T-duality and Actions for Non-BPS D-branes

E. A. Bergshoeff\textsuperscript{1}, M. de Roo\textsuperscript{1}, T. C. de Wit\textsuperscript{1}, E. Eyras\textsuperscript{2} and S. Panda\textsuperscript{3}

\textsuperscript{1}Institute for Theoretical Physics
Nijenborgh 4, 9747 AG Groningen
The Netherlands

\textsuperscript{2}DAMTP
University of Cambridge
Wilberforce Road
Cambridge CB3 9EW, UK

\textsuperscript{3}Mehta Research Institute of Mathematics and Mathematical Physics
Chhatnag Road, Jhoosi
Allahabad 211019, India

Abstract

We employ T-duality to restrict the tachyon dependence of effective actions for non-BPS D-branes. For the Born-Infeld part the criteria of T-duality and supersymmetry are satisfied by a simple extension of the D-brane Born-Infeld action.
1 Introduction

Dp-branes [1] have played a crucial rule in the understanding of the relations between different string theories. They are stable extended objects, which preserve half of the maximal supersymmetry. They are known in terms of explicit solutions to low-energy supergravity equations, and their effective actions and its symmetries are by now well understood [2].

D-branes also give rise to other stable and unstable objects, such as brane-antibrane configurations, and non-BPS D-branes [3, 4, 5]. These objects and their descendants could potentially play an equally important role, since they extend the relations between string theories to a different domain\(^1\). Non-BPS branes in the Type II theories are unstable, and can decay to the stable D-branes. Much work has already been done on the classification of these objects [9, 10], and this has clarified their relation to D-branes, as well as the structure of the hierarchy of D-branes themselves.

A proposal for an effective action for non-BPS D-branes in the Type II theories was given by Sen [11]. In this action the instability is due to the presence of a tachyon. The tachyon dependence must be such that condensation to the D-brane is possible, which puts severe restrictions on the tachyon dependence of this action. The general structure is that a non-BPS Dp-brane in the Type IIA (IIB) theory, will condense to a BPS D(p-1)-brane in the same IIA (IIB) theory. The non-BPS Dp-branes in IIA (IIB) are related to BPS D(p+1)-brane and antibrane configurations in IIA (IIB) by condensation of the complex tachyon living on this brane-antibrane pair. This interrelationship between BPS and non-BPS branes implies that the T-duality map between Dp-branes in IIA (IIB) to D(p±1)-branes in IIB (IIA) must also hold for the non-BPS branes. This T-duality map gives further information about the tachyon dependence of the effective action of the non-BPS brane. It is our aim to investigate the T-duality properties of non-BPS branes in the Type II theories.

In this introduction we will give a short overview of the effective action for non-BPS D-branes, mainly following [11] for the Born-Infeld contribution, and [12, 13] for the Wess-Zumino term. Then we will discuss aspects of T-duality, first for the Born-Infeld term in Section 2, then for the Wess-Zumino term in Section 3. In Section 4 we speculate about the analogue of tachyon condensation in non-BPS D-branes in the context of \(D = 11\) M-branes.

The Born-Infeld term in the action for a non-BPS Dp-brane in a nontrivial background should be of the following form:

\[
S_{BI}^{(p)} = - \int d^{p+1}\sigma e^{-\phi} \sqrt{|\det G_{ij}|} f(T, \partial T, ..),
\]

where

\[
G_{ij} = g_{ij} + \mathcal{F}_{ij}.
\]

Here \(g\) is the metric induced by the supersymmetric line-element, and \(\mathcal{F}\) involves the Born-Infeld vector \(F_{ij} = 2\partial_i V_j\) and the Neveu-Schwarz \(B\)-superfield. The function \(f\) contains the dependence on the tachyon and its derivatives, and may also depend on other worldvolume and background fields.

