative corrections are expected to be much smaller than the ones at \( 1/m_b^2 \) order and their effects appear to be no more than 1% in a conservative estimation.

(2). From the hadronic matrix elements, i.e., \( \kappa_1, \kappa_2 \) and the charm quark mass \( m_c \).
As mentioned above, the extraction of \( \kappa_2 \) could have an accuracy up to 5%, while the value of \( \kappa_1 \) has not been well determined. As the “dressed bottom quark” mass enters to the decay rates in powers of \( \bar{m}_Q^5 \), the uncertainties of \( \kappa_1 \) become the main sources of the uncertainty. From Fig. 1 and Fig. 2 one can see that the resulting uncertainties are no more than 3% for \( |V_{ub}| \) and 2.7% for \( |V_{ub}|/|V_{cb}| \). As for the charm quark mass \( m_c \), we have considered the range \( 1.55 \text{ GeV} \leq m_c \leq 1.80 \text{ GeV} \), it leads to an uncertainty of 3.5% for \( |V_{ub}|/|V_{cb}| \) (here we have limited the case that \( m_c \leq 1.65 \text{ GeV} \) for \( \kappa_1 = -0.6 \text{ GeV}^2 \) and \( m_c \leq 1.8 \text{ GeV} \) for \( \kappa_1 = -0.5 \text{ GeV}^2 \) from the consideration of lifetime differences among \( B^0 \) and \( B_s^0 \) mesons as well as \( \Lambda_b \) baryon).

(3). From the perturbative QCD corrections. The first order QCD corrections have been calculated in ref. [14] and the second order results have been presented in ref. [15,16,18]. Since the size of the second order QCD corrections have been found to be comparable with the first order ones, the unknown higher order QCD corrections may lead to a sizable theoretical uncertainties. The additional uncertainties could arise from the energy scale \( \mu \). Generally, the scale \( \mu \) in the \( b \) decays is taken from \( m_b/2 \) to \( 2m_b \), which could lead to a large theoretical uncertainty for the determination of \( |V_{ub}| \). One method to reduce the possible uncertainties arising from the perturbative corrections is to choose a
proper value of $\mu$ by fitting the experimental data of the semileptonic decays $B(b \to c\ell \bar{\nu})$.

When only one-loop QCD corrections are considered, it is seen that one should take lower values of $\mu$ with the range $m_b/4 \leq \mu \leq m_b$. From Fig. 1 one can see that this would lead to an uncertainty of about 3% for $|V_{ub}|$ and a similar one for $|V_{ub}|/|V_{cb}|$.

With above considerations and using the ALEPH experimental data [7] without the non-Gaussian errors, we obtain the following results

\begin{align}
|V_{ub}| &= (3.48 \pm 0.11_{th} \pm 0.62_{exp}) \times 10^{-3}, \quad (3.9a) \\
|V_{cb}| &= (3.89 \pm 0.20_{th} \pm 0.05_{exp}) \times 10^{-2}, \quad (3.9b) \\
|V_{ub}|/|V_{cb}| &= 0.089 \pm 0.005_{th} \pm 0.015_{exp}. \quad (3.9c)
\end{align}

by choosing the parameters $m_c$ and $\kappa_1$ to be within the range: $1.55$ GeV $\leq m_c \leq 1.80$ GeV, $-0.6$ GeV$^2 \leq \kappa_1 \leq -0.1$ GeV$^2$ and considering the lifetime ratio $\tau(\Lambda_b)/\tau(B^0)$ to be $0.70 \leq \tau(\Lambda_b)/\tau(B^0) \leq 0.85$ as well as the ratio between the $\tau$ and $\beta$ decay $Br(b \to c\tau\nu)/Br(b \to c\ell\nu) \leq 0.285$.

C. Ratios $r_{uu}$, $r_{\tau u}$ and $r_{cu}$

The nonperturbative corrections in the $b \to u$ transitions have a very simple form and their effects are also quite small, thus the uncertainties induced by them are much smaller than those of the perturbative corrections. Fixing $B(b \to c\ell\bar{\nu})$ to be 10.48%, the ratios defined in eq. (1.1) are found to be

\begin{align}
    r_{uu} &= 4.8 \pm 0.5 , \quad (3.10a) \\
    r_{cu} &= 2.4 \pm 0.3 , \quad (3.10b) \\
    r_{\tau u} &= 0.44 \pm 0.02 . \quad (3.10c)
\end{align}

