CM-P00060420

Ref.TH.1669-CERN

Archives

HIGH ENERGY BEHAVIOUR OF TREE DIAGRAMS IN GAUGE THEORIES

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A B S T R A C T

Tree diagram scattering amplitudes may behave extra badly at high energy when vector mesons are involved. The cancellation, among bad terms, implied by gauge invariance is discussed. It is shown that in the renormalizable spontaneously broken gauge theories such cancellation is complete.

Ref.TH.1669-CERN

7 May 1973

It is well known that in quantum field theories with vector mesons tree diagrams for scattering amplitudes may behave badly at high energy, and that this is related somehow to non-renormalizability ¹⁾. It is well known that gauge invariance leads to cancellation among the bad terms ²⁾ and it is natural to conjecture that these cancellations are complete in spontaneously broken gauge (SBG) theories which are supposed to be renormalizable ³⁾. This will be demonstrated here. It seems that no one has bothered to do this already ; but now C.H. Llewellyn Smith ⁴⁾ has studied the (more interesting) inverse question : does the requirement of good high energy behaviour (for physical scattering amplitudes in tree approximation) dictate SBG structure ? The following may be a useful preface to his paper.

First, for ease of reference, it will be recalled how gauge invariance permits the complete elimination of the bad terms in the Abelian case - i.e., for "massive photons". Then the point at which such arguments fail in the non-Abelian case of massive Yang-Mills fields will be indicated. Finally it will be shown that related arguments again go through in the SBG case. Since we are concerned only with tree diagrams it is not necessary to invoke the full machinery of quantized gauge theories ; classical field theory is sufficient ⁵⁾.

Consider a Lagrangian

$$L = -\frac{1}{2}(\partial_\mu W_\nu - \partial_\nu W_\mu)^2 - \frac{1}{2}M^2 W_\mu^2 + J_\mu W_\mu - R \quad (1)$$

where R is a polynomial residual Lagrangian not containing W in linear or bilinear terms. The equation of motion for W is

$$\partial_\mu \partial_\mu W_\nu - \partial_\nu \partial_\mu W_\mu - M^2 W_\nu = \frac{\delta R}{\delta W_\nu} - J_\nu \quad (2)$$

With Feynman boundary conditions this can be cast into integral form

$$W_\mu = \int d\gamma G_{\mu\nu}(x,\gamma) \left\{ J_\nu(\gamma) - \frac{\delta R}{\delta W_\nu}(\gamma) \right\} \quad (3)$$

where $G_{\mu\nu}$ is the Fourier transform of

$$(\delta_{\mu\nu} + k_\mu k_\nu / M^2) / (M^2 + k^2 - i\epsilon) \quad (4)$$

Treating all fields and J as small, iterative solution of (3) yields a series of terms which can be represented in an obvious way à la Feynman by tree diagrams, such as

The end x_μ is associated with the point x_μ at which the field is evaluated, and the other ends are associated with various arguments $y, z, \dots$ of the source J . In this way

$$\begin{aligned} W_\mu(x) &= \int dy G_{\mu\nu}(x, y) J_\nu(y) \\ &+ \frac{1}{2!} \int dy dz G_{\mu\nu\lambda}(x, y, z) J_\nu(y) J_\lambda(z) \\ &+ \dots \end{aligned} \quad (6)$$

where

$$G_{\mu\nu\lambda\dots}(x, y, z, \dots) \quad (7)$$

is the sum of contributions from all tree diagrams with the appropriate end points. It is important to note that despite the unsymmetric construction described here, which seems to distinguish the arguments (μ, x) , the functions (7) are actually fully symmetric in their end points. We identify the quantities (7) with the Feynman n point functions (in tree approximation) of the quantum theory. Physical scattering matrix elements are then obtained by the following steps :

Fourier transform :

$$\tilde{G}_{\mu\nu\dots}(k, l, \dots) = \int dx dy \dots e^{ikx + il y + \dots} G_{\mu\nu\dots}(x, y, \dots) \quad (8)$$

Cancel the propagator denominators (M^2+k^2) associated with external lines :

$$\Gamma_{\mu\nu\cdots}(k,l,\cdots) = \tilde{G}_{\mu\nu\cdots}(k,l,\cdots)(M^2+k^2)(M^2+l^2)\cdots \quad (9)$$

