Determination of equation of state of quark matter from $J/\psi$ and $\Upsilon$ suppression at RHIC and LHC

Dipali Pal, Binoy Krishna Patra, Dinesh Kumar Srivastava

Variable Energy Cyclotron Centre
1/AF Bidhan Nagar, Calcutta 700 064
India

Abstract

The long life-time of the quark-gluon plasma likely to be created in the relativistic heavy ion collisions at RHIC and LHC energies renders it sensitive to the details of the equation of state of the quark-matter. We show that the $p_T$ dependence of the survival probability of the directly produced $J/\psi$ at RHIC energies and that of the directly produced $\Upsilon$ at LHC energies is quite sensitive to the speed of sound in the quark matter, which relates the pressure and the energy density of the plasma. The transverse expansion of the plasma is shown to strongly affect the $J/\psi$ suppression at LHC energies.

PACS: 12.38.M
1 Introduction

Statistical quantum chromodynamics predicts that at sufficiently high densities or temperatures the quarks and gluons confined inside hadrons undergo a deconfining phase transition to a plasma of quarks and gluons. The last two decades of high energy nuclear physics activity has been directed towards the production of this new state of matter through relativistic heavy ion collisions. This has led to experiments at BNL AGS and CERN SPS and to the building of the BNL Relativistic Heavy Ion Collider and the CERN Large Hadron Collider. With the reported confirmations of the quark-hadron phase transition at the relativistic heavy ion collision experiments at the CERN SPS [1], the first step in the search for quark-gluon plasma, which pervaded the early universe, microseconds after the big bang and which may be present in the core of neutron stars, is complete.

The emphasis of the experiments at the BNL RHIC and the CERN LHC will now necessarily shift to an accurate determination of the properties of the quark matter. An important observable for this is the speed of sound in the plasma, defined through:

\[ c_s^2 = \frac{\partial p}{\partial \epsilon} \]  

Often one writes,

\[ p = c_s^2 \epsilon \]  

for the equation of state of the quark matter, where \( \epsilon \) is the energy density and \( p \) is the pressure. For the simplest bag-model equation of state (with \( \mu_B = 0 \)), we write

\[ \epsilon = 3aT^4 + B, \]  
\[ p = aT^4 - B, \]  
\[ a = \left[ 2 \times 8 + \frac{7}{8} \times 2 \times 2 \times 3 \times N_f \right] \frac{\pi^2}{90}, \]

with \( B \) as the bag pressure and \( N_f \) as the number of flavours, so that \( c_s^2 = 1/3 \). In general \( \Delta = \epsilon - 3p \) measures the deviation of the equation of state from the ideal gas of massless quarks and gluons (when it is identically zero) and depends sensitively on the interactions present in the plasma. The lattice QCD calculations show that \( \Delta \geq 0 \), till the temperature is several times the critical temperature [2]. This implies that in general \( c_s^2 \leq 1/3 \). Any experimental information on this will be most welcome.

We show in the present work that the transverse momentum dependence of the survival probability of the \( J/\psi \) and \( \Upsilon \) at RHIC and LHC energies are quite sensitive to the value of the speed of sound. The very long life-time of the plasma likely to be
attained at LHC makes it even more sensitive to the details of the equation of state of the quark matter through the transverse expansion of the plasma.

2 Formulation

The theory of quarkonium suppression in QGP [3] is very well studied and several excellent reviews exist [4, 5, 6], which dwell both on the phenomenology as well as on the experimental situation. We recall the basic details which are relevant for the present demonstration.

