Radiative symmetry breaking and Higgs mass bound in the NMSSM

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We study the upper mass bound of the lightest neutral Higgs scalar in the NMSSM using the RGE analysis. We require the successful occurrence of the electroweak radiative symmetry breaking to restrict the parameter space. As a result the upper mass bound $m_{h^0}$ is largely restricted compared with the one estimated without imposing this condition. We point out some features of $m_{h^0}$ related to the initial value of $h_t$ and discuss why the models with more extra matters $\tilde{5} + \tilde{\bar{5}}$ of SU(5) could bring the larger maximum value of $m_{h^0}$.

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The existence of a rather light CP-even neutral Higgs scalar is a common feature of the supersymmetric extension of the standard model [1, 2]. It is an important evidence of this kind of models and it is very useful to know its possible upper bound for the judgement of the consistency of the models.

The next to the minimal supersymmetric standard model (NMSSM) is the simplest extension of the minimal supersymmetric standard model (MSSM) [3]. In this model a singlet chiral supermultiplet $S$ is introduced and a $\mu$ term in the MSSM is replaced by a Yukawa coupling $\lambda S H_1 H_2$ with the usual Higgs doublet chiral superfields $H_1$ and $H_2$. The superpotential of the Higgs sector in this model is expressed as

$$W_{\text{NMSSM}} = \lambda S H_1 H_2 + \frac{1}{3} \kappa S^3 + \cdots. \quad (1)$$

If the scalar component $\tilde{S}$ of $S$ gets a vacuum expectation value, the $\mu$ scale appears as $\lambda \langle \tilde{S} \rangle$. In this model the $\mu$ problem in the MSSM is potentially solvable when the tree level $\mu$ term is forbidden due to a suitable symmetry [4].

An interesting feature of this model is the fact that the mass bound of the lightest neutral Higgs scalar can be estimated with no dependence on the soft supersymmetry breaking parameters at the tree level [5],

$$m_{h^0}^{(0)} \leq m_Z^2 \left[ \cos^2 2\beta + \frac{2\lambda^2}{g_1^2 + g_2^2} \sin^2 2\beta \right], \quad (2)$$

where $\tan \beta = \langle H_2 \rangle / \langle H_1 \rangle$. This bound is mainly controled by the value of $\tan \beta$ and the bound of $\lambda$. Its dependence on the soft supersymmetry breaking parameters appears through the loop correction to the effective potential as a result of the large top Yukawa coupling $h_t$ [2]. All of these effects are related through the renormalization group equations (RGEs). Many works have been done on this aspect [6, 7]. In Ref.[7] it has been suggested that the additional extra matters such as $5 + \overline{5}$ of SU(5) can heavily affect the mass bound by changing the running of gauge couplings, $h_t$ and $\lambda$ but not affecting the scale of the gauge coupling unification, which is a great success of the MSSM.

In the NMSSM the radiative generation of nonzero $\langle \tilde{S} \rangle$ or the $\mu$ scale is a very important aspect. From this point of view it seems to be rather crucial to study how the requirement of the successful occurrence of the radiative symmetry breaking [8] affects on the upper mass bound of the lightest neutral Higgs scalar. In this note we investigate this problem by finding the radiatively induced minimum of the effective potential parameterized by $(\tan \beta, \langle \tilde{S} \rangle)$ in the models with extra matters $n(5 + \overline{5})$ of SU(5).
The one-loop effective potential due to the large top Yukawa coupling [9, 10] is,

\[ V_1 = \frac{1}{64\pi^2} \left[ -12m_t^4 \left( \ln \frac{m_t^2}{Q^2} - \frac{3}{2} \right) + \sum_{i=1}^{2} 6\tilde{m}_{ti}^4 \left( \ln \frac{\tilde{m}_i^2}{Q^2} - \frac{3}{2} \right) \right], \]

where \( Q \) is a renormalization point and \( \tilde{m}_{ti}^2 \) is the eigenvalue of the stop mass matrix

\[
\begin{pmatrix}
\tilde{m}_Q^2 + m_t^2 & m_t(-A_t + \lambda \langle \tilde{S} \rangle \cot \beta) \\
-m_t(-A_t + \lambda \langle \tilde{S} \rangle \cot \beta) & \tilde{m}_Q^2 + m_t^2
\end{pmatrix}.
\]

The correction to Eq. (1) due to this one-loop effective potential is expressed by using these mass eigenvalues as

\[ \Delta m^2_{H^0} = \frac{1}{2} \left( \frac{\partial^2 V_1}{\partial v_1^2} - \frac{1}{v_1} \frac{\partial V_1}{\partial v_1} \right) \cos^2 \beta + \frac{1}{2} \left( \frac{\partial^2 V_1}{\partial v_2^2} - \frac{1}{v_2} \frac{\partial V_1}{\partial v_2} \right) \sin^2 \beta. \]

Here we used the potential minimum condition to eliminate the soft scalar masses of Higgs fields and took a field basis which is used to derive Eq. (2). The mass matrix in Eq. (4) depends on \((\tan \beta, \langle \tilde{S} \rangle)\) other than the Yukawa couplings and the soft SUSY breaking parameters.

