A Central Engine for Cosmic Gamma-Ray Burst Sources

M.A. Ruderman\textsuperscript{1} and L. Tao
Departments of Physics and Astronomy,
Columbia University, New York, NY 10027

and

W. Kluzniak
Copernicus Astronomical Center,
Warsaw, Poland

Received \textemdash; accepted \textemdash;

\textsuperscript{1}Corresponding author. \textit{E-mail address:} mar@carmen.phys.columbia.edu
One of a family previously proposed “central engines” for cosmic gamma-ray burst sources (Kluźniak & Ruderman 1998) is considered in some detail. A steadily accreting $10^6$ Gauss magnetic white dwarf should ultimately collapse to a strongly differentially rotating, millisecond-rotation-period neutron star for a wide range of steady accretion rates and initial masses if the accreting white dwarf has an evolved O-Ne-Mg composition. A similar neutron star could also result from an initial C-O white dwarf but only for more constrained accretion rates. Because the collapsing white dwarf begins as a $\gamma = 4/3$ polytrope, the final neutron star’s spin-rate increases strongly with cylindrical radius. A stable wind-up of the neutron star’s poloidal magnetic field then produces buoyant magnetic toroids which grow, break loose, rise, and partly penetrate the neutron star surface to form a transient, $B \approx 10^{17}$ G millisecond spin-period pulsar with a powerful pulsar wind (Usov 1992). This pulsar wind emission is then rapidly suppressed by the surface shear motion from the strong stellar differential rotation. This wind-up and transient pulsar formation can occur at other times on different cylinders and/or repeat on the same one, with (re-)wind up and surface penetration time scales hugely longer than the neutron star’s millisecond spin period. In this way, differential rotation both opens and closes the doors which allow neutron star spin-energy to be emitted in powerful bursts of pulsar wind. Predictions of this model compare favorably to needed central engine properties of gamma-ray burst sources (total energy, duration, sub-burst fluctuations and time scales, variability among burst events, and baryon loading).

Subject headings: gamma rays: bursts — instabilities — magnetic fields —
stars: neutron — stars: rotation
1. Introduction

Gamma-ray bursts (GRBs) are observed daily from sources at distances extending out to those of the oldest galaxies in our Universe. To account for details of these bursts, “central engines” (CEs) of the GRB sources should have the following properties (see Kluźniak & Ruderman 1998, hereafter KR, for details and references).

(a) Energy. Some CEs must store and release of order $10^{53}$ ergs (assuming modest beaming of the energy outflow).

(b) Fluctuations. There are often large temporal variations in the CE power output. A CE should be capable of attaining peak power within tens of milliseconds and exhibiting large fluctuations thereafter. The main power emission is often in sub-bursts between which the CE is relatively dormant, typically for about 10 seconds, but sometimes for as long as several $10^2$ seconds or as short as $10^{-1}$ seconds.

(c) CE lifetimes, typically seconds to tens of seconds, extend from less than a second to greater than $10^3$ seconds. (There is also some indication of an association of greater total energy release with longer CE lifetimes.)

(d) Baryon loading. The energy released from the CE of a GRB source carries with it at most only a tiny baryon load of mass $\lesssim 10^{-4} M_\odot$.

(e) The birth rate of GRB sources $\gtrsim 10^{-6}$/galaxy/yr (see, for example, Böttcher & Dermer 2000).

(f) There is a very great variability among observed GRB events: durations, time scales within a burst, and pulse shape structures, sub-burst numbers, etc., vary so much that one cannot really specify a typical GRB.

The shortest time scales of (b) together with the total energy emission (a) suggest a
CE formation involving stellar collapse to a neutron star or to a black hole, or a very tight binary of such collapsed objects, or as part of some exotic supernova which would form such objects. However, the lifetimes (c), baryon loading (d), the commonly observed repeated widely separated fluctuations (b), and perhaps the birthrate (e) may raise special problems for such CE models. Particularly significant is why, if the CEs are collapsed objects whose periods of rotation and vibration are expected to be milliseconds, energy emission from them so often involves several timescales which can be up to $10^6$ times longer.

A promising way of constructing CE models based upon collapsed objects, which incorporates this needed family of relatively long time scales, begins by converting the most of the released collapse energy into rotational energy of the collapsed objects. The subsequent transfer of that energy to emitted power in a form useful for ultimate $\gamma$-ray production may then be accomplished relatively slowly. It is generally necessary to have CE magnetic fields $B \gtrsim 10^{15}$ G to extract the rotationally stored energy fast enough. Such a CE model was long ago proposed by Usov (1992). A millisecond spin-period pulsar with a magnetic field $B \approx 10^{15}$ G was assumed to be formed from an accretion induced collapse of a strongly magnetized ($B \approx 10^9$ G) white dwarf. This simple CE model would be expected to have the needed energy (a), lifetime (c) and baryon loading (d) properties, but a sufficiently high birthrate (e) may be questionable and the required fluctuation property (b) does not seem to be realized.

