We explore the Tully-Fisher relation over five decades in stellar mass in galaxies with circular velocities ranging over $30 < V_c < 300$ km s$^{-1}$. We find a clear break in the optical Tully-Fisher relation: field galaxies with $V_c < 90$ km s$^{-1}$ fall below the relation defined by brighter galaxies. These faint galaxies are however very gas rich; adding in the gas mass and plotting baryonic disk mass $M_d = M_* + M_{gas}$ in place of luminosity restores a single linear relation. The Tully-Fisher relation thus appears fundamentally to be a relation between rotation velocity and total baryonic mass of the form $M_d \propto V_c^4$.

Subject headings: galaxies: dwarf — galaxies: formation — galaxies: fundamental parameters — galaxies: kinematics and dynamics — galaxies: spiral — dark matter

1. Introduction

The relation between luminosity and rotation velocity for galaxies is well known (Tully & Fisher 1977). It has been used extensively in estimating extragalactic distances (e.g., Sakai et al. 2000, Tully & Pierce 2000), and it provides a critical constraint on galaxy formation theory (Dalcanton, Spergel, & Summers 1997; McGaugh & de Blok 1998; Mo, Mao, & White 1998; Steinmetz & Navarro 1999; van den Bosch 1999). However, the physical basis of the Tully-Fisher relation remains unclear.

The requirements of the empirical Tully-Fisher relation are simple, but the steep slope and small scatter are difficult to understand. Luminosity must trace total (dark plus luminous) mass, which in turn scales exactly with circular velocity. Considerable fine-tuning is required to obtain these strict proportionalities (McGaugh & de Blok 1998). The intrinsic properties of dark halos are not expected to be as tightly correlated as observed (Eisenstein & Loeb 1995). The mapping from the properties of dark matter halos to observable quantities should introduce more scatter, not less. Somehow the baryons “know” precisely how many stars to form.

Let us suppose that, for whatever fundamental reason, there does exist a universal relationship between total mass and rotation velocity of the form $M_{tot} \propto V_c^b$. The empirical Tully-Fisher relation then follows if luminosity traces mass:

$$ L = \Upsilon_*^{-1} f_* f_d f_b M_{tot}, \quad (1) $$

where $f_b$ is the baryon fraction of the universe, $f_d$ is the fraction of the baryons associated with a particular galaxy halo which reside in the disk, $f_*$ is the fraction of disk baryons in the form of stars, and $\Upsilon_*$ is
the mass-to-light ratio of the stars. Each of the pieces which intervene between $L$ and $M_{\text{tot}}$ must be a nearly universal constant shared by all disks in order to maintain the strict proportionality the Tully-Fisher relation requires. Cast in this form, the traditional luminosity-linewidth relation is a sub-set of a more fundamental relation between \textit{baryonic mass} and \textit{rotational velocity}. In this context, one would expect to find galaxies which deviate from the luminosity-linewidth relation because much of their baryonic mass is not in the form of stars. For example, a gas rich galaxy should appear underluminous for its circular velocity, but would, after correction for the gas content, fall on the underlying “Baryonic Tully-Fisher relation” (cf. Freeman 1999).

In this paper, we specifically test this premise by constructing the luminosity-linewidth and Baryonic Tully-Fisher relations for a sample of late type galaxies that span a much larger range of luminosities than any previously available sample. Section 2 describes the data we employ. Section 3 discusses the results and §4 explores some of their implications. A summary is given in §5. All distance dependent quantities assume $H_0 = 75 \text{ km s}^{-1} \text{ Mpc}^{-1}$.

2. Data

We employ several data sets to maximize the dynamic range over which we can explore the Tully-Fisher relation. The different data sets have photometry in different pass bands. To put the data on the same system, and get at the question of the underlying mass, we assume a stellar mass-to-light ratio for each pass band. Stellar mass is most directly traced by the redder pass bands, so we adopt these when possible.

For galaxies with $V_c \gtrsim 100 \text{ km s}^{-1}$ we use the extensive $H$-band data for late type cluster spirals of Bothun et al. (1985). Circular velocities are estimated as half of the linewidth $W_{20}$. For galaxies of lower rotation velocity, we use the data for late type dwarf low surface brightness galaxies from the survey of Schombert, Pildis, & Eder (1997). This is currently the largest sample of field dwarf galaxies with both linewidths $W_{20}$ and HI masses (Eder & Schombert 1999) and red band photometry (Pildis, Schombert, & Eder 1997). The photometry provides $I$-band magnitudes and axial ratios for inclination estimates. Of these galaxies, only those with axial ratios $b/a < 0.71$, corresponding to $i > 45\degree$ for an intrinsic axial ratio of $q_0 = 0.15$, are used in order to minimize $\sin(i)$ errors. These nevertheless contributes substantially to the scatter, as inclinations estimated from the axial ratios of dim galaxies are intrinsically uncertain. The data for these faint galaxies extend the Tully-Fisher relation to much lower luminosities and circular velocities than have been explored previously.

