On the particle acceleration near the light surface of radio pulsars

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ABSTRACT

The two-fluid effects on the radial outflow of relativistic electron–positron plasma are considered. It is shown that for large enough Michel (1969) magnetization parameter $\sigma \gg 1$ and multiplication parameter $\lambda = n/n_{GJ} \gg 1$ one-fluid MHD approximation remains correct in the whole region $|E| < |B|$. In the case when the longitudinal electric current is smaller than the Goldreich–Julian one, the acceleration of particles near the light surface $|E| = |B|$ is determined. It is shown that, as in the previously considered (Beskin Gurevich & Istomin 1983) cylindrical geometry, almost all electromagnetic energy is transformed into the energy of particles in the narrow boundary layer $\Delta \varpi / \varpi \sim \lambda^{-1}$.

Key words: two-fluid relativistic MHD:radio pulsars—particle acceleration

1 INTRODUCTION

Despite the fact that the structure of the magnetosphere of radio pulsars remains one of the fundamental astrophysical problems, the common view on the key theoretical question — what is the physical nature of the neutron star braking — is absent (Michel 1991, Beskin Gurevich & Istomin 1993, Mestel 1999). Nevertheless, very extensive theoretical studies in the seventies and the eighties allowed to obtain some model-independent results. One of them is the absence of magnetodipole energy loss. This result was first obtained theoretically (Henriksen & Norton 1975, Beskin et al 1983). It was shown that the electric charges filling the magnetosphere screen fully the magnetodipole radiation of a neutron star for an arbitrary inclination angle $\chi$ between the rotational and magnetic axes if there are no longitudinal currents flowing in the magnetosphere. Later this result was also confirmed by observations. The direct detections of the interaction of the pulsar wind with a companion star in close binaries (see e.g. Djorgovsky & Evans 1988, Kulkarni & Hester 1988) have shown that it is impossible to explain the heating of the companion by a low–frequency magnetodipole wave.

On the other hand, the detailed mechanism of particle acceleration remains unclear. Indeed, a very high magnetization parameter $\sigma$ (Michel 1969) in the pulsar magnetosphere demonstrates that within the light cylinder $r < R_L = c/\Omega$ the main part of the energy is transported by the Poynting flux. It means that the additional mechanism of particle acceleration must work in the vicinity of the light cylinder. It is necessary to stress that an effective particle acceleration can only take place for small enough longitudinal electric currents $I < I_{GJ}$ when the plasma has no possibility to pass smoothly through the fast magnetosonic surface and when the light surface $|E| = |B|$ is located at a finite distance. As to the case of the large longitudinal currents $I > I_{GJ}$, both analytical (Tomimatsu 1994, Begelman & Li 1994, Beskin et al 1998) and numerical (Bogovalov 1997) considerations demonstrate that the acceleration becomes ineffective outside the fast magnetosonic surface, and the particle-to-Poynting flux ratio remains small: $\sim \sigma^{-2/3}$ (Michel 1969, Okamoto 1978).

The acceleration of an electron–positron plasma near the light surface was considered by Beskin Gurevich and Istomin (1983) in the simple 1D cylindrical geometry for $I \ll I_{GJ}$. It was shown that in a narrow boundary layer $\Delta \varpi / \varpi \sim 1/\lambda$ almost all electromagnetic energy is actually converted to the particles energy. Nevertheless, cylindrical geometry does not provide the complete picture of particle acceleration. In particular, it was impossible to include self–consistently the disturbance of a poloidal magnetic field and an electric potential, the later playing the main role in the problem of the plasma acceleration (for more details see e.g. Mestel & Shibata 1994). Hence, a more careful 2D consideration is necessary.
In Sect. 2 we formulate a complete system of 2D two-fluid MHD equations describing the electron-positron outflow from a magnetized body with a monopole magnetic field. The presence of an exact analytical force-free solution (Michel 1973) allows us to linearize this system which results in the existence of invariants (energy and angular momentum) along unperturbed monopole field lines similar to the ideal one-fluid MHD flow. In Sect. 3 it is shown that for $\sigma \gg 1$ and $\lambda \gg 1$ ($\lambda = n/ncL$ is the multiplication factor) the one-fluid MHD approximation remains true in the entire region within the light surface. Finally, in Sect. 4 the acceleration of particles near the light surface $|E| = |B|$ is considered. It is shown that, as in the case of cylindrical geometry, in a narrow boundary layer $\Delta \psi / \psi \sim \lambda^{-1}$ almost all the electromagnetic energy is converted into the energy of particles.

2 BASIC EQUATIONS

Let us consider a stationary axisymmetric outflow of a two-component plasma in the vicinity of an active object with a monopole magnetic field. It is necessary to stress that, of course, the monopole magnetic field is a rather crude approximation for a pulsar magnetosphere. Nevertheless, even for a dipole magnetic field near the origin, at large distances $r \gg R_L$ in the wind zone the magnetic field can have a monopole-like structure. For this reason the disturbance of a monopole magnetic field can give us an important information concerning particle acceleration far from the neutron star.

The structure of the flow is described by the set of Maxwell’s equations and the equations of motion

$$\nabla E = 4\pi \rho_e, \quad \nabla \times E = 0,$$

$$\nabla B = 0, \quad \nabla \times B = \frac{4\pi}{c} j,$$

$$(v^\pm \nabla)p^\pm = \pm e \left( E + \frac{v^\pm}{c} \times B \right).$$

Here $E$ and $B$ are the electric and magnetic fields, $\rho_e$ and $j$ are the charge and current densities, and $v^\pm$ and $p^\pm$ are the speed and momentum of particles. In the limit of infinite particle energy

$$\gamma = \infty, \quad v^0_r = c, \quad v^0_\varphi = 0, \quad v^0_\theta = 0,$$

and for charge and current density

$$\rho^0_e = \rho_e \frac{R_s^2}{r^2} \cos \theta, \quad j_r = \rho_e \frac{R_s^2}{r^2} \cos \theta,$$

the monopole poloidal magnetic field

$$B^0_\varphi = B_s \frac{R_s^2}{r^2}, \quad B^0_\theta = 0,$$

is the exact solution of Maxwell’s equations. In this case

$$B^0_\varphi = E^0_\theta = -B_s \frac{R_s \Omega R_s}{c} \sin \theta, \quad E^0_\varphi = E^0_\theta = 0,$$

which coincides with the well-known Michel (1973) solution. Here $\gamma$ is the Lorentz-factor of particles, $B_s$ is the magnetic field on the surface of the sphere $r = R_s$, and $\rho_e = \text{const}$. As a result, the angular velocity can be rewritten in a form $\Omega = 2\pi c/|\rho_e|/B_s$. The limit $\gamma \to \infty$ just corresponds to zero particle mass in the force-free approximation.

