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**PHOTON - AXION CONVERSION CROSS SECTIONS
IN AN ELECTROMAGNETIC FIELD**

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United Nations Educational Scientific and Cultural Organization
and
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IN AN ELECTROMAGNETIC FIELD**

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Abstract

Photon - axion conversions in static magnetic fields and in a periodic field with frequency equal to the axion mass are reconsidered in detail by Feynman methods. The differential cross sections are presented and numerical evaluations are given. It is shown that there is a resonant conversion for the considered process. Some estimates for experiments are given from our results.

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In the 1970's, it was shown that the strong CP problem can be solved [1] by the introduction of a light pseudoscalar particle, called the axion [2]. At present, the axion mass is constrained by laboratory searches [3] and by astrophysical and cosmological considerations [4] to be between 10^{-6} eV and 10^{-3} eV. Besides that, an axino (the fermionic partner of the axion) naturally appears in SUSY models [5,6], which acquire a mass from three - loop Feynman diagrams in a typical range between a few eV up to a maximum of 1 keV [7,8].

A particle, if it has a two - photon vertex, may be created by a photon entering an external electromagnetic (EM) field, the axion is one such particle. So far almost all experiments designed to search for light axions make use of the coupling of the axion to photons. Conversion of the axions into EM power in a resonant cavity was first suggested by Sikivie [9]. He suggested that this method can be used to detect the hypothetical galactic axion flux that would exist if axions were the dark matter of the Universe. Various terrestrial experiments to detect invisible axions by making use of their coupling to photons have been proposed [10,11], and the new results of such experiments recently appeared in Refs. [12,13].

EM detection of axions in experiments is briefly described as follows: The initial photon of energy q_0 from the laser (may be better from X ray) interacts with a virtual photon from the EM field to produce the axion of energy p_0 and momentum. The photon beam is then blocked to eliminate everything except the axions, which penetrate the wall because of their extremely weak interaction with ordinary matter. (Such shielding is straightforward for a low-energy laser beam.) The axion then interacts with another virtual photon in the second EM field to produce a real photon of energy q'_0 , whose detection is the signal for the production of the axion. For details see Refs.[10,12]. The purpose of this paper is to consider conversions of photons into axions in an external EM field, including static magnetic fields and the periodic EM field with frequency equal to the axion mass.

The axion mass and its couplings to ordinary particles are all inversely proportional to the magnitude v of the vacuum expectation value that spontaneously breaks the $U_{PQ}(1)$ quasisymmetry which was postulated by Peccei and Quinn and of which the axion is the pseudo-Nambu - Goldstone boson. For the axion- photon system a suitable Lagrangian density is given by [3,4]:

$$L = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + g_\gamma \frac{\alpha}{4\pi} \frac{\phi_a}{f_a} F_{\mu\nu} \tilde{F}^{\mu\nu} + \frac{1}{2}\partial_\mu\phi_a\partial^\mu\phi_a - \frac{1}{2}m_a^2\phi_a^2[1 + O(\phi_a^2/v^2)] \quad (1)$$

where ϕ_a is the axion field, m_a its mass, $\tilde{F}_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\rho\sigma}F^{\rho\sigma}$, and f_a is the axion decay constant and is defined in terms of the axion mass m_a by [9,12]: $f_a = f_\pi m_\pi \sqrt{m_u m_d} [m_a(m_u + m_d)]^{-1}$. The coupling constant in (1) is model dependent, in particular in the Dine- Fischler - Srednicki - Zhitnitskii model [14]: $g_\gamma(DFFSZ) \simeq 0.36$, and in the Kim- Shifman- Vainshtein- Zakharov model [15]: $g_\gamma(KSVZ) \simeq -0.97$.