\(^1\)For reviews, see [6, 7, 8]
In this paper we discuss the conjecture that (1.1) is of the form

$$S_{BI}^{(p)} = - \int d^{p+1}\sigma \sqrt{\det (G_{ij} + \partial_i T \partial_j T)} | g(T) | \ .$$

(1.3)

The argument which we will advance in support of this conjecture is that (1.3) agrees with T-duality and supersymmetry. Further arguments, based on a calculation of $S$-matrix elements, have been discussed by Garousi [14]. Of course, (1.3) is a special case of (1.1), and can be rewritten in that form by expanding the root. Our result implies that all terms in the expansion of (1.3) satisfy the requirement of T-duality. Independently of the validity of the conjecture we identify which contributions are allowed to appear in $f(T, \partial T, \ldots)$ by T-duality.

The Wess-Zumino term for a single non-BPS D-brane takes on the form:

$$S_{WZ}^{(p)} = \int d^{p+1}\sigma \ C \wedge dT \wedge e^\mathcal{F} \ .$$

(1.4)

The superfield $C$ contains the Ramond-Ramond fields. This generalizes the D-brane Wess-Zumino action given by [15], to which we refer for the notation. The leading form in $C$ is a $p$-form. The kink solution for the tachyon is expected to give a $\delta$-function from the $dT$-contribution and thus to produce the standard Wess-Zumino term for the resulting D$(p-1)$-brane.

The terms (1.1) and (1.4) are separately invariant under worldvolume reparametrizations and target space (super-)reparametrizations. It is assumed that the tachyon is a scalar under worldvolume reparametrizations, and that the function $f$ depends only on invariant combinations of worldvolume and background fields. However, the sum of (1.1) and (1.4) is not $\kappa$-symmetric as it would be for D-branes [16, 17, 18]. The relation between the non-BPS D$p$-brane and the BPS D$(p-1)$-brane then arises as follows. The tachyonic kink-solution effectively reduces the dimension of the worldvolume by one. Because the tachyon is then constant almost everywhere, terms with derivatives of tachyons vanish. The remainder of the action should then be the standard effective action of a D$(p-1)$-brane. The fermionic $\kappa$-symmetry, which is absent for the non-BPS brane, is restored by the tachyon condensation. In the resulting action the full set of background fields, as well as invariance under all (super-)reparametrizations, are still present.

In a flat background this can be made more explicit (see [16]). Then we have:\footnote{Worldvolume indices are denoted by $i, j = 0, \ldots, p$, target space indices by $\mu, \nu = 0, \ldots, D - 1$.}

$$g_{ij} = \eta_{\mu\nu} \Pi^\mu_i \Pi^\nu_j \ ; \ \ \ \ \Pi^\mu_i = \partial_i X^\mu - \bar{\partial} \Gamma^\mu \partial_i \theta \ ,$$

$$\mathcal{F}_{ij} = \partial_i V_j - \partial_j V_i - \{ \bar{T} \Gamma_{ij} \partial_i \theta \ (\partial_j X^\mu - \bar{\partial} \Gamma^\mu \partial_j \theta) - (i \leftrightarrow j) \} \ ,$$

$$C_{i_1 \ldots i_p} = \partial_{[i_1} X^{\mu_1} \ldots \partial_{i_{p-1}} X^{\mu_{p-1}} \bar{\theta} \mathcal{P}_{(p)} \Gamma_{i_1 \ldots i_p} \partial_{i_p]} \theta + \ldots \ .$$

(1.5)\quad (1.6)\quad (1.7)

In (1.5-1.7) we present the IIA case. The Majorana spinor $\theta$ can be expanded as $\theta = \theta_L + \theta_R$. To obtain the IIB case we should replace $\Gamma_{11}$ by $\sigma_3$ and write $\theta$ as a doublet $(\theta_{1R}, \theta_{2R})$. In the expression for the Ramond-Ramond field $\mathcal{P}_{(p)}$ equals $\Gamma_{11}$ for $p = 4k + 1$ and $1$ for $p = 4k + 3$. In the IIB case $p$ is even, and we must have $\mathcal{P}_{(p)}$ equal to $i\sigma_2$ for $p = 4k$, and equal to $\sigma_1$ for $p = 4k + 2$. In (1.7) we have not written higher-order contributions in the fermions, which are required for supersymmetry.
2 T-duality and the Born-Infeld term