In order to fix this matrix we need to determine these values as the ones at the potential minimum. It is a nontrivial problem whether such values of \((\tan \beta, \langle \tilde{S} \rangle)\) can be radiatively realized starting from certain sets of the Yukawa couplings and the soft SUSY breaking parameters. Our task is to estimate \( m^2_{H^0} \) numerically for the parameter sets which can radiatively realize the phenomenologically acceptable potential minimum. We determine such parameter sets so as to satisfy the following conditions:

(i) starting from the suitable initial values of parameters, the radiative symmetry breaking occurs successfully and the following phenomenologically required condition is satisfied at the potential minimum,

\[ \langle H_1 \rangle^2 + \langle H_2 \rangle^2 = (174 \text{ GeV})^2, \quad m_t = 174 \text{ GeV}, \]

(ii) \( m^2_{H^0} \) which corresponds to the one of diagonal elements of the \( 3 \times 3 \) neutral Higgs mass matrix should be smaller than other two diagonal components [5],

(iii) the experimental mass bounds on the charged Higgs bosons \( m_{H^\pm} \), charginos \( m_{\chi^\pm} \), stops \( \tilde{m}_{ti} \), and gluinos \( M_3 \) are satisfied. These masses, except for \( M_3 \), are dependent on \( \lambda \) and \( \langle \tilde{S} \rangle \). We require the following values for them:

\[ m_{H^\pm} > 65 \text{ GeV}, \quad m_{\chi^\pm} > 72 \text{ GeV} \quad \tilde{m}_{t2} > 67 \text{ GeV}, \quad M_3 > 173 \text{ GeV}, \]
(iv) the vacuum should be a color conserving one [11].

In this study we solve a set of RGEs which are composed of two-loop ones for dimensionless couplings and one-loop ones for dimensional SUSY breaking parameters, for simplicity. As the initial conditions for the SUSY breaking parameters we take

\[ \tilde{m}^2_{\phi_i} = (\gamma_i \tilde{m})^2, \quad M_a = M, \quad A_t = A_\kappa = A_\lambda = A, \]  

(8)

where \( \tilde{m} \) is the universal soft scalar mass. We introduce the nonuniversality represented by \( \gamma_i \) only among soft scalar masses of \( H_1, H_2 \) and \( S \) to make it easy to find the radiative symmetry breaking solutions. This will be taken as \( 0.8 \leq \gamma_i \leq 1.2 \). These initial conditions are assumed to be applied at the scale \( M_X \) where the coupling unification of \( SU(2)_L \) and \( U(1)_Y \) occurs. We donot require the regolous coupling unification of \( SU(3)_C \) but only impose the realization of the low energy experimental value [7]. The initial values of the parameters are surveyed through the following region,

\[
\begin{align*}
0 &\leq h_t \leq 1.2 \quad (0.1), \\
-2.0 &\leq \kappa \leq 0 \quad (0.2), \\
0 &\leq \lambda \leq 3.0 \quad (0.2), \\
0 &\leq M/M_S \leq 0.8 \quad (0.3), \\
0 &\leq \tilde{m}/M_S, \quad |A|/M_S \leq 3.0 \quad (0.5),
\end{align*}
\]

(9)

where in the parentheses we give the interval which we use in the survey of these parameter regions.\(^1\) We also assume that the RGEs of the model are changed from the supersymmetric ones to the nonsupersymmetric ones at a supersymmetry breaking scale \( M_S \), for which we take \( M_S = 1 \) TeV as a typical numerical value [6, 7].

To estimate the one-loop effect Eq. (5), it is necessary to know the values of \( (\tan \beta, \langle \tilde{S} \rangle) \) at the potential minimum. If we impose the radiative symmetry breaking condition, such a potential minimum has to be realized as a result of RGEs solution. In order to see the effect of the radiative symmetry breaking condition on the upper mass bound \( m_{h^0} \) we calculate it under two situations. We impose the full condition of (i) in a case (I). On the other hand, in a case (II) we require only Eq. (6) but donot require that it is realized at the potential minimum\(^2\). We take the number of extra matters as \( n = 3 \). The perturbative unification of the gauge coupling requires \( n \leq 4 \). Later we will also discuss other cases.

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\(^1\)Since the sign of \( \kappa \) and \( A \) affects the scalar potential, we need to investigate both sign of them. However, a negative \( \kappa \) seems to cover almost solutions for the positive \( \kappa \). Here we give the only result in the case of the negative \( \kappa \).