It has been proposed more recently that very large differential rotation plays an essential role in CE models (KR). One such model has significant similarities to Usov’s proposed millisecond-period “magnetars”, but the initial white dwarf’s pre-collapse history and magnetic field strengths differ, and there are essential differences in what happens within the neutron star and on its surface. This strongly differentially rotating CE would form and evolve in the following, quite different, way.
1) A common “garden-variety” magnetic white dwarf ($B \approx 10^6$ G) in a tight binary is spun up to its equilibrium spin-period ($P \approx 10^3$ s) by an accretion disk fed by its companion.

2) The accreting white dwarf is either an evolved one (O-Ne-Mg), or a canonical (C-O) dwarf, with accretion rates such that the accreting white dwarf increases its mass, implodes before its growing stellar mass reaches $1.4M_\odot$, and collapses to a neutron star.

3) A neutron star is then formed with an initial spin-period $P \approx 10^{-3}$ seconds, a nearly canonical pulsar polar magnetic surface dipole component $B_p \approx 10^{12}$ G, and, most importantly, a spin-rate which increases very greatly with distances from the star’s spin-axis. It is this crucial last feature which is the reason for choosing here to discuss this particular CE model from among the previously suggested possibilities for CEs with large initial differential rotation (KR).

4) An interior toroidal field ($B_\phi$) is then stably wound up from the poloidal field ($B_p$) by this differential rotation until $B_\phi \approx 10^{17}$ G. After that $B_\phi$ is achieved, the wound-up (and probably slightly twisted) toroid’s magnetic buoyancy for the first time exceeds interior anti-buoyancy forces (from compositional stratification). The buoyant toroid pushes up to the surface by moving parallel to the spin-axis up to, and then partly penetrating the stellar surface, within about $10^{-2}$ seconds after its initial release.

5) For as long as some of this magnetic field sticks out of the rapidly spinning neutron star’s surface, this will be an extreme realization of an Usov pulsar, a hyper-magnetar powered by the star’s spin energy. It is, however, extremely transient because of surface movements.

6) This surface dipole field (and higher multipoles) can survive for only a very brief time ($\approx 10^{-2}$ seconds): it is continually smeared out around the spin-axis, and thus diminished
by the strong on-going differential rotation shearing the surface below any protruding field. (There may also be considerable surface field reconnection after this.)

7) After the first break-out of wound-up toroidal field, surface penetration by some of it, and the resulting transient Usov pulsar, a similar wind-up of the $B_p \approx 10^{12}$ G may begin again as in Step 4, around the same cylinder or a somewhat slower wind-up may exist on some other cylinder). In tiehr case, a new toroid grows until its $B_\phi$ reaches $B_\phi(max) \approx 10^{17}$ G when another sub-burst occurs as in Steps 5-6. The characteristic interval between the first and second sub-burst would be

$$\tau_{sb} \approx \frac{2\pi B_\phi(max)}{(\Delta\Omega)B_p} \approx 10 \text{ seconds},$$

where $\Delta\Omega$ is the spin-frequency difference between the inner and outer parts of the differentially rotating neutron star.

8) The GRB source’s CE finally turns off completely when either of two stages is reached by the engine:

a) the differential rotation ($\Delta\Omega$) which drives the wind-up of $B_\phi$ becomes so diminished by the conversion of the differential rotation energy into toroidal field energy that it can no longer cause build-up to the critical $B_\phi \approx 10^{17}$ G needed for a pulsar wind sub-burst, or

b) the stellar spin ($\Omega$ of the outer region) becomes so reduced in the transient pulsar phases sustained by it that pulsar wind emission is almost extinguished even if a huge protruding field were to survive.

In this present note, we consider the above GRB source CE proposal in more detail and discuss why and how it should have all of the desired properties.
2. Accretion Induced Collapse of Magnetic White Dwarfs to Neutron Stars

Some white dwarfs (WDs) in tight binaries can accrete enough mass from their companions to initiate implosions (because of electron capture by nuclei) as they approach (but just before they reach) their Chandrasekhar limits. After such an implosion begins, there is a competition between energy release from nuclear fusion reactions which act to explode the star and a growing rate of electron capture—which removes pressure support and accelerates collapse. The winner in this competition, which depends upon these relative rates, determines whether such WDs end as Type Ia supernovae (no remnant star) or as neutron stars. Figure 1 shows how the ultimate fate of such accreting WDs is determined by the mass of the WD when accretion begins ($M$) and the steady accretion rate ($\dot{M}$) which brings it to the initial implosion instability (typically when the accreting stellar mass is about $1.35M_\odot$). There are three possibilities. 1) The accreting WD has $M$ and $\dot{M}$ in the cross-hatched region. Then nova explosions continually eject at least as much mass as is accreted between these nova explosions and the implosion mass is not reached. 2) The WD’s $M$ and $\dot{M}$ begin in the unmarked region. It then ends its life by an accretion induced collapse (AIC) to a neutron star. 3) When $M$ and $\dot{M}$ are in the dotted region, the accreting WD ends in an explosion with no remnant—a Type Ia supernova (SN)—if its composition is initially C+O. If it approaches implosion with a more evolved O+Ne+Mg composition, sustained $\dot{M}$ ultimately causes it to collapse to a neutron star (Nomoto & Kondo 1991; see also Bailyn & Grindlay 1990).