Before coming to non-BPS D-branes, let us briefly recall how T-duality works for D-branes [19]. In this example we work in a flat background, but we keep the worldvolume fermions to identify later the possible couplings between fermions and tachyons. For a Dp-brane we have the following Born-Infeld term

\[ \mathcal{L}_{\text{BI}} = - \sqrt{|\det G_{10ij}^{(p)}|}, \]  

with \( G = g + \mathcal{F} \). We will reduce a IIA Dp-brane and a IIB D(p+1)-brane to a nine dimensional Dp-brane. T-duality amounts to the fact that the resulting worldvolume actions should be the same in the two cases. The reduction of the fermions in the IIA case is as follows [20]:

\[ \theta_R \rightarrow \begin{pmatrix} \theta_1 \\ 0 \end{pmatrix}, \quad \theta_L \rightarrow \begin{pmatrix} 0 \\ \theta_2 \end{pmatrix}. \]  

The \( \Gamma \)-matrices reduce as

\[ \Gamma^\mu \rightarrow \begin{pmatrix} 0 & \gamma^\mu \\ \gamma^\mu & 0 \end{pmatrix}, \quad (\mu = 0, \ldots, 8), \quad \Gamma^9 \rightarrow \begin{pmatrix} 0 & \mathbb{I} \\ -\mathbb{I} & 0 \end{pmatrix}, \quad \Gamma^{11} = \begin{pmatrix} \mathbb{I} & 0 \\ 0 & -\mathbb{I} \end{pmatrix}. \]  

The reduction is over a transverse direction and the corresponding coordinate \( X^9 \) is written as a worldvolume scalar \( S \). The result is

\[ G_{10ij}^{(p)} \rightarrow G_{9ij}^{(p)} - \partial_i S \partial_j S + 2\bar{\theta}_2 \partial_i \theta_2 \partial_j S - 2\bar{\theta}_1 \partial_j \theta_1 \partial_i S + 2\bar{\theta}_2 \partial_i \theta_2 \bar{\theta}_1 \partial_j \theta_1, \]  

with

\[ G_{9ij}^{(p)} = g_{ij} + F_{ij} - 2\bar{\theta}_2 \gamma^\mu \partial_i \theta_2 \partial_j X^\mu - 2\bar{\theta}_1 \gamma^\mu \partial_j \theta_1 \partial_i X^\mu \\
+ \bar{\theta}_2 \gamma^\mu \partial_i \theta_2 \gamma^\nu \partial_j \partial_i X^\mu + \bar{\theta}_1 \gamma^\mu \partial_i \theta_1 \gamma^\nu \partial_j \theta_1 + 2\bar{\theta}_2 \gamma^\mu \partial_i \theta_2 \bar{\theta}_1 \gamma^\nu \partial_j \theta_1. \]  

In the IIB case we reduce a D(p+1)-brane over a worldvolume direction. We gauge-fix the corresponding coordinate \( X^9 \) equal to a worldvolume coordinate \( \sigma \), and the corresponding component of the Born-Infeld vector becomes a worldvolume scalar \( S \). The fermions now reduce as follows:

\[ \theta_1 \rightarrow \begin{pmatrix} \theta_1 \\ 0 \end{pmatrix}, \quad \theta_2 \rightarrow \begin{pmatrix} 0 \\ \theta_2 \end{pmatrix}. \]  

and we obtain the following:

\[ G_{10}^{(p+1)} \rightarrow \begin{pmatrix} G_{9ij}^{(p)} - 2\bar{\theta}_2 \partial_i \theta_2 \bar{\theta}_1 \partial_j \theta_1 & \partial_i S - 2\bar{\theta}_2 \partial_i \theta_2 \\ -\partial_j S - 2\bar{\theta}_1 \partial_j \theta_1 & -1 \end{pmatrix}. \]  