Put the external momenta on their respective mass shells :

$$k^2 \rightarrow -M^2, \quad l^2 \rightarrow -M^2, \quad \cdots \quad (10)$$

Contract the free tensor indices with physical polarization vectors

$$\epsilon_\mu \epsilon'_\nu \cdots \Gamma_{\mu\nu\cdots}(k,l,\cdots) \quad (11)$$

This is the scattering matrix element. Note that

$$k_\mu \epsilon_\mu = 0, \quad l_\mu \epsilon'_\mu = 0, \quad \cdots \quad (12)$$

and note that for zero helicity

$$\epsilon_\mu = \left(\frac{\vec{k}}{|\vec{k}|} \frac{k_0}{M}, i \frac{|\vec{k}|}{M} \right) = \frac{k_\mu}{M} + \vartheta_\mu \quad (13)$$

where ϑ_μ is of order (M/k_0) and

$$k_\mu \vartheta_\mu = M \quad (14)$$

The extension of all this to allow for other fields is straightforward (except that Fermion sources must anticommute).

Associated with vector mesons, as compared with scalar mesons, there are two potential sources of bad behaviour :

The propagator (4), because of the $k_\mu k_\nu / M^2$ in the numerator, does not fall off for large k_μ . (15)

The zero helicity polarization vector (14) increases for large k_μ . (16)

We have to study to what extent gauge invariance allows these dangers to be transformed away.

Actually it is possible (although not always very useful, as will be seen) to dispose of point (16) immediately by the Veltman trick. After step (10), in the formation of scattering matrix elements, there is still a factor

$$g_{\mu\nu} + k_\mu k_\nu / M^2 \quad (17)$$

associated with each external line. Note that

$$k_\mu (g_{\mu\nu} + k_\mu k_\nu / M^2) = k_\mu (M^2 + k^2) / M^2$$

is zero on mass shell. So the bad polarization vector (13) may systematically be replaced by the well-behaved g_μ :

$$\epsilon_\mu \rightarrow g_\mu \quad (18)$$

We will initially suppose this to be done and concentrate on difficulty (15).

Suppose that only the mass and source terms in (1) spoil invariance under infinitesimal variations

$$W_\mu \rightarrow W_\mu - \partial_\mu \omega \quad (19)$$

with associated variations of other fields. Then if this variation is made away from a solution of the equations of motion we must have ⁶⁾

$$\delta \int dx (\mathcal{J}_\mu W_\mu - \frac{1}{2} M^2 W_\mu^2) = 0$$

- because the action as a whole is stationary and any variation could come only from the gauge breaking part. So

$$\int dx \partial_\mu W_\mu (\mathcal{J}_\mu - M^2 W_\mu) = 0$$

whence

$$\partial_\mu W_\mu = M^{-2} \partial_\mu \mathcal{J}_\mu \quad (20)$$

Introducing this into (2)

$$(\partial_\mu \partial_\mu - M^2) W_\nu = \frac{\delta R}{\delta W_\nu} - \mathcal{J}_\nu + M^{-2} \partial_\nu \partial_\mu \mathcal{J}_\mu \quad (21)$$

Starting from (21) rather than (2) we come to a diagram expansion involving the good propagator

$$\delta_{\mu\nu} / (M^2 + k^2 - i\epsilon) \quad (22)$$

rather than the bad (4). However, an extra source term has appeared

$$M^{-2} \partial_\nu \partial_\mu \mathcal{J}_\mu \quad (23)$$

When the current is finally replaced by a polarization vector this gives

$$- M^{-2} k_\nu k_\mu \epsilon_\mu = 0$$

or for the zero-helicity residual vector η of (13)

$$- M^{-2} k_\nu k_\mu \eta_\mu = -k_\nu / M$$

- i.e., the extra source term just restores the bad part of the zero helicity state (13), recreating difficulty (16) for all but one of the external lines. Accepting this, renouncing the Veltman trick, and anticipating (12), the extra source can be dropped.