Consider the conversion of the photon γ with momentum q into the axion a with momentum

p in an external electromagnetic field. For the above mentioned process, the relevant coupling is the second term in (1). Using the Feynman rules we get the following expression for the matrix element

$$\langle p|M|q\rangle = -\frac{g_{a\gamma}}{2(2\pi)^2\sqrt{q_0p_0}}\varepsilon_\mu(\vec{q},\sigma)\varepsilon^{\mu\nu\alpha\beta}q_\nu\int_V e^{i\vec{k}\vec{r}}F_{\alpha\beta}^{class}d\vec{r} \quad (2)$$

where $\vec{k} \equiv \vec{q}-\vec{p}$ the momentum transfer to the EM field, $g_{a\gamma} \equiv g_\gamma \frac{\alpha}{\pi f_a} = g_\gamma \alpha m_a(m_u+m_d)(\pi f_\pi m_\pi \sqrt{m_u m_d})^{-1}$ and $\varepsilon^\mu(\vec{q},\sigma)$ represents the polarization vector of the photon. Expression (2) is valid for an arbitrary external EM field. In the following we shall use it for cases, namely conversions in the static magnetic fields of the flat condensor and the selenoid and in the periodic field of the TE_{10} mode with frequency equal to the axion mass. Here we use the following notations: $q \equiv |\vec{q}|$, $p \equiv |\vec{p}| = (q_o^2 - m_a^2)^{1/2}$ and θ is the angle between $\vec{p}$ and $\vec{q}$.

Conversions in a static magnetic field of the size $a \times b \times c$.— Now we take that the EM field is the homogeneous magnetic field of the flat condensor of the size $a \times b \times c$. We shall use the coordinate system with the z axis parallel to the direction of the field, i.e., $F^{12} = -F^{21} = B$. Then the matrix element is given by

$$\langle p|M^m|q\rangle = \frac{g_{a\gamma}}{(2\pi)^2\sqrt{q_0p_0}}\varepsilon_\mu(\vec{q},\sigma)\varepsilon^{\mu\nu 01}q_\nu F_m(\vec{k}), \quad (3)$$

where a form factor for the magnetic region

$$F_m(\vec{k}) = \int_V e^{i\vec{k}\vec{r}}B(\vec{r})d\vec{r}.$$

For a homogeneous magnetic field of intensity B we have

$$F_m(\vec{k}) = 8B \sin(\frac{1}{2}ak_x) \sin(\frac{1}{2}bk_y) \sin(\frac{1}{2}ck_z)(k_x k_y k_z)^{-1}. \quad (4)$$

Substituting (4) into (3) we find finally the differential cross section (DCS) for the conversion of photons into axions

$$\frac{d\sigma^m(\gamma \rightarrow a)}{d\Omega} = \frac{g_{a\gamma}^2 q^2}{2(2\pi)^2} (1 - \frac{q_z^2}{q^2}) F_m^2 \quad (5)$$

From (5) we see that if the photon moves in the direction of the magnetic field i.e., $q^\mu = (q, 0, 0, q)$ then DCS vanishes. If the momentum of the photon is parallel to the y axis, i.e., $q^\mu = (q, 0, q, 0)$ then Eq.(5) becomes:

$$\begin{aligned} \frac{d\sigma^m(\gamma \rightarrow a)}{d\Omega''} &= \frac{32g_{a\gamma}^2 B^2 q^2}{(2\pi)^2} \left[\sin\left(\frac{cp \sin \theta \sin \varphi''}{2}\right) \sin\left(\frac{a}{2}(q - p \cos \theta)\right) \right. \\ &\quad \times \left. \sin\left(\frac{bp \sin \theta \cos \varphi''}{2}\right) \right]^2 (p^2 \sin^2 \theta \sin \varphi'' \cos \varphi'' (q - p \cos \theta))^{-2} \end{aligned} \quad (6)$$

where φ'' is the angle between the z axis and the projection of $\vec{p}$ on the xz plane. From (6) we have

$$\frac{d\sigma^m(\gamma \rightarrow a)}{d\Omega''} = \frac{2g_{a\gamma}^2 B^2 b^2 c^2}{(2\pi)^2 \left(1 - \sqrt{1 - \frac{m_a^2}{q^2}}\right)^2} \sin^2 \left[\frac{qa}{2} \left(1 - \sqrt{1 - \frac{m_a^2}{q^2}}\right) \right] \quad (7)$$

for $\theta \approx 0$ and

$$\frac{d\sigma^m(\gamma \rightarrow a)}{d\Omega''} = \frac{8g_{a\gamma}^2 b^2 B^2}{(2\pi)^2(q^2 - m_a^2)} \sin^2\left(\frac{aq}{2}\right) \sin^2\left(\frac{cq}{2} \sqrt{1 - \frac{m_a^2}{q^2}}\right) \quad (8)$$

for $\theta = \frac{\pi}{2}, \varphi'' = \frac{\pi}{2}$.