It is convenient to introduce the electric field potential $\Phi(r, \theta)$, so that $E = -\nabla \Phi$ and

$$\Phi^0 = -\frac{\Omega R_s^2}{c} B_s \cos \theta,$$

and the flux function $\Psi(r, \theta)$, so that

$$B_\varphi^0 = \frac{\nabla \Psi \times e_\varphi}{2\pi c \sin \theta},$$

and $\Psi^0 = 2\pi B_s R_s^2(1 - \cos \theta)$. Then one can seek the first-order corrections for the case $v \neq c$ in the following form:

$$n^+ = \frac{\Omega B_s R_s^2}{2\pi c e} \left[ \lambda - \frac{1}{2} \cos \theta + \eta^+(r, \theta) \right],$$

$$n^- = \frac{\Omega B_s R_s^2}{2\pi c e} \left[ \lambda + \frac{1}{2} \cos \theta + \eta^-(r, \theta) \right],$$

$$\Phi(r, \theta) = \frac{\Omega R_s^2 B_s}{c} \left[ -\cos \theta + \delta(r, \theta) \right],$$

$$\Psi(r, \theta) = 2\pi B_s R_s^2 \left[ 1 - \cos \theta + \varepsilon f(r, \theta) \right].$$
\[ v_\phi^+ = c \left[ 1 - \xi_\phi^+(r, \theta) \right], \quad v_\phi^- = c \xi_\phi^-(r, \theta), \quad v_\theta^+ = c \xi_\theta^+(r, \theta), \quad v_\theta^- = c \xi_\theta^-(r, \theta), \]

\[ B_r = B_0 \frac{R_0^2}{r^2} \left( 1 + \frac{\varepsilon}{\sin \theta} \frac{\partial f}{\partial \theta} \right), \]

\[ B_\theta = -\frac{B_0 R_0^2}{r} \frac{\partial f}{\sin \theta \partial \theta}, \]

\[ B_\phi = B_0 \frac{\Omega R_0}{c} \frac{\partial f}{\partial \theta} \left[ -\sin \theta - \zeta(r, \theta) \right], \]

\[ E_r = -\frac{\Omega B_0 R_0^2}{c} \frac{\partial f}{\partial \theta}, \quad \frac{\partial f}{\partial \theta} = 0, \]

\[ E_\theta = \frac{\Omega R_0 B_0}{cr} \left( -\sin \theta - \frac{\partial \delta}{\partial \theta} \right). \]

Such a choice corresponds to a constant particle-to-magnetic flux ratio. Here \( \lambda \gg 1 \) is the multiplication parameter \( (\lambda = e n_0/|\rho_n|) \), where \( n_0 \) is the concentration of particles on the surface \( r = R_0 \) which is \( 10^5 - 10^7 \) for radio pulsars. In what follows we consider for simplicity the case \( \lambda = \text{const.} \)

Substituting (9)-(18) into equations (1)-(2), we obtain to the first-order approximation the following system of equations:

\[ -\frac{1}{\sin \theta} \frac{\partial}{\partial \theta} \left( \xi \sin \theta \right) = 2(\eta^+ - \eta^-) - 2 \left[ \left( \lambda - \frac{1}{2} \cos \theta \right) \xi^+ - \left( \lambda + \frac{1}{2} \cos \theta \right) \xi^- \right], \]

\[ 2(\eta^+ - \eta^-) + \frac{\partial}{\partial \theta} \left( \frac{\partial f}{\partial \theta} \right) + \frac{1}{\sin \theta} \frac{\partial}{\partial \theta} \left( \sin \theta \frac{\partial \delta}{\partial \theta} \right) = 0, \]

\[ -\frac{\varepsilon}{\sin \theta} \frac{\partial^2 f}{\partial \theta^2} + \frac{\varepsilon}{\sin \theta} \frac{\partial f}{\partial \theta} = \frac{2\Omega}{\sin \theta} \left[ \left( \lambda - \frac{1}{2} \cos \theta \right) \xi^+ - \left( \lambda + \frac{1}{2} \cos \theta \right) \xi^-, \right] \]

\[ \frac{\partial}{\partial \theta} \left( \xi^+ \gamma^+ \right) + \frac{\xi^+ \gamma^+}{r} = 4\lambda \sigma \left( -\frac{1}{\sin \theta \partial \theta} + \frac{\xi^+}{r} - \frac{\sin \theta}{r} \xi^+ + \frac{c}{\Omega r^2} \xi^+ \right), \]

\[ \frac{\partial}{\partial \theta} \left( \xi^- \gamma^- \right) + \frac{\xi^- \gamma^-}{r} = -4\lambda \sigma \left( -\frac{1}{\sin \theta \partial \theta} + \frac{\xi^-}{r} - \frac{\sin \theta}{r} \xi^- + \frac{c}{\Omega r^2} \xi^- \right), \]

\[ \frac{\partial}{\partial \theta} \left( \gamma^+ \right) + \frac{\gamma^+}{r} = 4\lambda \sigma \left( -\frac{\partial \delta}{\partial \theta} \sin \theta \frac{\partial f}{\partial \theta} - \frac{c}{\Omega r^2} \xi^+ \right), \]

\[ \frac{\partial}{\partial \theta} \left( \gamma^- \right) = -4\lambda \sigma \left( -\frac{\partial \delta}{\partial \theta} \sin \theta \frac{\partial f}{\partial \theta} - \frac{c}{\Omega r^2} \xi^- \right), \]

\[ \frac{\partial}{\partial \theta} \left( \xi^+ \gamma^- \right) + \frac{\xi^+ \gamma^-}{r} = 4\lambda \sigma \left( -\frac{\varepsilon}{\Omega r \sin \theta \partial \theta} + \frac{c}{\Omega r^2} \xi^+ \right), \]

\[ \frac{\partial}{\partial \theta} \left( \xi^- \gamma^+ \right) + \frac{\xi^- \gamma^+}{r} = -4\lambda \sigma \left( -\frac{\varepsilon}{\Omega r \sin \theta \partial \theta} - \frac{c}{\Omega r^2} \xi^- \right). \]

Here

\[ \sigma = \frac{\Omega e B_0 R_0^2}{4\lambda mc^2} \]

is the Michel’s (1969) magnetization parameter, \( m \) is the electron mass, and all deflecting functions are supposed to be \( \ll 1 \).