In the present case one can go further. For this purpose it is useful to note that the modified equation

$$(\partial_\mu \partial_\mu - M^2) W_\nu = \frac{\delta R}{\delta W_\nu} - J_\nu \quad (24)$$

follows directly from the modified Lagrangian

$$L = \frac{1}{2} (\partial_\mu W_\mu)^2$$

Here the extra term, as well as the original mass term, breaks gauge invariance. The argument giving (20) now gives instead

$$(M^2 - \partial_\mu \partial_\mu) \partial_\nu W_\nu = \partial_\nu J_\nu \quad (25)$$

or

$$\partial_\nu W_\nu(x) = \int d\gamma G(x, \gamma) \partial_\nu J_\nu$$

where G is the Feynman propagator. It follows that $\partial_\nu W_\nu$ is again just linear in the source, and so accounted for entirely by the trivial diagram

$$X_\mu \text{ --- }$$

For interesting diagrams we have effectively

$$\partial_\mu W_\mu = 0$$

Then

$$k_\mu \Gamma_{\mu\nu \dots}(k, l, \dots) = 0$$

with $\Gamma_{\mu\nu\dots}$ defined as in (9) but in the modified formalism of (25). By symmetry

$$l_\nu \Gamma_{\mu\nu\dots}(k, l, \dots) = 0$$

etc. So once again the bad terms in the zero helicity states can be dropped, and there are no bad terms left. So much for the Abelian case.

Note by the way that in this modified formalism the limit $M \rightarrow 0$ presents no problem. In particular scattering matrix elements involving zero helicity states go to zero. A massless theory can be defined in this way.

So far the only external lines considered were associated with a single species of vector meson. The rôle of other particles in general will be sufficiently illustrated by that of other vector mesons, which in any case are of particular interest. Suppose then that in the Lagrangian the quadratic and bilinear terms involving vector mesons are those displayed explicitly in

$$L = -\frac{1}{4} (\partial_\mu W_\nu^n - \partial_\nu W_\mu^n)^2 - \frac{1}{2} (M W_\mu^n)^2 + J_\mu^n W_\mu^n - R \quad (26)$$

where the summation convention operates, and

$$M W_\mu^n \equiv M_n W_\mu^n$$

Suppose that only the W mass and source terms in (26) spoil invariance under

$$\delta W_\mu^l = -\partial_\mu \omega^l + \Lambda_{lm}^n \omega^n W_\mu^m \quad (27)$$

(and related variations of other fields) where the Λ 's are a set of numbers antisymmetric for exchange of any pair of indices.

Consider now the application of the previous argument to a particular W^n , say W^0 . The relevant gauge transformation is with

$$\omega^l = \delta_{20} \omega \quad (28)$$

Because of the antisymmetry of the Λ 's, (27) then reduces for $l=0$ to (19). The subsequent argument, however, must be modified because the extra source terms do, and the other mass terms may, also spoil gauge invariance. As a result (20) and (21) are replaced by

$$\begin{aligned} \partial_\mu W_\mu^0 &= M_0^{-2} \left\{ \partial_\mu J_\mu^0 + \Lambda_{em}^0 J_\mu^l W_\mu^m - \Lambda_{em}^0 (M^2 W_\mu^l) W_\mu^m \right\} \\ &= D, \text{ say} \end{aligned} \quad (29)$$

$$(\partial_\mu \partial_\mu - M_0^2) W_\nu^0 = \frac{\delta R}{\delta W_\nu^0} - J_\nu^0 + \partial_\nu D \quad (30)$$

The extra source term $\partial_\mu J_\mu$ can again be dropped at the expense of renouncing the Veltman trick (18). There is now another extra source in (29), linear in field strength W . This gives rise to diagrams in which at least one source is connected to the rest of the diagram by two lines rather than one. Such contributions do not have the one particle poles required to survive steps (9) and (10) above, and so do not contribute to physical scattering matrix elements. So reasoning for each vector meson in turn the original equations of motion

$$\partial_\mu \partial_\mu W_\nu^n - \partial_\nu \partial_\mu W_\mu^n - M^2 W_\nu^n = \frac{\delta R}{\delta W_\nu^n} - J_\nu^n \quad (31)$$

can be replaced by

$$\partial_\mu \partial_\mu W_\nu^n - M^2 W_\nu^n = \frac{\delta R}{\delta W_\nu^n} - J_\nu^n - M_n^{-2} \partial_\nu \left\{ \Lambda_{em}^n (M^2 W_\mu^l) W_\mu^m \right\} \quad (32)$$

Then danger (15) has again been eliminated at the expense of having some new vertices in the diagrams [and of restoring difficulty (16) for all but one external line]. The extra vertex terms in (32) cancel against one another for given n (because of the antisymmetry of Λ) when the masses of all other mesons W^l , with $l \neq n$, are equal. In the completely symmetric case, all masses m_n equal, the extra vertices go away completely. More generally they do not pose a new threat of bad high energy behaviour, because their coupling constants

$$(M_l^2 / M_n^2) \Lambda_{lm}^n$$

are dimensionless.