The fundamental rotation velocity of interest here is the flat portion of the rotation curve, $V_{\text{flat}}$. Presumably, the line width $W_{20}$ commonly employed in Tully-Fisher work is an adequate indicator of $V_{\text{flat}}$. As a check on this, we also employ the data of Verheijen (1997) and McGaugh & de Blok (1998) for which $V_{\text{flat}}$ is measured from resolved rotation curves. The data of Verheijen (1997) are $K'$-band data for spiral galaxies in the UMa cluster (Tully et al. 1996), while the data discussed by McGaugh & de Blok (1998) are $B$-band data drawn from a variety of sources.

The two red-band data sets, the $H$-band data of Bothun et al. (1985) at the bright end, and the $I$-band dwarf galaxy sample at the faint end, together suffice to define a Tully-Fisher relation over five decades in stellar mass. The rotation curve samples are consistent with these data. For comparison, we also examine the gas rich, late type galaxy sample of Matthews, van Driel, & Gallagher (1998). Their $B$-band data are entirely consistent with our own data, provided we make the same inclination cut, $i > 45\degree$. Though this inclination limit is of obvious importance, it is interesting to note that including or excluding the galaxies
they note as having strongly asymmetric or single-horned HI profiles makes no difference to the result.

In all cases, we have simply taken the data as given by each source. Aside from the necessary inclination correction, we have not made any corrections for internal extinction or for non-circular motions (shown to be small for late type systems by Rix & Zaritsky 1995 and by Beauvais & Bothun 1999). That the data treated in this way produce a good Tully-Fisher relation indicates, to first order, that these effects are not important.

3. Results

Figure 1 illustrates the Tully-Fisher relation for the combined data sets. Two versions are shown: in (a) the stellar mass is plotted in place of luminosity, and in (b) the total luminous baryonic mass is shown. In order to place the data sets using different band passes for photometry on the same scale, we convert luminosity to stellar mass assuming a fixed mass-to-light ratio $\Upsilon_*$ for each band. The value of the mass-to-light ratio appropriate to the stellar populations of late type galaxies with ongoing star formation has been examined in detail by de Jong (1996). We adopt his model for a 12 Gyr old, solar metallicity population with a constant star formation rate and Salpeter IMF. The adopted mass-to-light ratios are $\Upsilon^B_*=1.4$, $\Upsilon^I_*=1.7$, $\Upsilon^H_*=1.0$, and $\Upsilon^K_*=0.8$. These $K'$ and $I$-band mass-to-light ratios are consistent with the maximum disk fits to the bright galaxies of Verheijen (1997) and Palunas (1996). We do of course expect variation in stellar populations and their mass-to-light ratios. This should be modest in the redder bands, especially $H$ and $K$, which are not very sensitive to differences in star formation history. The $I$-band mass-to-light ratio is not very sensitive to metallicity (Worthey 1994), so this should suffice for the fainter galaxies which in any case are dominated by gas mass. The $B$-band is a less robust indicator of stellar mass, so we do not include these data in the fit in Figure 1(b). While the absolute normalization of stellar mass-to-light ratios remains uncertain, tweaking the adopted values has no effect on the basic result.

The stellar mass plotted in Figure 1(a) is simply $M_*=\Upsilon_* L$, so this plot is directly analogous to the conventional luminosity-linewidth diagram. The baryonic disk mass plotted in Figure 1(b) is the sum of stars and gas, $M_d=M_*+M_{\text{gas}}$. The mass in gas is taken from the observed HI mass with the standard correction for helium and metals: $M_{\text{gas}}=1.4 M_{\text{HI}}$. It appears that molecular gas is not a significant mass component in these late type galaxies (Schombert et al. 1990; de Blok & van der Hulst 1998; Mihos, Spaans, & McGaugh 1999; Gerritsen & de Blok 1999).