In obtaining the above results, the range $-0.6$ GeV$^2 \leq \kappa_1 \leq -0.1$ GeV$^2$ has been used. The uncertainties mainly arise from that of the energy scale $\mu$. It is interesting to note that the ratio between $\tau$ and $\beta$ decays approaches to be about half in the $b \to u$ transitions, while it is only about quarter in the $b \to c$ transitions.
D. Nonspectator Effects

The contributions from the nonspectator effects could vary in the decays of different bottom hadrons. Using the values given in eq.(3.7) for various parameters, the magnitude could be from $-6\%$ to $-11\%$ in $B^+$ decays, and about $(0.6-0.7)\%$, $(0.2-0.3)\%$ and $(0.6-0.75)\%$ in $B^0$, $B^0_s$ and $\Lambda_b$ decays, respectively.

As a consequence, we arrive at the following results for the ratios of the lifetimes

$\frac{\tau(B^-)}{\tau(B_d)} = 1.08 \pm 0.05$, \hfill (3.11a)

$\frac{\tau(B_s)}{\tau(B_d)} = 0.96 \pm 0.06$, \hfill (3.11b)

$\frac{\tau(\Lambda_b)}{\tau(B_d)} = 0.78 \pm 0.05$. \hfill (3.11c)

which are in good agreement with the experimental data.

Note that the contributions to the ratios in eqs.(3.11b) and (3.11c) from the nonspectator effects are rather small when the relevant parameters take values in eq.(3.7). Thus the usual HQET fails in explaining the lifetime differences among $B^0$, $B^0_s$ and $\Lambda_b$ hadrons. This is because the lifetime differences in the usual HQET arise mainly from the nonspectator effects.

E. More Numerical Results

We show in Fig.3 the correlation between the charm counting $n_c$ and the branching ratio $B_{SL}$ of the semileptonic $B \to X_c e\nu$ decay with different values of the charm quark mass $m_c$ ($m_c = 1.55 \sim 1.80$ GeV), the energy scale $\mu$ ($\mu = m_b/2 \sim 2m_b$) and the parameter $\kappa_1$ (the solid curve for $\kappa_1 = -0.5GeV^2$ and the dotted curve for $\kappa_1 = -0.2GeV^2$). It is seen that the would average values of the charm counting $n_c$ and the branch ratio $B_{SL}$ lie in the allowed region predicted from the HQEFT. The lower value of $\mu = m_b/2 \sim m_b$ and larger value of $m_c = 1.65 \sim 1.80$ GeV as well as the smaller value of $|\kappa_1|$ seem to be favorable. In Fig.4, we present the correlation among the three observables: the charm counting $n_c$, the branching ratio $B_{SL}$ and the lifetime ratio $\tau(\Lambda_b)/\tau(B^0)$. It is interesting to notice that the stable region is more favorable to the experimental data.
It is also seen from Fig. 3 that the charming counting $n_c$ has strong dependance on charm quark mass $m_c$ and $\kappa_1$. Within the allowed range of $m_c$ and $\kappa_1$, and by considering the lifetime ratio $\tau(\Lambda_b)/\tau(B^0)$ to be $0.70 \leq \tau(\Lambda_b)/\tau(B^0) \leq 0.85$ as well as the ratio between the $\tau$ and $\beta$ decay to be $B_{\tau}(b \rightarrow c\tau\nu)/Br(b \rightarrow ce\nu) \leq 0.285$, the charming $n_c$ is found to be

$$n_c = 1.19 \pm 0.04. \quad (3.12)$$

It is notice that small values of $\kappa_1$ and large $m_c$ will result in a low value of charm counting $n_c = 1.15$.

For more clear, we provide in Table 1 and Table 2 the most reliable values for the various interesting observables. The agreement with the experimental data must be regarded as a success of QCD since both the perturbative corrections and nonperturbative contributions described by HQEFT are resulted from QCD. We hope that a better agreement can be arrived by considering higher order contributions.