To deal again with (16), one might note again that in the fully symmetric case the modified equations

$$(\partial_\mu \partial_\mu - M_n^2) W_\nu^n = \frac{\delta R}{\delta W_\nu^n} - J_\nu^n \quad (33)$$

again follow from the modified Lagrangian

$$L = \frac{1}{2} (\partial_\mu W_\mu^n)^2 \quad (34)$$

Then we again have

$$\delta \int dx \left(-\frac{1}{2} (\partial_\mu W_\mu^n)^2 - \frac{1}{2} (M W_\mu^n)^2 + J_\mu^n W_\mu^n \right) = 0$$

under gauge variations (27). In the particular case (28) for example this gives

$$0 = \int dx \left\{ M^2 W_\mu^0 \partial_\mu \omega - J_\mu^0 \partial_\mu \omega + J_\mu^l \Lambda_{lm}^0 \omega W_\mu^m + \partial_\mu W_\mu^0 \partial_\nu \partial_\nu \omega - \partial_\mu W_\mu^l \partial_\nu (\Lambda_{lm}^0 \omega W_\nu^m) \right\}$$

whence

$$(M^2 - \partial_\nu \partial_\nu) \partial_\mu W_\mu^0 = \partial_\mu J_\mu^0 + \Lambda_{em}^0 J_\mu^L W_\mu^m + \Lambda_{em}^0 W_\nu^m \partial_\nu \partial_\mu W_\mu^L \quad (35)$$

Because of the last term here, which is not a source term, this is much less immediately useful than (25). It does permit the conclusion that $\partial_\mu W_\mu$ is zero when all sources are finally on-mass shell and divergenceless [i.e., anticipating (12)]. This permits the Veltman trick in one external line, but this we had already.

So for massive Yang-Mills fields we can eliminate the propagator difficulty (15), but have not entirely disposed of the zero helicity difficulty (16). There could remain as many bad powers of energy as there are zero helicity external lines less two. Less two, because the Veltman trick for one line disposes of two powers of k_0 .

$$\xi_\mu = O\left(\frac{M}{k_0}\right)^2 \epsilon_\mu$$

It is notable that symmetry breaking mass terms do not weaken this conclusion. Note, however, that symmetry breaking Fermion masses, m , of spin $\frac{1}{2}$ fields could be more serious, introducing extra vertices in (32) with dangerously dimensional coupling constants

$$\propto m / M^2$$

The fact that we have not succeeded in explicitly eliminating the bad terms does not of course prove that they do not cancel among one another in some way that could be exhibited by being smarter. That they do not cancel has, however, been shown for particular examples by explicit calculation ⁴⁾.

The persistence of powers of (k/M) when all momenta are scaled up implies of course the persistence of such powers when the masses are scaled down. With dimensionless coupling constants these limits are equivalent. Thus the zero mass limit of the Yang Mills theory does not exist, even at the tree diagram level. That such a limit does not give the zero mass theory is known ⁷⁾.

Consider finally SBG theories. We start from a Lagrangian

$$L = -\frac{1}{4}(\partial_\mu W_\nu^n - \partial_\nu W_\mu^n)^2 - \frac{1}{2}(M W_\mu^n + \partial_\mu \theta^n)^2 - R \quad (36)$$

It is supposed that a scalar field θ occurs in association with each massive W ; that the only linear or bilinear terms involving W or θ are those displayed explicitly ; that the residual terms R are polynomial and have no dangerously dimensional couplings (i.e., no inverse masses). There is supposed to be exact gauge symmetry (i.e., unbroken even by masses) under

$$\begin{aligned} \delta W_\mu^n &= -\partial_\mu \omega^n + \dots \\ \delta \theta^n &= M \omega^n + \dots \\ \delta \phi &= \dots \end{aligned} \quad (37)$$

where the residual terms --- are field dependent, and ϕ stands for any other of the fields in the theory.

By a change of variables

$$\begin{aligned} W_\mu^n &\rightarrow W_\mu^n - M_n^{-1} \partial_\mu \theta^n + \dots \\ \theta^n &\rightarrow \theta^n + \dots \\ \phi &\rightarrow \phi + \dots \end{aligned} \quad (38)$$

(where the residual terms --- are non-linear in fields) the variables θ can be eliminated from the Lagrangian. Thus these fields are considered as "ghost" fields, not associated with the asymptotic propagation of free particles. It is at this stage, with the ghost free Lagrangian, that we add source terms (for fields other than θ) and define n point functions and scattering matrix elements in the usual way.