$$\frac{d\sigma^m(\gamma \rightarrow a)}{d\Omega''} = \frac{8g_{a\gamma}^2 c^2 B^2}{(2\pi)^2(q^2 - m_a^2)} \sin^2\left(\frac{aq}{2}\right) \sin^2\left(\frac{bq}{2} \sqrt{1 - \frac{m_a^2}{q^2}}\right) \quad (9)$$

for $\theta = \frac{\pi}{2}, \varphi'' = 0$. For the high energy limit $m_a \ll q$, Eqs.(7),(8) and (9) become, respectively,

$$\frac{d\sigma^m(\gamma \rightarrow a)}{d\Omega'} = \frac{g_{a\gamma}^2 q^2 V^2 B^2}{2(2\pi)^2}, \quad (10)$$

$$\frac{d\sigma^m(\gamma \rightarrow a)}{d\Omega'} = \frac{8g_{a\gamma}^2 b^2 B^2}{(2\pi)^2 q^2} \sin^2\left(\frac{aq}{2}\right) \sin^2\left(\frac{cq}{2}\right), \quad (11)$$

and

$$\frac{d\sigma^m(\gamma \rightarrow a)}{d\Omega'} = \frac{8g_{a\gamma}^2 c^2 B^2}{(2\pi)^2 q^2} \sin^2\left(\frac{aq}{2}\right) \sin^2\left(\frac{bq}{2}\right), \quad (12)$$

From (10) we see that DCS in the direction of the photon motion depends quadratically on the *intensity* B , the *volume* V of condensor, and the *photon momentum* q . Therefore, since the external EM field is classical we can increase the scattering probability as much as possible by increasing the intensity of the field or the volume containing the field. In the C.G.S units, Eqs. (10), (11) and (12) become, respectively,

$$\frac{d\sigma^m(\gamma \rightarrow a)}{d\Omega'} \approx 7.4 \times 10^{-44} B^2 V^2 \lambda^{-2}, \quad (13)$$

$$\frac{d\sigma^m(\gamma \rightarrow a)}{d\Omega'} \approx 3.7 \times 10^{-46} b^2 B^2 \lambda^2 \sin^2\left(\frac{a\pi}{\lambda}\right) \sin^2\left(\frac{c\pi}{\lambda}\right), \quad (14)$$

and

$$\frac{d\sigma^m(\gamma \rightarrow a)}{d\Omega'} \approx 3.7 \times 10^{-46} c^2 B^2 \lambda^2 \sin^2\left(\frac{a\pi}{\lambda}\right) \sin^2\left(\frac{b\pi}{\lambda}\right), \quad (15)$$

where λ is a wavelength of photons. For $a = b = c = 100$ cm, the intensity of the magnetic field $B = 10T$, the photon length $\lambda = 10^{-5}$ cm, and $m_a \simeq 10^{-5} eV$ [12] then in the DFSZ model the cross section given by (13) is $\frac{d\sigma(\gamma \rightarrow a)}{d\Omega} \simeq 7.4 \times 10^{-12} cm^2$, while by (14) and (15): $\frac{d\sigma(\gamma \rightarrow a)}{d\Omega} \simeq 3.7 \times 10^{-42} cm^2$. This implies that *the axion is mainly created in the direction of photon motion*.

Conversions in a magnetic field of the Solenoid.— Now we consider conversions in the homogeneous magnetic field of the selenoid with a radius R and a length h , and without loss of generality, suppose that direction of the magnetic field is parallel to the z axis. In this case, the form factor for the magnetic region is

$$F_m(\vec{k}) = \frac{4\pi BR}{k_z \sqrt{k_x^2 + k_y^2}} J_1(R \sqrt{k_x^2 + k_y^2}) \sin\left(\frac{hk_z}{2}\right), \quad (16)$$

where J_1 is the one-order spherical Bessel function. From Eqs. (5) and (16) we obtain the DCS for the conversion of photons into axions as

$$\frac{d\sigma^m(\gamma \rightarrow \sigma)}{d\Omega'} = \frac{2g_{a\gamma}^2 R^2 B^2 q^2}{k_z^2(k_x^2 + k_y^2)} J_1^2 \left(Rq \sqrt{k_x^2 + k_y^2} \right) \sin^2 \left(\frac{k_z h}{2} \right) \left(1 - \frac{q_z^2}{q^2} \right) \quad (17)$$