There have long been hints (e.g., Romanishin, Strom, & Strom 1983) that faint galaxies fall below the extrapolated Tully-Fisher relation for bright galaxies. Matthews et al. (1998) and Stil & Israel (1999) claim to see this in their samples. However, it is not clear from their data in Figure 1. The apparent discrepancy in our results stems not from a difference in the data, but from what is taken to define the Tully-Fisher relation. Matthews et al. (1998) and Stil & Israel (1999) compare their data to lines fit to the $B$-band data of brighter galaxies. These fiducial lines have a shallow slope which considerably overpredicts the luminosities of faint galaxies when extrapolated to low circular velocity. It is not clear that it is safe to extrapolate the slope in this fashion. Extinction appears to be relatively more important in brighter galaxies, with careful corrections giving steeper slopes (Tully et al. 1998). Samples of galaxies with low intrinsic extinctions also give considerably steeper $B$-band slopes (Sprayberry et al. 1995; Verheijen 1997; McGaugh & de Blok 1998). The $H$-band data of Bothun et al. (1985) and the $K'$-band data of Verheijen

\footnote{For the mean $H-K'$ color of late type galaxies given by de Jong (1996), $\Upsilon^H_*=1.2\Upsilon^K_$.}
two bands where extinction is minimal, also indicate steep slopes. A steep slope is also supported by the calibration of the Tully-Fisher relation from the HST Key Project (Sakai et al. 2000). Such a slope eliminates the discrepancy reported by Matthews et al. (1998) and by Stil & Israel (1999).

Nevertheless, it is now clear from the larger dwarf sample employed here that there is indeed a break in the Tully-Fisher relation for faint field6 galaxies. For \( V_c < 90 \text{ km s}^{-1} \), galaxies are underluminous for their rotation velocity as predicted by the extrapolation of a linear fit to the bright galaxy data. There is a great amount of scatter here too — the relation bends and flares. There have been concerns that there might be curvature in the Tully-Fisher relation (e.g., Bothun & Mould 1987) but the data in Figure 1(a) are probably better described by a broken power law, if it makes sense to fit anything to the faint end at all.

A break in the Tully-Fisher relation would have important ramifications for its application and interpretation. However, many of these faint galaxies are very gas rich. So much so, in fact, that the gas outweighs the stars in most of them for any reasonable choice of stellar mass-to-light ratio (Schombert, McGaugh, & Eder 2000). Therefore, we examine in Figure 1(b) the effects of including the gas mass in the ordinate by plotting the total observed baryonic disk mass, \( M_d = M_* + M_{\text{gas}} \). This has the remarkable effect of restoring a single linear relation over the entire span of the observations.

It appears that the fundamental relation underpinning the Tully-Fisher relation is one between rotation velocity and total baryonic disk mass. This relation has the form

\[
M_d = A V_c^b. \tag{2}
\]

An unweighted fit to the red (\( I, H, \) and \( K' \)-band) data gives \( \log A = 1.57 \pm 0.25 \) and \( b = 3.98 \pm 0.12 \). The precise value of the normalization would of course change if we assumed a different distance scale or different stellar mass-to-light ratios. The slope is indistinguishable from \( b = 4 \). If we fix the slope to this value, the normalization is \( A \approx 35 \left( \frac{\Upsilon_{K'}^*}{0.8} \right) h_{75}^{-2} M_{\odot} \text{ km}^{-4} \text{ s}^4 \).

4. Implications

The basic result seen in Figure 1(b) falls directly out of the observations. All we have done is assume a plausible mass-to-light ratio for the stars, added in the gas mass, and plotted the data. This simple result has a number of interesting implications.

First, there is an apparently universal relation between baryonic mass and rotation velocity, with a single normalization. While this relation specifically applies to our sample of late type spiral galaxies, it seems plausible that it might also apply to early type spirals, provided appropriate consideration is given to the bulge component, which might require a different \( \Upsilon_* \), and to any other baryonic components which might be significant (like molecular gas).

The logarithmic slope of the relation is indistinguishable from 4. While this slope is often attributed to the virial theorem, it is possible to derive other slopes as well depending on the assumptions one makes (Mo et al. 1998). Current cold dark matter models predict a slope of 3 (Mo et al. 1998; Steinmetz & Navarro 1999) which is excluded at 8\( \sigma \). Significant tweaking is required to obtain the observed slope. Feedback from supernovae is often invoked in this context (van den Bosch 1999), but it is not obvious that

---

6The \( K' \)-band data of Pierini & Tuffs (1999) shows a steep slope with no break down to \( V_c \approx 60 \text{ km s}^{-1} \). These are cluster galaxies, so this makes sense if these objects are less gas rich than the field sample.
the modest amount of feedback required by the Tully-Fisher relation is consistent with the large amount needed to explain the luminosity function (Lobo & Guiderdoni 1999). The correct slope and normalization is predicted by one alternative to cold dark matter (Milgrom 1983). In this alternative there is no dark matter — all of the mass is baryonic.