Add now to the starting Lagrangian (36), following 't Hooft ³⁾, a term

$$-\frac{1}{2}(\partial_\mu W_\mu^n + M \theta^n)^2 \quad (39)$$

At the "ghost-free" stage this would add

$$-\frac{1}{2} F_n F_n \quad (40)$$

with

$$F_n = \partial_\mu W_\mu^n + M \theta^n - M_n^{-1} \partial_\mu \partial_\mu \theta^n + \dots \quad (41)$$

This would restore some θ dependence to the otherwise ghost free Lagrangian. Varying (40) with respect to θ 's would yield the equations of motion

$$\sum_m \left\{ (M_n^{-1} \partial_\mu \partial_\mu - M_n) \delta_{nm} + \dots \right\} F_m = 0 \quad (42)$$

(not summed over n). Iterative solution of these along with the other equations of motion yields

$$F_n = 0 \quad (43)$$

Thus the extra term (40) serves only to determine the otherwise undetermined and unimportant θ , without changing the other fields - because (43) implies that the variation of the extra term (40) with respect to any field is zero.

An important special case is that in which there is a zero mass photon, say W^0 , and a corresponding Abelian gauge invariance persisting at the "ghost free" stage. In this case there is no field θ^0 which can be eliminated; the θ^0 terms in the above equations are omitted. The addition of the $n=0$ term in (40) is not then justified as simply determining a particular redundant field θ , but it is justified as a legitimate choice of electromagnetic gauge.

With the extra term (39) the starting Lagrangian (36) becomes

$$L = -\frac{1}{4} (\partial_\mu W_\nu^\mu - \partial_\nu W_\mu^\mu)^2 - \frac{1}{2} (M W_\mu^\mu + \partial_\mu \theta^\mu)^2 - \frac{1}{2} (\partial_\mu W_\mu^\mu + M \theta^\mu)^2 - R \quad (44)$$

In the action integral the $W\theta$ cross terms just cancel, so this is equivalent to

$$L = -\frac{1}{2} (\partial_\mu W_\nu^n)^2 - \frac{1}{2} (M W_\mu^n)^2 - \frac{1}{2} (\partial_\mu \theta^n)^2 - \frac{1}{2} (M \theta^n)^2 - R \quad (45)$$

If we were to add source terms

$$J_\mu^n W_\mu^n + J \phi$$

to (45) and define n point functions in the usual way, they would differ of course from those of the ghost free formalism. But they would give the same physical scattering matrix elements. The extra terms due to the nonlinearities --- in (38) would not have the right pole structure to survive steps (9), (10). The linear term $\partial_\mu \theta$ would also be ineffective if we anticipate (12). Actually it is convenient not to anticipate (12), but to assume rather the Veltman trick (18). Therefore we choose to define n point functions in terms of the combination

$$U_\mu^n = W_\mu^n + M_n^{-1} \partial_\mu \theta^n \quad (46)$$

which differs from the ghost free W only by non-linear terms. So we use the source terms

$$J_\mu^n U_\mu^n + J \phi \quad (47)$$

From (45) with (47)

$$\begin{aligned} (\partial_\mu \partial_\mu - M_n^2) W_\nu^n &= \frac{\delta R}{\delta W_\nu^n} - J_\nu^n \\ (\partial_\mu \partial_\mu - M_n^2) \theta^n &= \frac{\delta R}{\delta \theta^n} + \frac{\partial_\mu J_\mu^n}{M_n} \end{aligned} \quad (48)$$

Iterative solution yields a diagram expansion in which the only dangerous looking terms come from those explicitly containing reciprocal masses and gradients in (46) and (48). However, these dangers disappear when the currents are finally replaced by, and the field U contracted with, polarization vectors ϵ or φ . The $M^{-1}\partial\theta$ and $M^{-1}\partial J$ then give factors

$$M^{-1} k_\mu \epsilon_\mu = 0$$

for non-zero helicity, or

$$M^{-1} k_\mu \varphi_\mu = 1$$

for zero helicity. Thus the dangers associated with vector mesons are in SBG theories only apparent.

In connection with this work, I thank Professors G. 't Hooft, C.H. Llewellyn Smith, R. Stora and M.J.G. Veltman for guidance.

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