(Below we shall consider WDs with magnetic fields $B \approx 10^6$ G and, mainly because of that magnetic field, spinning with periods $P \approx 10^3$ seconds. Neither are of much consequence in the early stages of collapse of these WDs when the ultimate fate of the WD is determined. The magnetic field energy density is approximately $10^{-9}$ that of the WD’s rotational kinetic energy, which, in turn, is approximately $10^{-3}$ that of its gravitational
binding energy. Therefore Figure 1 should not be sensitive to a WD’s possible $10^6$ G field or $10^3$ second spin period.)

A WD with a $B \approx 10^6$ G has a relatively modest field among the “magnetic white dwarfs” in the local WD population. Based upon those, they might be expected to number several percent of the WD population. Because of this field, an accretion disk around such a WD, fed by mass pulled from its companion, should spin up the accreting WD to a steady state angular spin rate

$$\Omega \approx \frac{\dot{M}^{3/7} (GM)^{5/7}}{(BR^3)^{6/7}} \approx \dot{M}_{18}^{3/7} 10^{-2} \text{ seconds}^{-1},$$

with $R$ the WD radius and $\dot{M}_{18}$ the WD accretion rate in $10^{18}$ grams/second ($\dot{M}_{18} = 1$ when $\dot{M} = 2 \times 10^{-8} M_\odot$ per year). For equation (2) to hold, it is assumed that $\dot{M}$ is small enough to keep the inner edge of the accretion disk above the stellar surface, i.e.,

$$\dot{M} < \left( \frac{R^5 B^2}{GM} \right)^{1/2} \approx 10^{20} \text{ g/s} = 2 \times 10^{-6} M_\odot \text{ year}^{-1}$$

The total mass which would have to be accreted to reach the $\Omega$ of equation (2) is about $10^{-2} M_\odot$. Thus before magnetic WDs with a dipole field $B \approx 10^6$ G accrete enough to collapse, it is reasonable to expect a good fraction of them, probably most, to have been spun-up to a period $P_{WD} \approx 10^3$ seconds. After they have collapsed to neutron stars with $R \approx 10^6$ cm, those neutron stars would then have

$$P_{NS} \approx 10^{-3} \text{ seconds}.$$  

If magnetic flux is conserved during the collapse, a very plausible approximation because of the short time for collapse (on the order of seconds), these millisecond period neutron stars are formed with (poloidal) fluxes

$$B_{NS} \approx 10^{12} \text{ G}$$

(5)
If such neutron stars are to be candidates for GRB source CE’s, their formation rate must be \( \gtrsim 10^{-6}\text{yr}^{-1}\text{-galaxy}^{-1} \). Type Ia supernovae are observed to occur at a rate \( 2 \times 10^{-3}\text{yr}^{-1}\text{-galaxy}^{-1} \). A plausible guess for the fraction of \( B \approx 10^6 \text{ G} \) exploding WDs among them \( \approx 2 \times 10^{-2} \), if this fraction is about the same as that for such fields to be found in the local WD population. The fraction of such moderately magnetized WDs in cataclysmic variables (accreting WDs in tight binaries) is very much greater than this. (However, their number statistics are subject to significant but still unquantified selection effects.) A fraction of \( 2 \times 10^{-2} \) assumed above, for WDs which become Type Ia SNe, seems rather conservative based on present knowledge about these WDs. Then if more than only \( 3 \times 10^{-2} \) of the WDs which accrete enough to explode as Type Ia SNe had a composition, or a combination of initial \( M \) and \( \dot{M} \), to implode to neutron stars, the formation rate for neutron stars satisfying equation (4) and equation (5) would be enough for them to be a candidate population for CEs if the other required properties are met.

The simplest of these and most necessary to satisfy is the maximum energy requirement (a). The spin-energy of a neutron star with an average rotation rate \( (\tilde{\Omega} \approx 10^4/\text{second}) \) about \( \approx 10^{53} \text{ ergs} \) is difficult to compare precisely with CE requirements because of the still unknown GRB beaming and some uncertainties in the neutron star equation of state and moment of inertia. We turn next to other special properties of these particular AIC formed neutron stars which determine CE fluctuation timescales (b), lifetimes (c) and baryon loading (d) and variability among the family of these engines (e).