The interquark potential for (non-relativistic) quarkonium states at zero temperature may be written as:

\[ V(r, 0) = \sigma r - \frac{\alpha}{r} \]

where \( r \) is the separation between \( Q \) and \( \overline{Q} \). The bound-states of \( cc \) and \( bb \) are well described if the parameters \( \sigma = 0.192 \) GeV\(^2 \), \( \alpha = 0.471 \), \( m_c = 1.32 \) GeV, and \( m_b = 4.746 \) GeV are used [7]. At finite temperatures the potential is modified due to colour screening, and evolves to:

\[ V(r, T) = \frac{\sigma}{\mu(T)} \left( 1 - e^{-\mu(T)} \right) - \frac{\alpha}{r} e^{-\mu(T)r} \]

The screening mass increases with temperature. When \( \mu(T) \to 0 \), the equation (6) is recovered. At finite temperature, when \( r \to 0 \) the \( 1/r \) behaviour is dominant, while as \( r \to \infty \) the range of the potential decreases with \( \mu(T) \). This makes the binding less effective at finite temperature. Semiclassically, one can write for the energy of the pair,

\[ E(r, T) = 2m_Q + \frac{c}{m_Q r^2} + V(r, T) \]

where \( <p^2> = <r^2> = c = O(\infty) \). Radius of the bound state at any temperature is obtained by minimizing \( E(r, T) \). Beyond some critical value \( \mu_D \) for the screening mass \( \mu(T) \), no minimum is found. The screening is now strong enough to make the binding impossible and the resonance can not form in the plasma. The ground state properties of some of the quarkonia reported by authors of Ref. [7] are given in table 1. We have also listed the formation time of these resonances defined in Ref.[8] as the time taken by the heavy quark to traverse a distance equal to the radius of the quarkonium in its rest frame \( \sim m_Q r_{Q\overline{Q}}/p_{Q\overline{Q}} \), where \( p_{Q\overline{Q}} \) is the momentum of either of the quarks of the resonance. It may be recalled that somewhat different values for the formation time are reported by Blaizot and Ollitrault [9] who solve the bound-state
problem within the WKB approximation and define the formation time as the time spent by a quark in going between the two classical turning points.

Now let us consider a central collision in a nucleus-nucleus collision, which results in the formation of quark gluon plasma at some time $\tau_0$. Let us concentrate at $z = 0$ and on the region of energy density, $\epsilon \geq \epsilon_s$ which encloses the plasma which is dense enough to cause the melting of a particular state of quarkonium. We assume the plasma to cool, according to Bjorken’s boost invariant (longitudinal) hydrodynamics and then generalize our results to include the transverse expansion of the plasma. We assume that the $Q\overline{Q}$ pair is produced at the transverse position $r$ at $\tau = 0$ on the $z = 0$ plane with momentum $p_T$. In the collision frame, the pair would take a time equal to $\tau_F E_T/M$ for the quarkonium to form, where $E_T = \sqrt{p_T^2 + M^2}$ and $M$ is the mass of the quarkonium. During this time, the pair would have moved to the location $(r + \tau_F p_T/M)$. If at this instant, the plasma has cooled to an energy density less than $\epsilon_s$, the pair would escape and quarkonium would be formed. If however, the energy density is still larger than $\epsilon_s$ the resonance will not form and we shall have a quarkonium suppression [9, 10, 11].

It is easy to see that the $p_T$ dependence of the survival probability will depend on how rapidly the plasma cools. If the initial energy density is sufficiently high, the plasma will take longer to cool and only the pairs with very high $p_T$ will escape. If however the plasma cools rapidly, then even pairs with moderate $p_T$ will escape. The transverse expansion of the plasma can further accelerate the rate of cooling giving us an additional handle to explore the equation of state, which as we know, will control the expansion of the plasma.

### 2.1 Longitudinal expansion of the plasma

As indicated, we first take the Bjorken’s boost-invariant longitudinal hydrodynamics to explore the expansion of the plasma. Thus, the energy momentum tensor of the plasma is written as [12];

$$T^{\mu\nu} = (\epsilon + p)u^\mu u^\nu + g^{\mu\nu}p,$$  \hspace{1cm} (9)

where $\epsilon$ is the energy-density, $p$ is the pressure, and $u^\mu$ is the four velocity of the fluid, in a standard notation. If the effects of viscosity are neglected, the energy-momentum conservation is given by

$$\partial_\mu T^{\mu\nu} = 0.$$  \hspace{1cm} (10)