It is necessary to stress that for applications the magnetic field \( B_0 \) is to be taken near the light cylinder \( R_0 \approx R_1 \) because in the internal region of the pulsar magnetosphere \( B \propto r^{-3} \). As it has already been mentioned, only outside the light cylinder the poloidal magnetic field may have quasi monopole structure. As a result, \( B_0 \) is the Michel’s (1969) magnetization parameter, \( m \) is the electron mass, and all deflecting functions are supposed to be \( \ll 1 \).

\[ \sigma = \frac{\Omega^2 e B_0 R_0^3}{4\lambda mc^4} \approx 10^8 B_{12} \lambda^{-1} P^{-2}, \]

where \( B_0 \) is the magnetic field on the neutron star surface \( r = R \). Hence, for ordinary pulsars (\( P \sim 1 \text{s}, B_0 \sim 10^{12} \text{G} \)) we have \( \sigma \sim 10^3 - 10^5 \), and only for fast ones (\( P \sim 0.1 - 0.01 \text{s}, B_0 \sim 10^7 \text{G} \)) we have \( \sigma \sim 10^6 - 10^7 \).

Formally, this system of equations requires twelve boundary conditions. We consider for simplicity the case \( \Omega R/c \ll 1 \) when the star radius \( R \) is much smaller than the light cylinder. As a result, one writes down the first six boundary conditions as

\[ \xi_\phi^+(R_1, \theta) = 0, \]

\[ \xi_\phi^-(R_1, \theta) = 0, \]

\[ \gamma^+(R_1, \theta) = \gamma_n, \]

i.e. \( \xi_\phi^+(R_1, \theta) = 1/(2\gamma_n^2) \). According to all theories of particle generation near the neutron star surface (Ruderman Sutherland
1975, Arons Scharlemann 1979), \( \gamma_{\text{in}} \lesssim 10^2 \) for secondary plasma. For this reason in what follows we consider in more details the case
\[ \gamma_{\text{in}}^3 \ll \sigma, \tag{34} \]
when the additional acceleration of particles inside the fast magnetosonic surface takes place (see e.g. Beskin Kuznetsova Ralfikov 1998). It is this case that can be realized for fast pulsars. Moreover, it has more general interest because the relation (34) may be true also for AGNs. As to the case \( \gamma_{\text{in}}^3 \gg \sigma \) corresponding to ordinary pulsars, the particle energy remains constant (\( \gamma = \gamma_{\text{in}} \)) at any way up to the fast magnetosonic surface (see Bogovalov 1997 for details).

Further, one can put
\[ \delta(R_{\text{e}}, \theta) = 0, \tag{35} \]
\[ \varepsilon f(R_{\text{e}}, \theta) = 0, \tag{36} \]
\[ \eta^+(R_{\text{e}}, \theta) - \eta^-(R_{\text{e}}, \theta) = 0. \tag{37} \]

These conditions result from the relation \( cE_a + \Omega R c_{\varphi} \times B_s = 0 \) corresponding rigid rotation and perfect conductivity of the surface of a star. Finally, as will be shown in Sect. 3.2, the derivative \( \partial \delta / \partial r \) actually determines the phase of plasma oscillations only and plays no role in the global structure. Finally, the determination of the electric current and, say, the derivative \( \partial f / \partial r \) depend on the problem under consideration. Indeed, as is well known, the cold one-fluid MHD outflow contains two singular surfaces, Alfvénic and fast magnetosonic ones. It means that for the transonic flow two latter functions are to be determined from critical conditions (Heyvaerts 1996). In particular, the longitudinal electric current within this approach is not a free parameter. On the other hand, if the electric current is restricted by some physical reason, the flow cannot pass smoothly through the fast magnetosonic surface. In this case, which can be realized in the magnetosphere of radio pulsars (Beskin et al 1983, Beskin & Malyshkin 1998), near the light surface \( |E| = |B| \) an effective particle acceleration may take place. Such an acceleration will be considered in Sect. 4.

3 THE ELECTRON–POSITRON OUTFLOW

3.1 The MHD Limit

In the general case Eqns. (19)–(28) have several integrals. Firstly, Eqns.(21), (25), and (26) result in
\[ \zeta - \frac{2}{\tan \theta} \frac{\delta}{\delta t} - \left( \lambda - 1/2 \cos \theta \right) \gamma_{\text{in}} = \frac{1}{\sigma \sin \theta} \gamma_{\text{in}} + \frac{I(\theta)}{\sin \theta}. \tag{38} \]
where \( I(\theta) \) describe the disturbance of the electric current at the star surface by the equation \( I(R, \theta) = I_A \left[ \sin^2 \theta + l(\theta) \right] \).

Expression (38) corresponds to conservation of the total energy flux along a magnetic field line. Furthermore, Eqns. (25)–(28) together with the boundary conditions (35), (36) result in
\[ \delta = \varepsilon f - \frac{1}{4 \lambda \sigma} \gamma_{\text{in}} \left( 1 - \frac{\Omega r \sin \theta}{c} \xi_{\varphi} \right) + \frac{1}{4 \lambda \sigma} \gamma_{\text{in}}; \tag{39} \]
\[ \delta = \varepsilon f + \frac{1}{4 \lambda \sigma} \gamma_{\text{in}} \left( 1 - \frac{\Omega r \sin \theta}{c} \xi_{\varphi} \right) - \frac{1}{4 \lambda \sigma} \gamma_{\text{in}}. \tag{40} \]

They correspond to conservation of the \( z \)-component of the angular momentum for both types of particles. It is necessary to stress that the complete nonlinearized system of equations contains no such simple invariants.

As \( \sigma \lambda \gg 1 \), we can neglect in Eqns. (23)–(28) their left-hand sides. In this approximation we have \( \xi_{\text{in}} = \xi_{\text{in}} \) i.e. \( \gamma_{\text{in}} = \gamma_{\text{in}} \), so that
\[ \frac{1}{r} \frac{\partial \delta}{\partial \theta} + \frac{\zeta}{r} \frac{\sin \theta}{r} \xi_{\varphi} + \frac{c}{1 \Omega r^2} \xi_{\varphi} = 0, \tag{41} \]
\[ \varepsilon c \frac{\partial f}{\partial r} + \frac{1}{1 \Omega r^2} \xi_{\varphi} = 0, \tag{42} \]
and
\[ \gamma \left( 1 - \frac{\Omega r \sin \theta}{c} \xi_{\varphi} \right) = \gamma_{\text{in}}. \tag{43} \]