\(^2\)In other words, this case corresponds to the situation that Eq. (6) may be satisfied at the minimum which is obtained from the unnatural initial soft scalar masses.
At first, we show how the radiative symmetry breaking condition restricts the parameters strongly relevant to $m_{h^0}$. In Fig. 1 we give the plots of the solutions in $(\tan \beta, \langle \tilde{S} \rangle)$ and $(\tan \beta, \lambda)$ planes. Since we assume that only the top Yukawa coupling is large, the consistent $\tan \beta$ cannot be so large. We take it as $\tan \beta \leq 15.0$. In the case (II), $\langle \tilde{S} \rangle$ can take the value in such a wide range as $70 \, \text{GeV} \lesssim \langle \tilde{S} \rangle \lesssim 45 \, \text{TeV}$, which is not plotted in Fig. 1(a). These show that the value of $\lambda(m_t)$ and $\langle \tilde{S} \rangle$ are heavily restricted in the case (I) compared with the case (II). We should remind that $\lambda(m_t)$ and $\langle \tilde{S} \rangle$ affect $m_{h^0}$ through Eqs. (2) and (4). As seen from Fig. 1, the $\langle \tilde{S} \rangle$ scale can take a value in a rather wide range to cause the successful radiative symmetry breaking.

In Fig. 2 we show the allowed region of $m_{h^0}$ in both cases for the corresponding parameter sets to the ones of Fig. 1. This shows that the imposition of the consistent occurrence of the radiative symmetry breaking can strongly affect the estimation of $m_{h^0}$. The boundary value of $m_{h^0}$ can be changed by a few percent to ten percent. We can also find some parameter dependences of $m_{h^0}$ in these figures. In Fig. 2(b) we restrict the initial soft scalar mass as $\tilde{m} = 1 \, \text{TeV}$. The larger values of soft scalar mass $\tilde{m}$ and $\langle \tilde{S} \rangle$ tend to realize the larger value of $m_{h^0}$. The value of $\langle \tilde{S} \rangle$ determines the off-diagonal

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3 To estimate $\tan \beta$ we take account of the translation of the pole mass to the running mass [12].
component of the stop mass matrix (4). This shows that the larger stop mixing tends to make $m_{h^0}$ larger.

It is useful to comment on the existence of two branches of the solutions which show a very different behavior in Figs. 1 and 2. This is a common feature in both case (I) and (II) and also in the models with a different $n$. For simplicity, we take the case (II) as an example to discuss this feature here. We refer the one of the smaller $\tan \beta$ as a branch B1 and the one of the larger $\tan \beta$ as a branch B2. They are divided by an initial value of $h_t$ as shown in Table 1. The interesting feature of two branches can be clearly seen in Fig. 1(b) and Table 1. The branch B2 corresponds to the smoothly extending solutions of $\lambda(m_t)$ to the large $\tan \beta$ region and the branch B1 comes from the ones which are confined in the small $\tan \beta$. The branch B2 has a similar lower bound of $\tan \beta$ and the similar maximum value of $m_{h^0} \sim 143\text{GeV}$ at $\tan \beta \gtrsim 9$ for the different $n$ values. If we note that at the larger $\tan \beta$ region the second term of Eq. (2) can be neglected and Eq. (2) reduces to the one of the MSSM, this behavior can be understood. On the other hand, the branch B1 strongly depends on $n$. There are solutions with a little bit larger maximum value of
\[ \lambda(\mu_t) \] at the smaller \( \tan \beta \) according to the increase of \( n \). This results in the larger maximum value of \( m_{h^0} \) for the larger \( n \). The reason can be mainly found in the \( \beta \) dependence of Eq. (2). Decreasing \( n \), the maximum value of \( \tan \beta \) of the branch B1 increases, where the maximum value of \( \lambda(\mu_t) \) is realized. The branch B1 disappears finally at \( n = 0 \) since it is difficult to realize \( 0.95 \lesssim h_t(\mu_t) \lesssim 1.35 \) corresponding to \( 1 \lesssim \tan \beta \lesssim 15 \) starting from the initial value of \( h_t \) used here. We should remind that the value of \( h_t(\mu_t) \) is strictly restricted by Eq. (6). In the case (I) we also find the similar qualitative feature discussed here in the case (II).

Finally we want to present one comment. There is a rather big difference of the density of the radiative symmetry breaking solutions in the models with the different \( n \). For example, in the present parameter setting the number of solutions in \( n = 4 \) is very smaller than the ones in \( n = 3 \). The finer tuning of marameters seems to be necessary to find the solution in the \( n = 4 \) case compared with the \( n = 3 \) case.

In summary we studied the upper bound \( m_{h^0} \) of the lightest neutral Higgs scalar mass in the NMSSM using the RGE analysis. We required the successful occurence of the electroweak radiative symmetry breaking starting from the suitable initial values of parameters. This condition substantially constrains the allowable parameter space and as a result the mass bound \( m_{h^0} \) is heavily restricted compared with the one obtained without imposing this condition. We discussed the typical feature related to the initial value of \( h_t \) and also why the larger \( n \) models could bring the larger maximum value of \( m_{h^0} \).

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References