Whatever mechanism sets the observed relation is intimately connected to the observed baryonic mass. The interpretation of the standard luminosity-linewidth relation has long supposed that the stellar mass-to-light ratios of galaxies are a nearly uniform. Indeed, the error budget allowed by the modest amount of intrinsic scatter observed in the $K'$-band is easily consumed by variations in the star formation history (Verheijen 1997). There is little room left for variation in the IMF, or cosmic scatter in the underlying mass-rotation velocity relation.

We have now addressed another piece of this puzzle. In addition to the near constancy of $\Upsilon_*$, we have explicitly corrected for the stellar fraction $f_*$. Equation (1) now reduces to

$$M_d = f_d b M_{tot}.$$  

(3)

The presumed mass-rotation velocity relation can now show through in the observations provided both $f_b$ and $f_d$ are universal constants. The baryon fraction of the universe is constant by definition. But it is less obvious that the fraction of baryons which reside in the disk should be the same for all spirals. Indeed, it is frequently suggested (e.g., Navaroro, Eke, & Frenk 1996) that the sort of faint dwarfs studied here are likely to lose a significant portion of their baryons. This idea is blatantly at odds with the data, as the product $f_d b f_b$ would no longer be constant.

It seems to us implausible that $f_d$ could be some arbitrary yet universal fraction. While it is easy to imagine mechanisms which might prevent some of the baryons from cooling to join the disk, it is difficult to contemplate any which do so with the required precision. There is very little room in the budget for the intrinsic scatter for any scatter in $f_d$. Let us call the mass in baryons not already accounted for in the disk mass $M_{other}$. The disk fraction is then

$$f_d = \frac{M_* + M_{gas}}{M_* + M_{gas} + M_{other}}.$$  

(4)

If this other form of baryonic mass is significant ($M_{other} \sim M_*$), then $f_d < 1$, but there should be a lot of scatter in $f_d$ unless some magical mechanism strictly regulates the ratio $M_{other}/(M_* + M_{gas})$. This unlikely situation occurs naturally only if $M_{other} \ll M_* + M_{gas}$, so $f_d \rightarrow 1$. The modest intrinsic scatter in the Baryonic Tully-Fisher relation therefore suggests that the luminous mass in stars and gas represents nearly all the baryons associated with an individual galaxy and its halo, arguing against a significant mass of dark baryons in these systems.

5. Conclusions

We have explored the Tully-Fisher relation over five decades in luminous mass. This is a considerable increase in dynamic range over previous studies. We find clear evidence for a break in the optical Tully-Fisher relation around $V_c \approx 90$ km s$^{-1}$. Galaxies with rotation velocities less than this are underluminous relative

---

7One could contemplate a variable $f_d$ provided that it was a very finely tuned (zero scatter) function of circular velocity. For example, $f_d \propto V_c$ would recover the slope predicted by CDM.
to the extrapolation of the fit to more rapidly spinning galaxies. However, these faint galaxies are very gas rich. Considering both stellar and gas mass restores a linear relation over the entire observed range.

These observations strongly suggest that the Tully-Fisher relation is fundamentally a relation between rotation velocity and total baryonic disk mass. This relation has the form

$$M_d = AV_c^4$$

with $A \approx 35 \left( \frac{Y^K}{0.8} \right) h_{75}^{-2} M_\odot \text{ km}^{-4} \text{ s}^4$. The well known optical Tully-Fisher relation is an approximation to this more fundamental relation in the limit of galaxies dominated by stars.

The existence of the Baryonic Tully-Fisher relation has a number of important implications. That it works means that stars in spiral galaxies have mass-to-light ratios which are reasonable for composite stellar populations. The modest amount of scatter indicates that the IMF must be nearly universal in order to yield such uniform mass-to-light ratios. Only corrections for gas content are necessary to obtain the Baryonic Tully-Fisher relation. The data do not allow much room for any further significant baryonic mass components. Any component of dark baryons which does exist must do so in strict proportionality to the observed baryons, with effectively zero scatter. This unlikely situation argues against a significant mass in dark baryons in any form (be it very cold molecular gas in the disk, very hot ionized gas in the halo, or baryonic MACHOs). Any model which supposes a large mass of such baryons must explain why it contributes so little to the scatter in the Baryonic Tully-Fisher relation.