Hence, within this approximation
\[ \delta = \varepsilon f, \tag{44} \]
\[ \zeta = \frac{2}{\tan \theta} \varepsilon f + \frac{l(\theta)}{\sin \theta} - \frac{1}{\sigma \sin \theta} (\gamma - \gamma_{\text{in}}). \tag{45} \]
Substituting these expressions into (41) and using Eqns. (19)–(22), we obtain the following equation describing the disturbance of the magnetic surfaces

\[ \varepsilon (1 - x^2 \sin^2 \theta) \frac{\partial f}{\partial x} + \varepsilon (1 - x^2 \sin^2 \theta) \sin \theta \frac{\partial}{\partial \theta} \left( \frac{1}{\sin \theta} \frac{\partial f}{\partial \theta} \right) - 2\varepsilon x \sin^2 \theta \frac{\partial f}{\partial x} - 2\varepsilon \sin \theta \cos \theta \frac{\partial f}{\partial \theta} + 2\varepsilon (3 \cos^2 \theta - 1)f \]

(46)

\[ + \frac{1}{\sin \theta} \frac{d}{d\theta} \left( \frac{x^2 \sin^2 \theta}{\sin \theta} \right) - 2\frac{\cos \theta}{\sigma} (\gamma - \gamma_{in}) - \frac{\sin \theta \partial \gamma}{\partial \theta} - 2\lambda \sin^2 \theta (\xi_\psi - \xi_\varphi) + \frac{2\lambda}{x} \sin \theta (\xi_\psi - \xi_\varphi) = 0, \]

where \( x = \Omega r/c \). One can see that it actually coincides with the one-fluid MHD Eqns.(32), (52) from Beskin et al (1998), but contains the two last additional nonhydrodynamical terms. Nevertheless, as will be shown in the next subsection, at small distances \( r \ll r_F \) where \( r_F \) is the radius of the fast magnetosonic surface we have

\[ -\lambda \sin^2 \theta (\xi_\psi - \xi_\varphi) + \frac{\lambda}{x} \sin \theta (\xi_\psi - \xi_\varphi) \approx 0, \]

(47)

so actually there is perfect agreement with the MHD approximation

\[ \varepsilon (1 - x^2 \sin^2 \theta) \frac{\partial f}{\partial x} + \varepsilon (1 - x^2 \sin^2 \theta) \sin \theta \frac{\partial}{\partial \theta} \left( \frac{1}{\sin \theta} \frac{\partial f}{\partial \theta} \right) - 2\varepsilon x \sin^2 \theta \frac{\partial f}{\partial x} - 2\varepsilon \sin \theta \cos \theta \frac{\partial f}{\partial \theta} + 2\varepsilon (3 \cos^2 \theta - 1)f \]

(48)

\[ + \frac{1}{\sin \theta} \frac{d}{d\theta} \left( \frac{x^2 \sin^2 \theta}{\sin \theta} \right) - 2\frac{\cos \theta}{\sigma} (\gamma - \gamma_{in}) - \frac{\sin \theta \partial \gamma}{\partial \theta} - 2\frac{\cos \theta}{\sigma} (\gamma - \gamma_{in}) - \frac{\sin \theta \partial \gamma}{\partial \theta} = 0. \]

As was shown earlier (Beskin et al 1998), to pass through the fast magnetosonic surface it’s necessary to have

\[ \left| \varepsilon \right| \leq \sigma^{-4/3}. \]

(49)

Hence, within the fast magnetosonic surface \( r \ll r_F \) one can neglect terms containing \( \delta = \varepsilon f \) and \( \zeta \). Then, relations (41) and (42) result in

\[ \gamma (1 - x \sin \theta \xi_\varphi) = \gamma_{in}, \]

(50)

\[ \xi_\varphi = \frac{\xi_\varphi}{x \sin \theta}, \]

(51)

\[ \xi_\psi = 0. \]

(52)

Finally, using the definition

\[ \gamma^2 = \frac{1}{2\xi_\varphi - \xi_\psi^2}, \]

(53)

we obtain for \( \sigma \gg \gamma_{in}^2 \) for \( r \ll r_F \)

\[ \gamma^2 = \frac{\gamma_{in}^2 + x^2 \sin^2 \theta}{\gamma_{in}^2 + x^2 \sin^2 \theta} \]

(54)

\[ \xi_\varphi = \frac{\sqrt{\gamma_{in}^2 + x^2 \sin^2 \theta} - \gamma_{in}}{x \sin \theta \sqrt{\gamma_{in}^2 + x^2 \sin^2 \theta}} \sim \frac{1}{x \sin \theta}, \]

(55)

\[ \xi_\psi = \frac{\sqrt{\gamma_{in}^2 + x^2 \sin^2 \theta} - \gamma_{in}}{x^2 \sin^2 \theta \sqrt{\gamma_{in}^2 + x^2 \sin^2 \theta}} \sim \frac{1}{x^2 \sin^2 \theta}. \]

(56)

in full agreement with the MHD results.

Next, to determine the position of the fast magnetosonic surface \( r_F \), one can analyze the algebraic equations (38) and (41) which give

\[ -\frac{\partial \delta}{\partial \theta} + \frac{2}{\tan \theta} \delta - \frac{1}{\sigma \sin \theta} \gamma - \sin \theta \xi_\varphi + \frac{1}{x} \xi_\psi = 0. \]

(57)

Using now expressions (43) and (53), one can find

\[ 2\gamma^3 - 2\sigma \left[ K + \frac{1}{2x^2} \right] \gamma^2 + \sigma \sin^2 \theta = 0, \]

(58)

where

\[ K(r, \theta) = 2 \cos \theta \delta - \sin \theta \frac{\partial \delta}{\partial \theta}. \]

(59)

Equation (58) allows us to determine the position of the fast magnetosonic surface and the energy of particles. Indeed, determining the derivative \( r \partial \gamma / \partial r \), one can obtain

\[ r \frac{\partial \gamma}{\partial r} = \frac{\gamma \sigma (r \partial K/\partial r - x^{-2})}{3\gamma - \sigma (2K + x^{-2})}. \]

(60)
As the fast magnetosonic surface is the X-point, both the nominator and denominator are to be equal to zero here. As a result, evaluating \( r \partial K / \partial r \) as \( K \), we obtain

\[
\delta \sim 1^{2/3};
\]
\[
r_F \sim 1^{1/3} R_L; \tag{62}
\]
\[
\gamma(r_F) = 1^{1/3} \sin^{2/3} \theta, \tag{63}
\]

where the last expression is exact. These equations coincide with those obtained within the MHD consideration. It is the self-consistent analysis when \( \delta = \varepsilon f \), and hence \( K \) depends on the radius \( r \) that results in the finite value for the fast magnetosonic radius \( r_F \). On the other hand, in a given monopole magnetic field, when \( \varepsilon f \) does not depend on the radius, the critical conditions result in \( r_F \rightarrow \infty \) for a cold outflow (Michel, 1969, Li et al 1992).