If the momentum of the photon is parallel to the x axis, i.e., $q^\mu = (q, q, 0, 0)$ then Eq.(17) gets the final form:

$$\begin{aligned} \frac{d\sigma^m(\gamma \rightarrow a)}{d\Omega'} &= 2g_{a\gamma}^2 R^2 B^2 J_1^2 \left(Rq \sqrt{\left(1 - \cos \theta \sqrt{1 - \frac{m_a^2}{q^2}} \right)^2 + \left(1 - \frac{m_a^2}{q^2} \right) \sin^2 \theta \cos^2 \varphi'} \right) \\ &\times \left[\left(1 - \cos \theta \sqrt{1 - \frac{m_a^2}{q^2}} \right)^2 + \left(1 - \frac{m_a^2}{q^2} \right) \sin^2 \theta \cos^2 \varphi' \right]^{-1} q^{-2} \\ &\times \sin^2 \left(\frac{hq}{2} \sqrt{1 - \frac{m_a^2}{q^2}} \sin \theta \sin \varphi' \right) \left[\left(1 - \frac{m_a^2}{q^2} \right) \sin^2 \theta \sin^2 \varphi'^2 \right]^{-1} \end{aligned} \quad (18)$$

where φ' is the angle between the y axis and the projection of $\vec{p}$ on the yz plane. It is easy to see that

$$\frac{d\sigma^m(\gamma \rightarrow a)}{d\Omega'} = \frac{1}{2} g_{a\gamma}^2 R^2 h^2 B^2 J_1^2 \left[Rq \left(1 - \sqrt{1 - \frac{m_a^2}{q^2}} \right) \right] \left(1 - \sqrt{1 - \frac{m_a^2}{q^2}} \right)^{-2} \quad (19)$$

for $\theta \approx 0$ and

$$\frac{d\sigma^m(\gamma \rightarrow a)}{d\Omega'} = \frac{1}{2} g_{a\gamma}^2 R^2 h^2 B^2 J_1^2 \left(Rq \sqrt{2 - \frac{m_a^2}{q^2}} \right) \left(2 - \frac{m_a^2}{q^2} \right)^{-1} \quad (20)$$

for $\theta = \frac{\pi}{2}, \varphi' = 0$.

For the limit $m_a \ll q$, note that

$$\lim_{p \rightarrow q} \frac{J_1(R(q-p))}{q-p} = \frac{R}{2}$$

then Eqs.(19) and (20) become, respectively,

$$\frac{d\sigma^m(\gamma \rightarrow a)}{d\Omega'} = \frac{g_{a\gamma}^2 V^2 B^2 q^2}{2(2\pi)^2}; \quad V \equiv \pi R^2 h \quad (21)$$

and

$$\frac{d\sigma^m(\gamma \rightarrow a)}{d\Omega'} = \frac{1}{4} g_{a\gamma}^2 R^2 h^2 B^2 J_1^2 (\sqrt{2} Rq). \quad (22)$$

From (21) we see that this result is the same as in the previous item. From (22) it follows that DCS vanishes when $p_n = \frac{\mu_n}{R\sqrt{2}}$ with $n = 0, \pm 1 \pm 2..$, and has its largest value

$$\frac{d\sigma^m(\gamma \rightarrow a)}{d\Omega'} = \frac{1}{4} g_{a\gamma}^2 R^2 h^2 B^2 J_1^2(\mu'_n) \quad (23)$$

for $p_n = \frac{\mu'_n}{R\sqrt{2}}$, where μ_n and μ'_n are the roots of $J_1(\mu_n) = 0$ and $J'_1(\mu'_n) = 0$.