The assumption of the boost-invariance provides that the energy density, pressure, and temperature become functions of only the proper time $\tau$ and that the Eq.(10)
simplifies to

\[ \frac{d\epsilon}{d\tau} = -\frac{\epsilon + p}{\tau}. \] (11)

The effect of the speed of sound is seen immediately. Using the Eq.(2), we can now write

\[ \epsilon(\tau)\tau^{1+c_s^2} = \epsilon(\tau_0)\tau_0^{1+c_s^2} = \text{const.} \] (12)

so that if \( c_s^2 \) is small, the cooling is slower. Chu and Matsui [10] explored the consequence of the extremes \( c_s^2 = 1/3 \) and \( c_s^2 = 0 \) on the \( p_T \) dependence of the survival probability.

We now have all the ingredients to write down the survival probability and we closely follow Chu and Matsui for this.

We take a simple parametrization for the energy-density profile:

\[ \epsilon(\tau_0, r) = \epsilon_0 \left[ 1 - \frac{r^2}{R^2} \right]^\beta \theta(R - r) \] (13)

where \( r \) is the transverse co-ordinate and \( R \) is the radius of the nucleus. One can define an average energy density \( <\epsilon_0> \) as

\[ \pi R^2 <\epsilon_0> = \int 2\pi r \, dr \epsilon(r) \] (14)

so that

\[ \epsilon_0 = (1 + \beta) <\epsilon_0>. \] (15)

We have taken \( \beta = 1/2 \), which may be thought as indicative of the energy deposited being proportional to the number of participants in the system. In case one feels that the energy deposited may be proportional to number of nucleon-nucleon collisions then one can repeat the calculations with \( \beta = 1 \), which will reflect the proportionality of the deposited energy to the nuclear thickness. The average energy-density is obtained from the Bjorken formula:

\[ <\epsilon_0> = \frac{1}{\pi R_T^2 \tau_0} \frac{dE_T}{dy} \] (16)

where \( E_T \) is the transverse energy deposited in the collision.

The time \( \tau_s \) when the energy density drops to \( \epsilon_s \) is easily estimated as

\[ \tau_s(r) = \tau_0 \left[ \frac{\epsilon(\tau_0, r)}{\epsilon_s} \right]^{1/(1+c_s^2)} \]

\[ = \tau_0 \left[ \frac{\epsilon_0}{\epsilon_s} \right]^{1/(1+c_s^2)} \left[ 1 - \frac{r^2}{R^2} \right]^{\beta/(1+c_s^2)} \] (17)
As discussed earlier [10], we can equate the duration of screening $\tau_s(r)$ to the formation time $t_F = \gamma \tau_F$ for the quarkonium to get the critical radius, $r_s$:

$$r_s = R \left[ 1 - \left( \frac{\gamma \tau_F}{\tau_{s0}} \right) \left( 1 + c_s^2 / \beta \right)^{1/2} \right] \theta \left[ 1 - \frac{\gamma \tau_F}{\tau_{s0}} \right], \quad (18)$$

where $\tau_{s0} = \tau_s(r = 0)$. This critical radius, is seen to mark the boundary of the region where the quarkonium formation is suppressed. As discussed earlier, the quark-pair will escape the screening region (and form quarkonium) if its position and transverse momentum $p_T$ are such that

$$|r + \tau_F p_T / M| \geq r_s. \quad (19)$$

Thus, if $\phi$ is the angle between the vectors $r$ and $p_T$, then

$$\cos \phi \geq \left[ (r_s^2 - r^2) M - \tau_F^2 p_T^2 / M \right] / [2 r \tau_F p_T], \quad (20)$$

which leads to a range of values of $\phi$ when the quarkonium would escape. We also realize that if the right hand side of the above equation is greater than 1, then no angle is possible when the quarkonium can escape. Now we can write for the survival probability of the quarkonium:

$$S(p_T) = \left[ \int_0^R r \, dr \int_{-\phi_{\max}}^{+\phi_{\max}} d\phi \, P(r, p_T) \right] / \left[ 2 \pi \int_0^R r \, dr \, P(r, p_T) \right], \quad (21)$$