Near the fast magnetosonic surface \( r \sim 1^{1/3} R_L \) the MHD solution gives

\[
\gamma \sim 1^{1/3}, \tag{64}
\]
\[
\varepsilon f \sim 1^{-2/3}. \tag{65}
\]

Hence, Equs. (53), (55), and (56) result in

\[
\xi_r \sim 1^{-2/3}, \tag{66}
\]
\[
\xi_\theta \sim 1^{-2/3}, \tag{67}
\]
\[
\xi_\phi \sim 1^{-1/3}. \tag{68}
\]

As we see, the \( \theta \)-component of the velocity plays no role in the determination of the \( \gamma \).

However, analyzing the left-hand sides of the Equs. (23)–(28) one can evaluate the additional (nonhydrodynamic) variations of the velocity components

\[
\Delta \xi_r^\pm \sim \lambda^{-1} \sigma^{-4/3}, \tag{69}
\]
\[
\Delta \xi_\theta^\pm \sim \lambda^{-1} \sigma^{-2/3}, \tag{70}
\]
\[
\Delta \xi_\phi^\pm \sim \lambda^{-1} \sigma^{-1}. \tag{71}
\]

Hence, for nonhydrodynamic velocities \( \Delta \xi_r^\pm \ll \xi_r \) and \( \Delta \xi_\phi^\pm \ll \xi_\phi \) to be small, it is necessary to have a large magnetization parameter \( \sigma \gg 1 \) only. On the other hand, \( \Delta \xi_\theta^\pm / \xi_\theta \sim \lambda^{-1} \). In other words, for a highly magnetized plasma \( \sigma \gg 1 \) even outside the fast magnetosonic surface the velocity components (and, hence, the particle energy) can be considered hydrodynamically. The difference \( \sim \lambda^{-1} \) appears in the \( \theta \) component only, but it does not affect the particle energy. Finally, one can obtain from

\[
\delta - \varepsilon f \sim \lambda^{-2} \sigma^{-2/3}. \tag{72}
\]

To put it differently, at large distances the nonhydrodynamical terms are much smaller than hydrodynamical ones.

As a result, at large distances where, according to (39)–(40), one can neglect the toroidal component \( \xi_\phi \), we obtain

\[
\delta = \varepsilon f, \tag{73}
\]
\[
\zeta = \frac{2}{\tan \theta} \sigma^{-1} \frac{1}{\sin \theta} \gamma. \tag{74}
\]

On the other hand, Eqn. (23) gives

\[
\zeta = \frac{\partial \delta}{\partial \theta} + \sin \theta \xi_r. \tag{75}
\]

Together with (21) one can obtain for \( r \gg r_F \)

\[
\gamma = \sigma \left( 2 \cos \theta \varepsilon f - \varepsilon \sin \theta \frac{\partial \varepsilon f}{\partial \theta} \right), \tag{76}
\]

which coincides with the MHD solution. Finally, using Equs. (19), (20), and neglecting the nonhydrodynamic term \( 4 \lambda (\xi_\phi^+ - \xi_\phi^-) \), one can find

\[
\varepsilon \frac{\partial}{\partial r} \left( r \frac{\partial f}{\partial r} \right) - 4 \cos \theta \xi_r - \sin \theta \frac{\partial}{\partial \theta} \xi_r + \frac{1}{\sin \theta} \frac{\partial}{\partial \theta} (\xi_\phi \sin \theta) = 0. \tag{77}
\]

Together with (76) this equation in the limit \( r \gg r_F \) coincides with the asymptotic version of the trans-field equation (Tomimatsu 1994, Beskin et al 1998)

\[
\varepsilon \frac{\partial^2 f}{\partial r^2} + 2 \sigma \frac{\partial f}{\partial r} - \sin \theta \frac{D}{D + 1} \frac{\partial q}{\partial \theta} = 0, \tag{78}
\]

where
where \( g(\theta) = K(\theta)/\sin^2 \theta \), and

\[
D + 1 = \frac{1}{\sigma^2 \sin \theta} g^{-1}(\theta) \ll 1.
\]

In this limit, none of the terms containing \( \xi^+ \) and \( \xi^- \) plays role in the asymptotic trans-field equation. Hence, it is not necessary to consider the effect of the nonhydrodynamical term \( 4\lambda(\xi_\theta^+ - \xi_\theta^-) \) either.

### 3.2 Plasma Oscillations

In the intermediate region \( r \ll r_F \) Eqn. (77) cannot be used. The point is that in the limit \( \lambda \gg 1 \) the important role in Eqns. (19) and (22) is played by the nonhydrodynamic terms (47) corresponding to different velocities of two components. As a result, the full version of Eqn. (77) has the form

\[
\frac{\partial}{\partial r} \left( r^2 \frac{\partial \delta}{\partial r} \right) - 4 \cos \theta \xi_r - \sin \theta \frac{\partial \xi_r}{\partial \theta} + \frac{1}{x \sin \theta} \frac{\partial}{\partial \theta} (\xi_\varphi \sin \theta) + 2\lambda(\xi_r^+ - \xi_r^-) = 0.
\]

Indeed, one can see from equations (19) and (20) that near the origin \( x = R_p \) in the case \( \gamma_\text{in}^+ = \gamma_\text{in}^- \) (and for the small variation of the current \( \zeta \sim \sigma^{-4/3} \) which is necessary, as was already stressed, to pass through a fast magnetosonic surface) the density variation on the surface is large enough: \( (\eta^+ - \eta^-) \sim \gamma_\text{in}^{-2} \gg \zeta \). Hence, the derivative \( \partial^2 \delta/\partial r^2 \) here is of the order of \( \gamma_\text{in}^{-2} \). On the other hand, according to (22), the derivative \( \varepsilon \partial^2 f/\partial t^2 \) is \( x^2 \) times smaller. This means that in the two-component system the longitudinal electric field is to appear resulting in a redistribution of the particle energy. Clearly, such a redistribution is impossible for the charge-separated outflow. In other words, for a finite particle energy a one-component plasma cannot be realized by a small redistribution of particle energy (Ruderman & Sutherland 1975, Arons & Scharlemann 1989).