**Table 1.** The quantities $V_{ub}$, $V_{cb}$ and lifetime ratios among bottom hadrons as well as the relative contributions between $b \rightarrow u + X_i$ (with $X_i = ud', cs', \tau\nu$) and $b \rightarrow u + e\nu$ transitions are given as functions of $m_c$ and $\kappa_1$. For the given values of $m_c$ and $\kappa_1$, the value of $\mu$ is yielded by fixing the semileptonic branching ratio $B_{SL} = 10.48\%$. The quantities except $r_{\tau u}$, $r_{ca}$ and $r_{uu}$ have been evaluated by including two-loop QCD corrections and nonspectator effects.
<table>
<thead>
<tr>
<th>$m_c$ (GeV)</th>
<th>1.55</th>
<th>1.65</th>
<th>1.75</th>
<th>1.80</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\kappa_1$ (GeV$^2$)</td>
<td>-0.2 -0.4 -0.6</td>
<td>-0.2 -0.4 -0.6</td>
<td>-0.2 -0.4 -0.6</td>
<td>-0.2 -0.4 -0.6</td>
</tr>
<tr>
<td>$\mu$ (GeV)</td>
<td>2.36 2.53 2.73</td>
<td>2.27 2.43 2.69</td>
<td>2.19 2.36 2.74</td>
<td>2.14 2.34 2.82</td>
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<tr>
<td>$</td>
<td>V_{ub}</td>
<td>(10^{-3})$</td>
<td>3.42 3.47 3.52</td>
<td>3.43 3.48 3.52</td>
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<tr>
<td>$</td>
<td>V_{cb}</td>
<td>(10^{-2})$</td>
<td>3.59 3.71 3.83</td>
<td>3.75 3.87 3.88</td>
</tr>
<tr>
<td>$</td>
<td>V_{ub}</td>
<td>/</td>
<td>V_{cb}</td>
<td>(10^{-2})$</td>
</tr>
<tr>
<td>$\tau(B^0_s)/\tau(B^0)$</td>
<td>0.92 0.92 0.93</td>
<td>0.92 0.92 0.98</td>
<td>0.92 0.94 1.06</td>
<td>0.92 0.96 1.12</td>
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<tr>
<td>$\tau(\Lambda_b)/\tau(B^0)$</td>
<td>0.76 0.74 0.73</td>
<td>0.75 0.73 0.78</td>
<td>0.73 0.75 0.88</td>
<td>0.72 0.76 0.97</td>
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<tr>
<td>$\tau(B^+)/\tau(B^0)$</td>
<td>1.07 1.07 1.07</td>
<td>1.08 1.08 1.07</td>
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</tr>
<tr>
<td>$n_c$</td>
<td>1.19 1.21 1.23</td>
<td>1.18 1.20 1.22</td>
<td>1.17 1.19 1.23</td>
<td>1.17 1.19 1.23</td>
</tr>
<tr>
<td>$r_{uu}$</td>
<td>4.94 4.79 4.64</td>
<td>5.00 4.85 4.66</td>
<td>5.06 4.89 4.63</td>
<td>5.09 4.91 4.60</td>
</tr>
<tr>
<td>$r_{cu}$</td>
<td>2.26 2.36 2.45</td>
<td>2.30 2.41 2.47</td>
<td>2.36 2.45 2.45</td>
<td>2.38 2.46 2.42</td>
</tr>
<tr>
<td>$r_{uu}$</td>
<td>0.46 0.45 0.45</td>
<td>0.46 0.45 0.45</td>
<td>0.46 0.45 0.45</td>
<td>0.46 0.45 0.45</td>
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</table>

Table 2. The quantities $V_{ub}$, $V_{cb}$ and lifetime ratios among bottom hadrons as well as the relative contributions between $b \rightarrow u + X_i$ (with $X_i = ud', cs', \tau\nu$) and $b \rightarrow u + e\nu$ transitions are given as functions of $m_c$ and $\kappa_1$. For the given values of $m_c$ and $\kappa_1$, the value of $\mu$ is fixed to be $\mu = 2.5$ GeV. The quantities except $r_{uu}$, $r_{cu}$ and $r_{uu}$ have been evaluated by including two-loop QCD corrections and nonspectator effects.
<table>
<thead>
<tr>
<th>$m_c$ (GeV)</th>
<th>1.55</th>
<th>1.65</th>
<th>1.75</th>
<th>1.80</th>
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<tbody>
<tr>
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<td>-0.2</td>
<td>-0.4</td>
<td>-0.6</td>
<td>-0.2</td>
</tr>
<tr>
<td>$B_{SL}$ (%)</td>
<td>10.70</td>
<td>10.44</td>
<td>10.18</td>
<td>10.85</td>
</tr>
<tr>
<td>$</td>
<td>V_{ub}</td>
<td>(10^{-3})$</td>
<td>3.40</td>
<td>3.47</td>
</tr>
<tr>
<td>$</td>
<td>V_{cb}</td>
<td>(10^{-2})$</td>
<td>3.57</td>
<td>3.72</td>
</tr>
<tr>
<td>$</td>
<td>V_{ub}</td>
<td>/</td>
<td>V_{cb}</td>
<td>(10^{-2})$</td>
</tr>
<tr>
<td>$n_c$</td>
<td>1.19</td>
<td>1.21</td>
<td>1.23</td>
<td>1.17</td>
</tr>
<tr>
<td>$r_{uu}$</td>
<td>4.85</td>
<td>4.81</td>
<td>4.76</td>
<td>4.85</td>
</tr>
<tr>
<td>$r_{cu}$</td>
<td>2.19</td>
<td>2.37</td>
<td>2.56</td>
<td>2.19</td>
</tr>
<tr>
<td>$r_{tu}$</td>
<td>0.46</td>
<td>0.45</td>
<td>0.45</td>
<td>0.46</td>
</tr>
</tbody>
</table>