Finally we have to prove that the determinants of the two nine-dimensional expressions are the same. This can be shown by using the identity

\[ \det \begin{pmatrix} A & B \\ C & D \end{pmatrix} = \det \begin{pmatrix} A - BD^{-1}C & B \\ 0 & D \end{pmatrix} \begin{pmatrix} \mathbb{I} & 0 \\ D^{-1}C & \mathbb{I} \end{pmatrix} = \det (A - BD^{-1}C) \det D. \]  

\[ \text{det} \begin{pmatrix} A & B \\ C & D \end{pmatrix} = \text{det} \begin{pmatrix} A - BD^{-1}C & B \\ 0 & D \end{pmatrix} \begin{pmatrix} \mathbb{I} & 0 \\ D^{-1}C & \mathbb{I} \end{pmatrix} = \text{det} (A - BD^{-1}C) \text{det} D. \]
To identify possible couplings between tachyons and the other worldvolume fields in the case of the non-BPS D-brane, we consider as the starting point the Lagrangian in (1.3):

\[
L_{\text{BI}}^{(p)} = -\sqrt{\det(G_{10}^{(p)ij} + \partial_i T \partial_j T)} g(T) .
\]  

(2.9)

If we assume that \( T \) becomes independent of the compact direction in which the T-duality transformation is performed, then the result of the calculation we performed above will be the same, with the replacement

\[
G_{ij}^{(p)} \rightarrow G_{ij}^{(p)} + \partial_i T \partial_j T .
\]

(2.10)

The equality between the determinants still holds, and T-duality will be preserved. If in addition we assume that \( T \) is inert under supersymmetry, (2.9) is also supersymmetric.

If we expand (2.9) in the tachyon field, we reobtain an action of the form (1.1), with a set of explicit couplings of the tachyon to worldvolume fields, and, in a general background, target space fields. Let us write explicitly the leading terms, including expressions quadratic in the fermions and in \( \partial T \), which result from this expansion for the case of a IIA flat background:

\[
L_{\text{BI}}^{(p)} = g(T) \sqrt{\det G_{ij}} \times \left\{ 1 + \frac{1}{2} G^{ij}(-2 \tilde{\theta}_L \Gamma_{\mu} \partial_i \theta_L \partial_j X^\mu - 2 \tilde{\theta}_R \Gamma_{\mu} \partial_i \theta_R \partial_j X^\mu + \partial_i T \partial_j T) \right. \\
- \frac{1}{2} G^{ki}(-2 \tilde{\theta}_L \Gamma_{\mu} \partial_i \theta_L \partial_k X^\mu - 2 \tilde{\theta}_R \Gamma_{\mu} \partial_i \theta_R \partial_k X^\mu) G^{lm} \partial_m T \partial_l T \\
\left. + \frac{1}{4} G^{ij}(-2 \tilde{\theta}_L \Gamma_{\mu} \partial_i \theta_L \partial_j X^\mu - 2 \tilde{\theta}_R \Gamma_{\mu} \partial_i \theta_R \partial_j X^\mu) G^{kl} \partial_k T \partial_l T + \ldots \right\} ,
\]

(2.11)

where \( G_{ij} = \eta_{\mu\nu} \partial_i X^\mu \partial_j X^\nu + F_{ij} \), and \( G^{ij} \) is its inverse. These couplings satisfy the requirement of T-duality. They are not supersymmetric by themselves, since we have expanded the supersymmetric combinations (1.5, 1.6). However, supersymmetry can be restored by the addition of quartic fermion terms, which follow from (1.5, 1.6). The expression (2.11) was written for the IIA case, for IIB replace \( \theta_L \rightarrow \theta_{2R} \), \( \theta_R \rightarrow \theta_{1R} \).

In the literature there has been some effort to include couplings between worldvolume fermions and tachyons in the function \( f \) [21]. This work suggests a coupling of the form

\[
\partial_i \tilde{\theta} \partial_\mu \theta \partial^ij ,
\]

(2.12)

multiplying a function of \( T \). Although this form is indeed nonzero in the IIA theory, it does not have a counterpart in the IIB theory because of the different chirality structure. Therefore it does not satisfy T-duality, and should not appear in the non-BPS brane action.

From (2.9) it is clear that the \((\partial T)^2\) terms couple only to the symmetric part of \( G_{10}^{(p)-1} \). Nevertheless, in the expansion (2.11) the terms mixing \((\partial T)^2\) with the fermionic contributions do couple to a nonvanishing NS-NS background field.