For the magnitude [12] $B=8T$, $R = h = 1m$, $\lambda = 10^{-5}cm$ and $m_a \simeq 10^{-5}eV$ by (21) we have $\frac{d\sigma(\gamma \rightarrow a)}{d\Omega} = 4.7 \times 10^{-11}cm^2$ (in DFSZ model). To obtain the results in the KSVZ model we only need to note that $g_{a\gamma}^2(KSVZ) \approx 7.26g_{a\gamma}^2(DFSZ)$. The new result of the experiment for the axion detection by using the strong magnetic field and X - ray detectors, the readers can see Ref.[13]. The one dimensional solution is basics for the experimental setups in Ref. [10]. However, in general cases, *the best axion detection is in the direction of the photon motion and at the high energies.*

Resonant conversions in a periodic EM field of the TE_{10} mode.— We move on to conversions in the external EM field of the TE_{10} mode with frequency equal to the axion mass. The non-trivial solution of the TE_{10} mode is given by [16]

$$\begin{aligned} H_z &= H_o \cos\left(\frac{\pi x}{a}\right) e^{ikz - i\omega_a t}, \\ H_x &= -\frac{ika}{\pi} H_o \sin\left(\frac{\pi x}{a}\right) e^{ikz - i\omega_a t}, \\ H_y &= i\frac{\omega_a a \mu}{\pi} H_o \sin\left(\frac{\pi x}{a}\right) e^{ikz - i\omega_a t}. \end{aligned} \quad (24)$$

Using the Feynman rules we get the following expression for the matrix element

$$\langle p|M|q \rangle = \frac{g_a \gamma}{(2\pi)^2 \sqrt{p_0 q_0}} [(\varepsilon_3(\vec{q}, \tau) q_1 - \varepsilon_1(\vec{q}, \tau) q_3) F_y + \varepsilon_1(\vec{q}, \tau) q_0 F_x + \varepsilon_3(\vec{q}, \tau) q_0 F_z], \quad (25)$$

where $p_0 \equiv q_0 + \omega_a$, $\omega_a = m_a$, and

$$\begin{aligned} F_x &= -\frac{8kaH_o(q_x - p_x) \cos[\frac{1}{2}a(q_x - p_x)] \sin[\frac{1}{2}b(q_y - p_y)] \sin[\frac{1}{2}c(q_z - p_z + k)]}{\pi[(q_x - p_x)^2 - \frac{\pi^2}{a^2}](q_x - p_x)(q_y - p_y)(q_z - p_z + k)}, \\ F_y &= -\mu F_x, \\ F_z &= -\frac{8\pi H_o \cos[\frac{1}{2}a(q_x - p_x)] \sin[\frac{1}{2}b(q_y - p_y)] \sin[\frac{1}{2}c(q_z - p_z + k)]}{a[(q_x - p_x)^2 - \frac{\pi^2}{a^2}](q_x - p_x)(q_y - p_y)(q_z - p_z + k)}. \end{aligned} \quad (26)$$

Substituting Eq.(26) into Eq.(25) we find finally the DCS for conversions

$$\begin{aligned} \frac{d\sigma(\gamma \rightarrow a)}{d\Omega} &= \frac{g_{a\gamma}^2 p_o}{2(2\pi)^2 q_0} \left[(q_x^2 + q_z^2) F_y^2 + \left(1 - \frac{q_x^2}{q^2}\right) q_0^2 F_x^2 + \left(1 - \frac{q_x^2}{q^2}\right) q_0^2 F_z^2 \right. \\ &\quad \left. - 2q_0 q_z F_x F_y - 2\left(\frac{q_0^2 q_x q_z}{q^2}\right) F_x F_z + 2q_0 q_z F_x F_y \right]. \end{aligned} \quad (27)$$

In the following we assume $\omega_a \ll q$. From (27) it follows that when the momentum of the photon is parallel to the z axis, DCS vanishes. It implies that *if the momentum of the photon is parallel to the external field propagation then there is no conversion.* Our results are the same as in previous items. If the momentum of the photon is parallel to the x axis, then Eq.(27) gets

the final form

$$\frac{d\sigma(\gamma \rightarrow a)}{d\Omega'} = \frac{8g_{a\gamma}^2 H_0^2 a^2 q^2}{\pi^4} \left(1 + \frac{\omega_a}{q}\right) \left[\omega_a(q - p \cos \theta) - \frac{\pi^2}{a^2} \right]^2 \times \left[\frac{\cos \frac{a}{2}(q - p \cos \theta) \sin \frac{b}{2}(p \sin \theta \cos \varphi') \sin \frac{c}{2}(-p \sin \theta \sin \varphi' + k)}{[(q - p \cos \theta)^2 - \frac{\pi^2}{a^2}] \cdot p \sin \theta \cos \varphi' (-p \sin \theta \sin \varphi' + k)} \right]^2. \quad (28)$$