**IV. CONCLUSIONS AND REMARKS**

Based on the new framework of HQEFT with including antiquark contributions [1–3], we have made a systematic analysis for the $b \rightarrow u(c)$ transitions with including the nonspectator effects. The important CKM quark mixing matrix elements $|V_{ub}|$ and $|V_{cb}|$ have been reliably extracted from the inclusive semileptonic decays $B \rightarrow X_u e\nu$ and $B \rightarrow X_c e\nu$. The resulting predictions for the various observables, such as the semileptonic branch ratio $B_{SL}$, the lifetime differences among $B^-$, $B^0$, $B_s$ and $\Lambda_b$ decays, and the charm counting $n_c$ are consistent with the present experimental data within the allowed region of parameters.

We would like to remark that it was thought before that the results like eqs.(3.8) may not be suitable to extract the CKM matrix element $|V_{ub}|$ and the ratio $|V_{ub}|/|V_{cb}|$ due to the difficulty of identifying the events of $b \rightarrow u l\nu$ from the overwhelming $b \rightarrow c l\nu$ background. However, the situation has been changed because of the progresses of the technique of using the invariant hadronic mass spectrum [5–7]. Especially, the ALEPH collaboration [7] has made it feasible. Our results given in eqs.(3.8) and (3.9) have been obtained by using the ALEPH data. Such results are more close to those obtained by
using the parton model [23], while they are somewhat smaller than those predicted from the usual HQET. It has been seen that the $b \rightarrow u$ transitions have different features in comparison with the $b \rightarrow c$ transitions, for instance, the ratio between the $\tau$ and $\beta$ decays in the $b \rightarrow u$ transitions is larger by a factor of two than the one in the $b \rightarrow c$ transitions. The nonspectator effects are in general not large enough to understand the observed lifetime ratio $\tau(\Lambda_b)/\tau(B^0)$ in the usual HQET, except one chooses unexpected large values of $r$ and $\bar{B}$, for such a choice, the nonspectator effects in $B^0$ decays are still small and their contributions to the total decay width are generally less than 1%, but the nonspectator effects in the $\Lambda_b$ decays must become unexpected large and their contributions to the total decay width have to be about 20% at the order of $1/m_b^3$ in order to explain the observed lifetime ratio $\tau(\Lambda_b)/\tau(B^0)$ in the usual HQET. As a consequence, the heavy quark expansion in the usual HQET may become unreliable if one insists such an explanation for the lifetime difference between $B^0$ and $\Lambda_b$.

It is very interesting to further explore the applications of the HQEFT with keeping both quark and antiquark fields [1], in particular for the processes concerning the quark and antiquark annihilations and productions, a special case with kinematic regimes of heavy quark pair production near the threshold has recently been discussed in ref. [24].

ACKNOWLEDGMENTS

We would like to thank Prof. Y. B. Dai and Dr. C. Liu for useful discussions. This work was supported in part by the NSF of China under the grant No. 19625514.
REFERENCES


FIGURES

FIG. 1. $|V_{ub}|$ as function of $\mu$ and $\kappa_1$.

FIG. 2. $|V_{ub}|/|V_{cb}|$ as function of $m_c$ and $\kappa_1$. In this figure we have chosen $\mu$ to normalize $B_r(B^0 \rightarrow c\ell\nu) = 10.48\%$.

FIG. 3. The correlation between $B_{SL}$ and $n_c$. The solid line is for $\kappa_1 = -0.5$ GeV$^2$ and the dash-line for $\kappa_1 = -0.2$ GeV$^2$

FIG. 4. The correlation between $\tau(\Lambda_b)/\tau(b^0)$, $B_{SL}$ and $n_c$. Here $x$ stands for $B_{SL}$, $y$ for $n_c$ and $z$ for $\tau(\Lambda_b)/\tau(B^0)$. 