Note that the expansion of the determinant in (2.9) gives a series of couplings between tachyons and other fields, with fixed relative coefficients. Supersymmetry and T-duality are not sufficient to fix these coefficients; any couplings of tachyons with (1.5, 1.6) would satisfy these two requirements. Also, the potential \( g(T) \) in (2.9) is not restricted by our arguments. Recently, remarkable progress has been made in constructing \( g(T) \) from open string field theory [22, 23, 24, 25].
3 T-duality and Wess-Zumino terms

For the Wess-Zumino terms we will restrict ourselves to contributions quadratic in the fermions, and again to a flat background. Let us first consider the case of a non-BPS \( p \)-brane \((p \text{ odd})\) in the IIA theory. The Lagrangian in (1.4) then takes on the explicit form

\[
\mathcal{L}_{\text{WZ}}^{(p)} = \epsilon^{i_1 \cdots i_{p+1}} \frac{(p-1)/2}{k=0} a_{p,k} C_{i_1 \cdots i_{p-2k}} (F^k)_{i_{p-2k+1} \cdots i_{p}} \partial_{i_{p+1}} T ,
\]

(3.1)

where \( C \) is given by (1.7). The coefficients \( a_{p,k} \) will be fixed by T-duality (see below). Writing out the Majorana spinor \( \theta \) in terms of chiral components, we find, after a partial integration

\[
C_{i_1 \cdots i_{p-2k}} = 2 \partial_{[i_1} X^{\mu_1} \cdots \partial_{i_{p-2k-1}} X^{\mu_{p-2k-1}} \bar{\theta}_L \Gamma_{\mu_1 \cdots \mu_{p-2k-1}} \partial_{i_{p-2k}} \theta_R .
\]

(3.2)

Upon reduction to \( D = 9 \) the total result becomes

\[
\mathcal{L}_{\text{WZ}}^{(p)} \rightarrow 2 \epsilon^{i_1 \cdots i_{p+1}} \frac{(p-1)/2}{k=0} a_{p,k} \left\{ \bar{C}_{i_1 \cdots i_{p-2k}} (F^k)_{i_{p-2k+1} \cdots i_{p}} \partial_{i_{p+1}} T \\
+ (p - 2k - 1) \bar{C}_{i_1 \cdots i_{p-2k}} \partial_{i_{p-2k}} S (F^k)_{i_{p-2k+1} \cdots i_{p}} \partial_{i_{p+1}} T \right\} .
\]

(3.3)

Here we have defined

\[
\bar{C}_{i_1 \cdots i_m} = \partial_{[i_1} X^{\mu_1} \cdots \partial_{i_{m-1}} X^{\mu_{m-1}} \bar{\theta}_2 \gamma_{\mu_1 \cdots \mu_{m-1}} \partial_{i_m]} \theta_1 ,
\]

(3.4)

which are the nine-dimensional RR-fields in a flat \( D = 9 \) background.

Again we should compare with the result obtained by reducing the WZ-term of a IIB non-BPS \( p + 1 \)-brane to nine dimensions. The starting point is of the same form as (3.1), except that we will use coefficients \( b_{p+1,k} \). Recall that the worldvolume scalar \( S \) now comes from the world-volume vector. The final result is

\[
\mathcal{L}_{\text{WZ}}^{(p+1)} \rightarrow -2 \epsilon^{i_1 \cdots i_{p+1}} s_{p-2k+1} b_{p+1,k} \left\{ \frac{(p-1)/2}{k=0} (p - 2k) \bar{C}_{i_1 \cdots i_{p-2k}} (F^k)_{i_{p-2k+1} \cdots i_{p}} \partial_{i_{p+1}} T \\
+ \frac{(p+1)/2}{k=1} 2k \bar{C}_{i_1 \cdots i_{p-2k+1}} (F^k)_{i_{p-2k+2} \cdots i_{p-1}} \partial_{i_p} S \partial_{i_{p+1}} T \right\} .
\]

(3.5)

Here \( s_m \) \((m \text{ even})\) is a sign which is \(+1\) for \( m = 4l + 2 \), and \(-1\) for \( m = 4l \).