where $\phi_{\max}$ is the maximum positive angle ($0 \leq \phi \leq \pi$) allowed by Eq.(20), and

$$\phi_{\max} = \begin{cases} \pi & \text{if } y \leq -1 \\ \cos^{-1}|y| & \text{if } -1 < y < 1 \\ 0 & \text{if } y \geq 1 \end{cases}, \quad (22)$$

where

$$y = \left[ (r_s^2 - r^2) M - \tau_F^2 p_T^2 / M \right] / [2 r \tau_F p_T], \quad (23)$$

and $P$ is the probability for the quark-pair production at $r$ with transverse momentum $p_T$, in a hard collision. Assuming that the $p_T$ and $r$ dependence for hard collisions factor out, we approximate

$$P(r, p_T) = P(r, p_T) = f(r) g(p_T), \quad (24)$$

where we take

$$f(r) \propto \left[ 1 - \frac{r^2}{R^2} \right]^{\alpha} \theta(R - r), \quad (25)$$

with $\alpha = 1/2$. The Eq.(21) can be solved analytically for some limiting cases of $p_T$ etc., see Ref. [10].
3 Transverse expansion of the plasma

It is generally accepted that the rarefaction wave from the surface of the plasma will reach the centre by \( \tau = R/c_s \). For the case of lead nuclei, this comes to about 12 fm. If the life time of the QGP is comparable to this time, the transverse expansion of the plasma can not be ignored. The transverse expansion of the plasma will lead to a much more rapid cooling than suggested by a purely longitudinal expansion.

For the four-velocity of the collective flow we write:

\[ u^\mu = (\gamma, \gamma v) \]  

(26)

where \( v \) is the collective flow velocity and \( \gamma = 1/\sqrt{1 - v^2} \). We further assume that the longitudinal flow of the plasma has a scaling solution, so that the boost-invariance along the longitudinal direction remains valid. Assuming cylindrical symmetry, valid for central collisions, it can be shown that the four-velocity \( u^\mu \) should have the form,

\[ u^\mu = \gamma_r(\tau, r)(t/\tau, v_r \cos \phi, v_r \sin \phi, z/\tau), \]  

(27)

with

\[
\begin{align*}
\gamma_r &= \left[1 - v_r^2(\tau, r)\right]^{-1/2} \\
\tau &= (t^2 - z^2)^{1/2} \\
\eta &= \frac{1}{2} \ln \frac{t + z}{t - z}.
\end{align*}
\]  

(28)

Thus all the Lorentz scalars are now functions of \( \tau \) and \( r \), and independent of the space-time rapidity \( \eta \). This reduces the (3+1) dimensional expansion with cylindrical symmetry and boost-invariance along the longitudinal direction to:

\[
\partial_\tau T^{00} + r^{-1} \partial_r (r T^{01}) + \tau^{-1} (T^{00} + p) = 0
\]  

(29)

and

\[
\partial_\tau T^{01} + r^{-1} \partial_r \left[r (T^{00} + p) v_r^2\right] + \tau^{-1} T^{01} + \partial_r p = 0
\]  

(30)

where

\[ T^{00} = (\epsilon + p) u^0 u^0 - p \]  

(31)

and

\[ T^{01} = (\epsilon + p) u^0 u^1. \]  

(32)

We solve these equations using well established methods [13] with initial energy density profiles as before and plot the constant energy density contours[14, 15] appropriate for \( \epsilon = \epsilon_s \) to get \( \tau_s(r) \). Rest of the treatment follows as before. In these calculations we have assumed the initial transverse velocity to be identically zero.
We only need to identify the initial conditions. We consider $Pb + Pb$ collisions ($Au + Au$, for RHIC) with the initial average energy densities:

$$< \epsilon_0 > = \begin{cases} 
6.3 \text{ GeV/fm}^3 & \text{SPS, } \tau_0 = 0.5 \text{ fm} \\
60 \text{ GeV/fm}^3 & \text{RHIC, } \tau_0 = 0.25 \text{ fm} \\
425 \text{ GeV/fm}^3 & \text{LHC, } \tau_0 = 0.25 \text{ fm}
\end{cases}$$

(33)

The estimate for SPS is obtained from assumption of QGP formation in $Pb + Pb$ experiments, while those for RHIC and LHC are taken from the self-screened parton cascade calculation [16].