The results presented here make sense in terms of a simple interpretation of the Tully-Fisher relation in which the mass of observed baryons is directly proportional to the total mass which in turn scales with the observed rotation velocity. This potentially includes the case where the mass observed in baryons is the total mass (Milgrom 1983). Matching these observations is a substantial challenge for modern structure formation theories based on cold dark matter. These predict a slope which is too shallow (3 rather than 4, different by 8σ), and fail to anticipate that effectively all the baryons associated with a halo have cooled into the disk.

We thank Ken Freeman for relevant conversations, and the referee, H.-W. Rix, for many insightful comments. The work of SSM is supported in part by NSF grant AST 99-01663.
ADDENDUM:
The text above is rigorously identical to that accepted by *ApJ Letters*. Two points were lost in the effort to meet the *Letters’* page limit.

1: Maximum Disk Mass-to-Light Ratios: We have assumed a constant $\Upsilon$, which is plausible for a composite stellar population. Presumably, there is scatter about this value which is reflected in the non-observational component of the scatter in Figure 1(b). If instead we adopted the stellar mass-to-light ratio suggested by maximum disk fits to rotation curves, the resultant relation would have a much larger scatter. The reason for this is that $\Upsilon^{\text{max}}$ increases systematically with decreasing disk central surface brightness (de Blok & McGaugh 1997, MNRAS, 290, 533; Swaters, Madore, & Trewella 2000, ApJ, 531, L107), yet there is a range of surface brightnesses at a given luminosity. For example, Swaters et al. discuss a number of galaxies with $M_B \approx -18$ for which $\Upsilon^{\text{max}}$ ranges from 1.5 to 17. Obviously, this factor would cause a large difference in the computed mass at a given circular velocity, and inflate the scatter in the Baryonic Tully-Fisher Relation. This argues against maximal disks in low surface brightness galaxies, though these remain plausible for high surface brightness galaxies.

2: Deviant Galaxies: Recently, O’Neil, Bothun, & Schombert (1999, AJ, 119, 136) have pointed out a population of red low surface brightness galaxies. Four of these galaxies deviate significantly from the Baryonic Tully-Fisher relation, falling roughly an order of magnitude below it (having too little mass for their circular velocity). Given their red colors, these galaxies might have high $\Upsilon$, or could contain some other component of baryonic mass. This point is tentative, so here we note only that the Baryonic Tully-Fisher relation is a good diagnostic for identifying unusual galaxies. This also applies to galaxies which might have very low $\Upsilon$, as a result of a young stellar population (e.g., Meurer et al. 1996, AJ, 111, 1551).
REFERENCES

Freeman, K.C. 1999, in The Low Surface Brightness Universe, IAU Col. 171, ASP Conference Series 170, 3
Fig. 1.— The Tully-Fisher relation plotted as a) stellar mass and b) baryonic disk mass against rotation velocity. Square symbols represent galaxies where the circular velocity is estimated from the linewidth by $V_c = \frac{1}{2} W_{20}$, while circles have $V_c = V_{flat}$ from resolved rotation curves. Data employed include the $H$-band data of Bothun et al. (1985; red), the $K'$-band data of Verheijen (1997; black), and the $I$-band data of Pildis et al. (1997) with velocities as reported by Eder & Schombert (1999; green). Also shown are the $B$-band data of McGaugh & de Blok (1998; light blue), and of Matthews et al. (1998; dark blue). The stellar mass is computed from the luminosity assuming a constant mass-to-light ratio: $M_* = \Upsilon_* L$, so (a) is directly analogous to the usual luminosity-linewidth diagram. We assume mass-to-light ratios for the stellar populations of late type galaxies of $\Upsilon_*^B = 1.4$, $\Upsilon_*^I = 1.7$, $\Upsilon_*^H = 1.0$, and $\Upsilon_*^{K'} = 0.8 \frac{M_\odot}{L_\odot}$ (see text). In (b), we plot the total baryonic disk mass $M_d = M_* + M_{gas}$ with $M_{gas} = 1.4 M_{HI}$. In (a), a clear break is apparent. Galaxies with $V_c < 90$ km s$^{-1}$ fall systematically below the Tully-Fisher relation defined by brighter galaxies. In (b), the deficit in mass apparent in (a) has been restored by including the gas mass. The solid line is an unweighted fit to the red band data in (b) with a correlation coefficient of 0.92 and a slope indistinguishable from 4.