For simplicity, let us consider only small distances \( x \ll 1 \). In this case one can neglect the changes of the magnetic surfaces. Using now (25) and (26), we have

\[
\begin{align*}
\gamma^+ &= \gamma_\text{in} - 4\lambda \sigma \delta; \\
\gamma^- &= \gamma_\text{in} + 4\lambda \sigma \delta.
\end{align*}
\]

Finally, taking into account that \( \xi_\theta \) and \( \xi_\varphi \) are small here, one can obtain from (20)

\[
r^2 \frac{\partial^2 \delta}{\partial r^2} + 2r \frac{\partial \delta}{\partial r} + \frac{1}{\sin \theta} \frac{\partial}{\partial \theta} \left( \sin \theta \frac{\partial \delta}{\partial \theta} \right) + A \delta = \frac{\cos \theta}{\gamma_\text{in}},
\]

where

\[
A = 16 \frac{\lambda^2 \sigma}{\gamma_\text{in}} \gg 1.
\]

Eqn. (83) has a solution

\[
\delta = \delta_0 + r^{-1/2} [C_1 \sin(\mu \ln r) + C_2 \cos(\mu \ln r)] \cos \theta,
\]

where

\[
\delta_0 \approx \frac{\gamma_\text{in} \cos \theta}{16\lambda \sigma},
\]

and \( \mu \approx \sqrt[4]{\lambda} \). As we see, Eqn. (85) describes plasma oscillations similar to those considered by Shibata (1997) for charge-separated flow. The decrease of oscillations results from a more accurate consideration of the Laplace operator in a 3D space.

One can easily check that the additional potential \( \delta_0 \) is small, and it is not necessary to add it in (38) and (39)–(40). Moreover, the nonhydrodynamic disturbance \( \Delta \xi_r \) (as well as \( \Delta \gamma \)) is also small, \( \Delta \xi_r/\xi_r \approx \lambda^{-1} \). Hence, as was already stressed, the boundary condition \( \partial \delta / \partial r \) (determining together with (35) the coefficients \( C_1 \) and \( C_2 \)) does not affect the general structure of the flow. On the other hand, the presence of an additional electric potential \( \delta_0 \) results in a full compensation of the last term in (19)

\[
2\lambda(\xi_r^+ - \xi_r^-) - \cos \theta \xi_r \approx 0.
\]

Next, as \( \varepsilon f \ll \sigma^{-2/3} \) for \( r \ll r_F \), a similar expression can be written for the \( \varphi \)-components as well

\[
2\lambda(\xi_\varphi^+ - \xi_\varphi^-) - \cos \theta \xi_\varphi \approx 0.
\]

Expressions (87) – (88) must hold for the whole region \( r < r_F \). In this case, the final version of Eqn. (80) in the internal region \( r < r_F \) can be rewritten as
\[ \frac{\partial}{\partial \tau} \left( x^2 \frac{\partial \bar{\psi}}{\partial \tau} \right) - 2 \cos \theta \frac{\partial \bar{\psi}}{\partial \theta} + \frac{1}{x \sin \theta} \frac{\partial}{\partial \theta} (\xi \sin \theta) = 0. \]  

(89)

As \( \delta \sim \varepsilon \ll \sigma^{-2/3} \) for \( r \ll r_F \), and \( \xi \sim \gamma_0^{-2} \gg \delta \), the first term in (89) can be omitted. As a result, the solution of Eqn. (89) coincides exactly with the MHD expression, i.e. \( \gamma^2 = \gamma_0^2 + x^2 \sin^2 \theta \) (54). Finally, using (87), (88), and (55)–(56), one can easily check that the nonhydrodynamical terms (47) in the trans-field equation (48) do actually vanish.

4 THE BOUNDARY LAYER

Let us now consider the case when the longitudinal electric current \( I(R, \theta) \) in the magnetosphere of radio pulsars is too small (i.e. the disturbance \( I(\theta) \) is too large) for the flow to pass smoothly through the fast magnetosonic surface. First of all, it can be realized when the electric current is much smaller than the Goldreich one. This possibility was already discussed within the Ruderman–Sutherland model of the internal gap (Beskin et al 1983, Beskin & Malyshkin 1998). But it may take place in the Arons model (Arons & Scharlemann 1979) as well. Indeed, within this model the electric current is determined by the Ruderman–Sutherland model of the internal gap (Beskin et al 1983, Beskin & Malyshkin 1998). But it may take place be realized when the electric current is much smaller than the Goldreich one. This possibility was already discussed within this statement).

For simplicity let us consider the case \( I(\theta) = h \sin^2 \theta \). Neglecting now the last terms \( \propto \sigma^{-1} \) in the trans-field equation (48), we obtain

\[ \varepsilon (1 - x^2 \sin^2 \theta) \frac{\partial^2 f}{\partial x^2} + \varepsilon (1 - x^2 \sin^2 \theta) \frac{\sin \theta}{x^2} \frac{\partial}{\partial \theta} \left( \frac{1}{\sin \theta} \frac{\partial f}{\partial \theta} \right) - 2x \sin^2 \theta \frac{\partial f}{\partial x} - 2 \varepsilon \sin \theta \cos \theta \frac{\partial f}{\partial \theta} + 2 \varepsilon (3 \cos^2 \theta - 1) f + 4h \sin^2 \theta \cos \theta = 0, \]  

(90)

which actually coincides with the force–free equation (Beskin et al 1998). This equation has an exact analytical solution

\[ \varepsilon f = h \sigma^2 \sin^2 \theta \cos \theta. \]  

(91)

For \( h < 0 \) (when the electric current is smaller than the Goldreich one) this solution results in the appearance of the light surface \( |E| = |B| \) at the finite distance

\[ \varpi_c = \frac{R_i}{(2|h|)^{1/4}}. \]  

(92)

As we see, for \( l(\theta) = h \sin^2 \theta \) this surface has the form of a cylinder. It is important that the disturbance of magnetic surfaces \( \varepsilon f \sim (|h|)^{1/2} \) remains small here.

Comparing now the position of the light surface (92) with that of the fast magnetosonic surface (62), one can find that the light surface is located inside the fast magnetosonic one if

\[ \sigma^{-4/3} \ll |h| \ll 1, \]  

(93)

which is opposite to (49). One can check that the condition (93) just allows to neglect the non-force–free term in Eqn. (48).

Using now the solution (91) and the MHD condition \( \delta = \varepsilon f \), one can find from (58)

\[ 2 \gamma^3 - 2 \sigma \left( h \sigma^2 \sin^2 \theta + \frac{1}{2x^2} \right) \gamma^2 + \sigma \sin^2 \theta = 0. \]  

(94)

This equation shows that near the force–free boundary \( x_{ff} = (2|h|)^{-1/4} \) (92)

\[ \gamma = \sigma^{1/3} \sin^{2/3} \theta - \frac{2}{3} \frac{2|h|^{3/8}}{\sigma^{1/3} \sin^{4/3} \theta \sqrt{x_0 - x \sin \theta}}, \]  

(95)

where

\[ x_0 = \frac{1}{(2|h|)^{1/4}} \left[ 1 - \frac{3}{4(2|h|)^{1/2}} \frac{1}{(\sigma \sin^2 \theta)^{2/3}} \right], \]  

(96)

(see Fig. 1). Hence, the real solution is absent for \( x \sin \theta > x_0 \). Here \( \gamma(x_0) = \sigma^{1/3} \sin^{2/3} \theta \), the condition \( \sigma^{-4/3} \ll |h| \) resulting in \( \varpi_c < r_F \), and the last term in (96) being small.