4 Results

4.1 Speed of sound vs. transverse expansion

It is quite clear that a competition between the speed of sound and the onset of the transverse expansion during the lifetime of the deconfining matter can lead to interesting possibilities. In order to illustrate this and to explore the consequences, we show in Fig. 1 the constant energy density contours which enclose the deconfining matter, which can dissociate the directly produced $J/\psi$ (see table 1), at RHIC energies. We see that owing to the (relatively) short time that the QGP would take to cool down to $\epsilon_{J/\psi}$, the effect of the transverse flow is marginal and for $c_s^2 = 1/3$, limited to large radii. Large changes in the contour are seen when the speed of sound is varied. This is very important indeed. Note that the cooling to the value appropriate for $\Upsilon$ suppression is attained too quickly to be affected by the transverse expansion, and even the change due to variation in the speed of sound is quite small. Of course, the duration of the deconfining medium is prolonged if the speed of sound is reduced.

The corresponding results for the LHC energies are shown in Fig. 2. Now we see that at $r = 0$ the duration of the deconfining medium reduces by a factor of 2 when the speed of sound is $1/\sqrt{3}$, and the transverse expansion of the plasma is allowed. This is a consequence of the longer time which the plasma takes to cool at LHC.

The scenario for the $\Upsilon$ dissociating matter at LHC is quite akin to the case of $J/\psi$ dissociating medium at RHIC; the results are affected by the speed of sound and not by the transverse flow (Fig. 3).
4.2 Consequences for survival of quarkonia

Now we return to the transverse momentum dependence of the survival probabilities. As a first step, we plot the survival of the directly produced $J/\psi$'s at SPS, RHIC and LHC energies (Fig. 4) when only longitudinal expansion is accounted for and the speed of sound is varied. We see that RHIC energies provide the most suitable environment to measure the speed of sound with the help of $J/\psi$ suppression. The variations in the $p_T$ dependence is too meagre at SPS (due to a very short duration of the deconfining medium) and at LHC (now, due to a very long duration!) when the speed of sound is varied.

This advantage of RHIC energies is maintained when the transverse expansion is accounted for (Fig. 5). As one could have expected from the contours (Fig. 1), the results are more sensitive to the variation of the speed of sound than to the transverse flow. The accuracy of the procedure is seen from the fact that the survival probability around $p_T = 15$ GeV for $c_s^2 = 1/3$ is identical for the longitudinal and the transverse expansion of the plasma. This is a direct reflection of the identity of the corresponding contours near $r = 0$ (Fig. 1).

The $J/\psi$ suppression at LHC energies, as indicated, becomes sensitive to the transverse flow, the shape of the survival probability changes and the largest $p_T$ for which the formation is definitely possible is enhanced by about 10 GeV (Fig. 6).

The $\Upsilon$ suppression, which in our prescription is possible only at the LHC energy, is seen to be clearly affected by the speed of sound but not by the transverse expansion of the plasma (Fig. 7).

5 Summary and Discussion

We have seen that the survival probability of $J/\psi$ at RHIC energies and that of $\Upsilon$ at LHC energies can provide valuable information about the equation of state of the quark matter, as the results are not affected by uncertainties of transverse expansion of the plasma. If the transverse expansion of the plasma takes place, it gives a distinct shape to the survival probability of the $J/\psi$ at LHC energies, whose detection will be a sure signature of the transverse flow of the plasma within the QGP phase.

Before concluding we would add that in these exploratory demonstrations we have chosen some specific values [8] for the deconfining matter which can dissociate quarkonia. These could be different, in particular the $c_{sJ/\psi}^2$ could be much larger than
the value used here. This, however, will not change the basic results as the time-scales involved in the $\Upsilon$ and the $J/\psi$ suppression are so very different.