T-duality determines the coefficients \( a_{p,k} \) and \( b_{p+1,k} \) up to an overall normalization. The result is

\[
a_{p,k} = \frac{(p-1)!}{2^k k! (p - 2k - 1)!} a_{p,0} , \\
b_{p+1,k} = \frac{(-1)^k p!}{2^k k! (p - 2k)!} b_{p+1,0} , \\
a_{p,0} = -s_{p+1} p b_{p+1,0} .
\]

(3.6)
The normalization relative to the Born-Infeld term can be fixed by requiring that the Dp-brane action which arises for the kink solution is $\kappa$-symmetric.

The coefficients $a$ and $b$ in this section turn out to be the same as those that are required by T-duality of Dp-branes. We conclude therefore that also the Wess-Zumino terms of non-BPS branes satisfy T-duality.

4 Discussion

We have shown, assuming a particular form of the tachyon coupling, that the worldvolume action for non-BPS D-branes proposed by Sen satisfies the criteria of T-duality and supersymmetry. The particular form (1.3) is interesting by itself. The same form has been suggested, from a different point of view, in [14]. It suggests a higher-dimensional structure for the non-BPS brane, a point which was remarked also by Hořava [10]. It is therefore natural to discuss non-BPS branes within the context of the eleven-dimensional M-theory. In M-theory open M2-branes can end on M5-branes. The quantization of the open M2-brane, in a certain low-energy limit, leads to a self-dual tensor multiplet living on the worldvolume of the M5-brane. An important difference with the ten-dimensional context is that it is not known what the field theory describing a set of coinciding M5-branes or a M5-M5 system should be.

On the side of classification, which depends mainly on the structure of the Wess-Zumino terms, a lot of work in $D = 11$ has already been done [26, 27]. In this context an interesting relation between K-theory and the Killing isometry direction of [28] has been pointed out [29].

As far as the dynamics is concerned the situation is more complicated. The field-theoretical approach which is suitable for a tachyon field, resulting from open strings, is replaced by a more complicated structure involving strings on the worldvolume, which represents interactions with membranes. A number of interesting points about the issue of “tachyonic string” condensation have been raised by Yi [30]. In this scenario the M5-M5 system decays into a BPS M2-brane. To achieve this Yi proposes a Higgs-mechanism for the non-selfdual tensor that should arise from the two selfdual tensors living on the M5-M5 system. An unattractive feature of this scenario is that the source for the three transverse scalars that are needed to describe the M2-brane remain unclear. It would be desirable if a Higgs-mechanism could be constructed for a single self-dual tensor only. The other remaining selfdual tensor exactly contains the three degrees of freedom which are required to describe the three transverse scalars.

In order to see whether a Higgs mechanism for a single selfdual tensor can be constructed it is convenient to use as a starting point the following action:

$$L_0 = -\frac{1}{24} e^{ijklm} H_{0ij} H_{klm} - \frac{1}{12} H_{ijk} H^{ijk},$$  \hspace{1cm} (4.1)

with $i = 1, \ldots, 5$. This action is a gauge-fixed version of the action constructed in [31]. Note that the Lagrangian is not Lorentz covariant but the equations of motion are. A
necessary condition for the existence of a Higgs mechanism is that a massive extension of
the Lagrangian \( \mathcal{L}_0 \) exists. The only local mass term one can write down is given by \(^3\)

\[
\mathcal{L}_m = m^2 B^\mu \nu B_{\mu \nu}.
\]

However, we have checked that, although the equations of motion corresponding to \( \mathcal{L}_0 \) and
\( \mathcal{L}_m \) are Lorentz covariant, the ones corresponding to the combination \( \mathcal{L} = \mathcal{L}_0 + \mathcal{L}_m \) violate
Lorentz symmetry. Therefore, within the context of a local field theory, a Higgs mechanism
for a single self-dual tensor seems not possible. It will be interesting to see how such a
mechanism is realized in M-theory.

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References


hep-th/9803194.

hep-th/9805019.

hep-th/9805170.

hep-th/9904207.

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\(^3\)One could consider the Lagrangian for the sum of a selfdual and an anti-selfdual tensor and then try
to add a mass term for the diagonal combination only. This would not change our discussion.