Although the energy of particles at the limiting point is finite, the derivative \( d\gamma/d\varpi \) moves to infinity. Hence, near the light surface the left-hand sides in the Equs. (23) – (28) are to be taken into consideration. Since in our case the light surface has the form of a cylinder, one can move to derivatives perpendicular to the boundary layer only by

\[ \partial/\partial \tau = \sin \theta \partial/\partial \varpi; \]  

(97)

\[ \partial/\partial \theta = \varpi \cos \theta \partial/\partial \varpi. \]  

(98)
As a result, $\zeta$ can be eliminated from (19) and (21). Together with (20) they give the equation for $\delta$ (see (104)). Next, the invariants (39) and (40) can be used to define $\xi_{\varphi}^\pm$:

$$\xi_{\varphi}^+ = \frac{1}{x \sin \theta} \left[ 1 + \frac{4 \lambda \sigma (\delta - \varepsilon f)}{\gamma} \right] :$$

$$\xi_{\varphi}^- = \frac{1}{x \sin \theta} \left[ 1 - \frac{4 \lambda \sigma (\delta - \varepsilon f)}{\gamma} \right].$$

Furthermore, one can define

$$2 \xi_{\varphi}^+ = \frac{1}{(\gamma^+)^2} + (\xi_{\phi}^+)^2 + (\xi_{\theta}^+)^2;$$

$$2 \xi_{\varphi}^- = \frac{1}{(\gamma^-)^2} + (\xi_{\phi}^-)^2 + (\xi_{\theta}^-)^2.$$

As to the energy integral (38), it determines the variation of the current $\zeta$. Now it can be rewritten as

$$\zeta = \frac{2}{\tan \theta} \delta - \frac{(\gamma^+ + \gamma^-)}{2 \sigma \sin \theta}.$$

Finally, Eqns. (19) – (28) look like

$$\frac{d^2 \delta}{d\varpi^2} = 2 \sin \theta \cos \theta \left[ \left( \lambda - \frac{1}{2} \cos \theta \right) \xi_{\phi}^+ - \left( \lambda + \frac{1}{2} \cos \theta \right) \xi_{\phi}^- \right] - 2 \sin^2 \theta \left[ \left( \lambda - \frac{1}{2} \cos \theta \right) \xi_{\varphi}^+ - \left( \lambda + \frac{1}{2} \cos \theta \right) \xi_{\varphi}^- \right],$$

$$\frac{d^2 f}{d\varpi^2} = -2 \sin^2 \theta \left[ \left( \lambda - \frac{1}{2} \cos \theta \right) \xi_{\varphi}^+ - \left( \lambda + \frac{1}{2} \cos \theta \right) \xi_{\varphi}^- \right],$$

$$\frac{d}{d\varpi} \left( \xi_{\phi}^+ \gamma^+ \right) = 4 \lambda \sigma \left( \frac{\gamma^+ + \gamma^-}{2 \sigma \sin \theta \frac{d \delta}{d\varpi}} - \frac{\cos \theta}{\sin \theta} \frac{d \delta}{d\varpi} - \sin \theta \xi_{\varphi}^+ + \frac{\sin \theta}{x_0} \xi_{\phi}^+ \right),$$

$$\frac{d}{d\varpi} \left( \xi_{\phi}^- \gamma^- \right) = -4 \lambda \sigma \left( \frac{\gamma^+ + \gamma^-}{2 \sigma \sin \theta \frac{d \delta}{d\varpi}} - \frac{\cos \theta}{\sin \theta} \frac{d \delta}{d\varpi} - \sin \theta \xi_{\varphi}^- + \frac{\sin \theta}{x_0} \xi_{\phi}^- \right).$$

Figure 1. The behavior of the Lorentz factor in the case $\sigma^{-1/3} \ll |h| \ll 1$. One can see that the one-fluid MHD solution (95) exists for $\gamma < \sigma^{1/3}$ only. But in the two-fluid approximation in the narrow layer $\Delta \varpi = \varpi_c/\lambda$ the particle energy increases up to the value $\sim \sigma$ corresponding to the full conversion of the electromagnetic energy to the energy of particles.
\[ \omega_c \frac{d}{d\omega} \gamma^+ = 4\lambda \sigma \left( -\omega_c \frac{d\delta}{d\omega} - \sin \theta \xi_\phi^+ \right), \]
\[ \omega_c \frac{d}{d\omega} \gamma^- = -4\lambda \sigma \left( -\omega_c \frac{d\delta}{d\omega} - \sin \theta \xi_\phi^- \right), \]
where we neglected the terms \( \propto \delta/r \) in (106) and (107).

Comparing the leading terms, we have inside the layer \( \Delta \omega/R_c \sim \lambda^{-1} \)
\[ \gamma^\pm \sim h_c^{1/2} \sigma, \]
\[ \xi_\phi^\pm \sim h_c^{1/4}, \]
\[ \xi_r^\pm \sim h_c^{1/2}, \]
\[ \Delta \delta \sim h_c^{3/4}/\lambda, \]
where \( h_c = |h| \). Then the leading terms in (99) – (103) for \( \Delta \omega > \lambda^{-1} R_c \) are
\[ \xi_\phi^+ = \frac{1}{x \sin \theta} \approx \frac{1}{x_0}, \]
\[ \xi_\phi^- = \frac{1}{x \sin \theta} \approx \frac{1}{x_0}, \]
\[ 2\xi_r^+ = (\xi_\phi^+)^2 + (\xi_r^+)^2, \]
\[ 2\xi_r^- = (\xi_\phi^-)^2 + (\xi_r^-)^2, \]
\[ \zeta = \frac{(\gamma^+ - \gamma^-)}{2\sigma \sin \theta}, \]
where \( x_0 = \omega_c/R_c = (2|h|)^{-1/4} \). Hence, one can totally neglect \( \epsilon f \) and \( \delta \) in (99) – (100), so Eqns. (104) – (109) in the region \( \Delta \omega > \lambda^{-1} R_c \) can be rewritten as
\[ \omega^2 \frac{d^2 \delta}{d\omega^2} = 2\lambda \sin \theta \cos \theta (\xi_\phi^+ - \xi_\phi^-), \]
\[ \omega_c \frac{d}{d\omega} (\xi_\phi^+ \gamma^+) = 4\lambda \sigma \left( \frac{\gamma^+ + \gamma^-}{2\sigma \sin \theta} - \omega_c \cos \theta \frac{d\delta}{d\omega} - \sin \theta \xi_r^+ \right), \]
\[ \omega_c \frac{d}{d\omega} (\xi_\phi^- \gamma^-) = -4\lambda \sigma \left( \frac{\gamma^+ + \gamma^-}{2\sigma \sin \theta} - \omega_c \cos \theta \frac{d\delta}{d\omega} - \sin \theta \xi_r^- \right), \]
\[ \omega_c \frac{d}{d\omega} (\gamma^+) = -4\lambda \sigma \sin \theta \xi_\phi^+, \]
\[ \omega_c \frac{d}{d\omega} (\gamma^-) = 4\lambda \sigma \sin \theta \xi_\phi^-, \]
with all the terms in the right–hand sides of (120) and (121) being of the same order of magnitude.