The other uncertainty comes from the usage of $\epsilon_s$ as the criterion for deconfinement. One could have as well used the Debye mass as the temperature and the fugacity changed in a chemically equilibrating plasma, to fix the deconfining zone. This can indeed be done, along with the other extreme of the Debye mass estimated from lattice QCD. This has been studied in great detail by authors of Ref. [17] and can be easily extended to the present case. We plan to do it in a future publication.

We do realize that we have only looked at the directly produced resonances and ignored the fact that they may be fed by decay from higher resonances. Work on this, along with the study of other resonances—for whom the competition between the formation times and the durations of the screening medium may be different, is in progress.

In brief, we have shown that the $J/\psi$ and $\Upsilon$ suppression at RHIC and LHC energies can be successfully used to map the equation of state for the quark-matter. As the two processes will map different but overlapping regions, taken together, these results will help us to explore a vast region of the equation of state.

Acknowledgement

We thank Pankaj Jain and Bikash Sinha for useful discussions.

References


Table 1: Critical screening masses, etc. for quarkonia [7, 8].

<table>
<thead>
<tr>
<th>State</th>
<th>$J/\psi$</th>
<th>$\chi_c$</th>
<th>$\Upsilon$</th>
<th>$\Upsilon'$</th>
<th>$\chi_b$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M$ (GeV)</td>
<td>3.1</td>
<td>3.5</td>
<td>9.4</td>
<td>10.0</td>
<td>9.9</td>
</tr>
<tr>
<td>$r$ (fm)</td>
<td>0.45</td>
<td>0.70</td>
<td>0.23</td>
<td>0.51</td>
<td>0.41</td>
</tr>
<tr>
<td>$\tau_F$ (fm)</td>
<td>0.89</td>
<td>2.0</td>
<td>0.76</td>
<td>1.9</td>
<td>2.6</td>
</tr>
<tr>
<td>$\mu_D$ (GeV)</td>
<td>0.70</td>
<td>0.34</td>
<td>1.57</td>
<td>0.67</td>
<td>0.56</td>
</tr>
<tr>
<td>$T_d/T_c$</td>
<td>1.17</td>
<td>1.0</td>
<td>2.62</td>
<td>1.12</td>
<td>1.0</td>
</tr>
<tr>
<td>$\epsilon_s$ (GeV/fm$^3$)</td>
<td>1.92</td>
<td>1.12</td>
<td>43.37</td>
<td>1.65</td>
<td>1.12</td>
</tr>
</tbody>
</table>
Figure 1: The contour of deconfining matter capable of suppressing direct $J/\psi$ formation at RHIC energies. The solid curves are results with transverse flow, while the dashed curves are for longitudinal flow.
Figure 2: The contour of deconfining matter capable of suppressing direct $J/\psi$ formation at LHC energies. The solid curves are results with transverse flow, while the dashed curves are for longitudinal flow.
Figure 3: The contour of deconfining matter capable of suppressing direct $\Upsilon$ formation at LHC energies. The solid curves are results with transverse flow, while the dashed curves are for longitudinal flow.
Figure 4: The survival probability for $J/\psi$ at SPS, RHIC, and LHC energies, with only longitudinal cooling, when speed of sound is changed.
Figure 5: The survival probability for $J/\psi$ at RHIC energy with longitudinal and transverse cooling when speed of sound is changed.
Figure 6: The survival probability for $J/\psi$ at LHC energy with longitudinal and transverse cooling when speed of sound is changed.
Figure 7: The survival probability for $\Upsilon$ at (RHIC and) LHC energy with longitudinal and transverse cooling when speed of sound is changed.
(fm)

\[ r \]

\[ \frac{t}{(fm/c)} \]

\[ \frac{\tau_s}{(fm/c)} \]

\[ \epsilon_s = 43.4 \text{ GeV} \]

\[ pp + p \otimes p \]

\[ \text{Trans. Exp. Long. Exp.} \]

\[ c_s^2 = 1/3 \]

\[ c_s^2 = 1/5 \]

Life-time of \( r \)-disсоssiating deconfined matter