where φ' is the angle between the y axis and the projection of $\vec{q}$ on the yz plane. From (28) we have

$$\frac{d\sigma(\gamma \rightarrow a)}{d\Omega'} \approx \frac{g_{a\gamma}^2 H^2 V^2 q^2 \left[\omega_a^2 + \frac{\pi^2}{a^2} \right]^2}{2\pi^4 \left[\omega_a^2 - \frac{\pi^2}{a^2} \right]^2} \left(1 + \frac{\omega_a}{q}\right) \times \cos^2\left(\frac{\omega_a a}{2}\right) \quad (29)$$

for $\theta \approx 0$, and

$$\frac{d\sigma(\gamma \rightarrow a)}{d\Omega'} \approx \frac{2g_{a\gamma}^2 H^2 a^2 c^2 \left(\omega_a q - \frac{\pi^2}{a^2} \right)^2}{\pi^4 \left(1 + \frac{\omega_a}{q}\right) \left(q^2 - \frac{\pi^2}{a^2} \right)^2} \cos^2\left(\frac{a}{2}q\right) \sin^2\left(\frac{b}{2}(q + \omega_a)\right) \quad (30)$$

for $\theta = \frac{\pi}{2}$, $\varphi' = 0$, and

$$\frac{d\sigma(\gamma \rightarrow a)}{d\Omega'} \approx \frac{2g_{a\gamma}^2 H^2 a^2 b^2 \left(\omega_a q - \frac{\pi^2}{a^2} \right)^2 \left(1 + \frac{\omega_a}{q}\right)}{\pi^4 \left(q^2 - \frac{\pi^2}{a^2} \right)} \cos^2\left(\frac{a}{2}q\right) \sin^2\left(\frac{c}{2}q\right) \quad (31)$$

for $\theta = \frac{\pi}{2}$, $\varphi' = \frac{\pi}{2}$. In the limit $\omega_a \rightarrow \frac{\pi}{a}$ then Eq. (29) becomes

$$\frac{d\sigma(\gamma \rightarrow a)}{d\Omega'} \approx \frac{g_{a\gamma}^2 V^2 H_0^2 q^2}{8\pi^2} \left(1 + \frac{\pi}{qa}\right). \quad (32)$$

It is easy to show that the cross section for the reverse process coincides exactly with the above results, so that for the conversion of photon - axion - photon, the cross section is the square of the previous evaluation.

In order to estimate photon - axion conversions, from Eqs.(30), and (31), if $\omega_a = 10^{-5}$ eV, $H_0 = 10^6 \text{ cm}^{-1/2} g^{1/2} s^{-1}$, and $a = b = c = 100$ cm then DCS's depend on the momentum of photons which is shown in Fig.1. The solid curve corresponds to $\theta = \frac{\pi}{2}$. It implies that when the momentum of photons is perpendicular to the momentum of axions then there exists a *resonant conversion* at the value $q = 2.8 \times 10^{-2}$ eV, *this is the best case for photon - axion conversions*. When $q \geq 6 \times 10^{-2}$ eV then DCS vanishes. From Eq.(32) we see that there are no resonant conversions and in the limit $\frac{\pi}{qa} \ll 1$ we have a result similar to that of static fields. However, it does not yield the same results, because in this case both the electric and magnetic components give contributions simultaneously. In C.G.S units Eq.(32) becomes

$$\frac{d\sigma(\gamma \rightarrow a)}{d\Omega'} \approx 5.6 \times 10^{-43} \frac{V^2 H_0^2}{\lambda^2} \left(1 + \frac{\lambda}{2a}\right). \quad (33)$$

From Eq.(33) it follows that order to get $\sigma \approx 10^{-30} \text{ cm}^2$ we need $H_0 \approx 1.3 \times 10^6 \lambda V^{-1} [(1 + 0.5\lambda a^{-1})]^{-1/2}$. We see again that *conversions are more probable at*

high energies.

Finally, it is known that the cutoff frequency of the TE_{10} is given by $\omega_o = \frac{\pi}{a}$ and at any given frequency only a finite number of modes can propagate [16]. It is often convenient to choose the dimensions of the guide such that in the operating frequency only lowest mode can occur. *This is an important point in order to apply it in experiments.*

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Fig.1