As a result, the nonlinear equations (119) – (123) and (105) give the following simple asymptotic solution
\[ \gamma^\pm = 4\sin^2 \theta \sigma (\lambda l)^2, \]
\[ \xi_\phi^\pm = \mp 2\sin \theta \lambda l, \]
\[ \Delta \delta = -\frac{4}{3} \sin^2 \theta \cos \theta \lambda^{-1} (\lambda l)^3, \]
\[ \Delta (\epsilon f) = \sin^2 \theta \cos \theta \lambda^{-2} (\lambda l)^2, \]
\[ \zeta = -4\sin \theta (\lambda l)^2, \]
where now \( l = \Delta \omega/\omega_c \). It is important that the last expressions are correct for arbitrary relations between \( \gamma_0^\pm \) and \( \sigma \). As we can see, in the narrow layer \( \Delta \omega = \omega_c/\lambda \) the particle energy increases up to the value \( \sim \sigma \) which corresponds to the full conversion of the electromagnetic energy to the energy of particles. For this reason we have here \( |\zeta| \sim 1 \), which just means the diminishing of the toroidal magnetic field determining the flux of the electromagnetic energy. On the other hand, the variation of the electric potential remains small \( \delta \sim \lambda^{-1} \), to say nothing about the variation of the magnetic surfaces \( \Delta \epsilon f \sim \lambda^{-2} \). These results coincide exactly with our previous evaluations (Beskin et al 1983) allowing us to neglect variations of the electric potential and the poloidal magnetic structure in the 1D cylindrical case.

The latter result has a simple physical explanation. Indeed, the diminishing of the toroidal magnetic field is connected with the \( \theta \)-component of the electric current which is produced by all the particles moving in opposite directions. On the other hand, the change of the electric potential depends on the small difference between the electron and positron densities. As a result, according to (127) and (128), the change of the toroidal magnetic field is just \( \lambda \) times larger than the change
of the electric potential. Unfortunately, it is impossible to consider this region more thoroughly because for $\lambda l \sim 1$ we have $\xi_\delta \sim 1$ and $\xi_\parallel \sim 1$, i.e. the linear approximation (19) – (28) itself becomes incorrect.

It is necessary to stress as well that we do not include into consideration the radiation reaction force

$$F_{x}^{(\text{rad})} = -\frac{2}{3} \frac{e^4}{m^2 c^4} \gamma^2 \left[(E_y - B_z)^2 + (E_z - B_y)^2\right],$$

(129)

which can be important for large enough particle energy. Comparing (129) with appropriate terms in (120) – (123) one can conclude that the radiation force can be neglected for $\sigma < \sigma_c$, where

$$\sigma_c = \left(\frac{c}{\lambda_r \Omega} \right)^{1/3} \approx 10^6,$$

(130)

and $r_e = e^2/me^2$ – classical electron radius. This relation can be rewritten in the form

$$\frac{\Omega R}{c} < 3 \times 10^{-3} B_{12}^{-3/7} \lambda_4^{2/7},$$

(131)

which gives

$$P > 0.06 B_{12}^{3/7} \lambda_4^{-2/7} \text{s}.$$  

(132)

Hence, for most radio pulsars the radiation force indeed can be neglected. As to the pulsars with $\sigma > \sigma_c$, it is clear that for $\gamma > \sigma_c$ the radiation force becomes larger than the electromagnetic one and strongly inhibits any further acceleration. As a result, we can evaluate the maximum gamma–factor which can be reached during the acceleration as

$$\gamma_{\text{max}} \approx \sigma_c \approx 10^6.$$

(133)

5 DISCUSSION

Thus, on a simple example it was demonstrated that for real physical parameters of the magnetosphere of radio pulsars ($\sigma \gg 1$ and $\lambda \gg 1$) the one–fluid MHD approximation remains true in the whole region within the light surface $|E| = |B|$. On the other hand, it was shown that in a more realistic 2D case the main properties of the boundary layer near the light surface existing for small enough longitudinal currents $I < I_{GJ}$ (effective energy transformation from electromagnetic field to particles, current closure in this region, smallness of the disturbance of electric potential and poloidal magnetic field) remain the same as in the 1D case considered previously (Beskin et al 1983).

It is necessary to stress the main astrophysical consequences of our results. First of all, the presence of such a boundary layer explains the effective energy transformation of electromagnetic energy into the energy of particles. As was already stressed, now the existence of such an acceleration is confirmed by observations of close binaries containing radio pulsars (as to the particle acceleration far from a neutron star, see e.g. Kennel & Coroniti 1984, Hoshino et al 1992, Gallant & Arons 1994). Simultaneously, it allows us to understand the current closure in the pulsar magnetosphere. Finally, particle acceleration results in the additional mechanism of high–energy radiation from the boundary of the magnetosphere (for more details see Beskin et al 1993).

Nevertheless, it is clear that the results obtained do not solve the whole pulsar wind problem. Indeed, as in the cylindrical case, it is impossible to describe the particle motion outside the light surface. The point is that, as one can see directly from Eqn. (126), for a complete conversion of electromagnetic energy into the energy of particles it is enough for them to pass only $\lambda^{-1}$ of the total potential drop between pulsar magnetosphere and infinity. It means that the electron–positron wind propagating to infinity has to pass the potential drop which is much larger than their energy. It is possible only in the presence of electromagnetic waves even in an axisymmetric magnetosphere which is stationary near the origin. Clearly, such a flow cannot be considered even within the two–fluid approximation. In our opinion, it is only a numerical consideration that can solve the problem completely and determine, in particular, the energy spectrum of particles and the structure of the pulsar wind. Unfortunately, up to now such numerical calculations are absent.

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