3. Initial Differential Rotation of the Neutron Star

The pressure support in a WD whose mass approaches \( 1.4M_\odot \) is from extreme relativistic electrons; the star is a \( \gamma = 4/3 \) polytrope. Such a star has a central density \( (\rho_c) \) very strongly peaked relative to its average density \( \bar{\rho}: \rho_c \approx 55\bar{\rho} \) (Shapiro & Teukolsky
The difference in $\Omega$ between the inner and outer parts of the newly formed neutron star will depend on the initial composition of the imploding WD. A C-O WD, when collapse begins from $^{16}O + e^- \rightarrow ^{16}C + \nu$, has $\rho_c \approx 2 \times 10^{10} \text{g/cm}^3$. A O-Ne-Mg WD, whose collapse is initiated by $^{24}Mg + e^- \rightarrow ^{24}Na + \nu$, has $\rho_c \approx 3 \times 10^9 \text{g/cm}^3$. If during collapse to a neutron star the angular momentum were to be conserved independently in each of the “rings” of matter circulating around the spin-axis, then the final spin-rates of rings which were originally in from the central region of the collapsing $\gamma = 4/3$ WD are much less than those of rings collapsing in the outer regions. Roughly, the average neutron star spin

$$\Omega_{\text{NS}} \approx \Omega_{\text{WD}} \times \left( \frac{\bar{\rho}_{\text{NS}}}{\bar{\rho}_{\text{WD}}} \right)^{\frac{2}{3}} \approx 10^{4} \text{/second},$$

where $\bar{\rho}_{\text{WD}}$ is the initial average WD density and $\bar{\rho}_{\text{NS}}$ that of the neutron star. However, for the central region of the WD

$$\rho_c(WD) \approx 55\bar{\rho}_{\text{WD}},$$

compared to the very much more modest peaking for the central region of a $1.3M_\odot$ neutron star,

$$\rho_c(NS) \approx 5\bar{\rho}_{NS}.$$ 

Insofar as the pressure support of a somewhat cooled neutron star can be approximated as that of a non-relativistic neutron kinetic energy ($\gamma = 4/3$), a neutron star’s $\bar{\rho}/\rho_c \approx 6$ (Shapiro & Teukolsky 1983). Additional contributions to stiffening the neutron star’s equation of state (which must be present to increase its maximum neutron star mass from the Oppenheimer-Volkoff $0.7M_\odot$ to the observed range which is at least twice as large) reduce this ratio further. Therefore the central regions of this newly born neutron star should initially be spinning much less rapidly than most of the matter in that star by a factor of about

$$\left[ \frac{\bar{\rho}_{\text{NS}} \rho_c(WD)}{\bar{\rho}_{\text{WD}} \rho_c(NS)} \right]^{-2/3} \approx 10^{-2/3} \approx 0.2,$$
but, other than the fact that this number is very considerably less than 1, its precise value
will not be important in the approximations considered below.

To the approximation that the pressure in the neustrop stellar matter depends only on
density, a dynamically stable steady state is finally achieved after fluid flow adjustments give
an $\Omega_{NS}$ which depends only on the distance from the spin axis ($r_\perp$) (the Taylor-Proudman
theorem). In this idealization, the newly formed neutron stars rotates on cylinders whose
angular speed, $\Omega(r_\perp)$, increases strongly with increasing $r_\perp$ because of the differences in
density distribution between the $\gamma = 4/3$ polytrope WD and the neutron star to which
it implodes. A crucial question is whether this differential rotation might first have been
dissipated during collapse and, if it has survived, what becomes of it in the next $10^3$ seconds
or so.

During collapse, canonical viscous coupling between distant parts of the star (e.g.,
by Ekman pumping) is far too weak to be important. However, exchanges of angular
momentum by transient energetic neutrino transfer needs special consideration. If this were
important, it would be expected to be most efficient for neutrinos whose mean-free path ($\lambda$)
is of order $R_{NS} \approx 10^6$ cm. These are emitted in canonical SNEs from the rapidly cooling
neutron star remnants (e.g. the neutrinos detected from the SN 1987A explosion) mainly
over a 10 second interval. In the implosion leading to the special model CE neutron stars of
interest here, most of the released energy should go into stellar rotation rather than thermal
heating. Thermal neutrino emission during and just after neutron star formation should,
therefore, be much less. If we approximate angular momentum transfers from neutrino
transport by assuming, say, a single absorption or scatter before escape (i.e., $\lambda \approx R_{NS}$),
then the ratio of angular momentum transfer by emitted neutrinos of total energy $\mathcal{E}_\nu$ to the
total angular momentum of the neutron star would be $\approx \mathcal{E}_\nu/Mc^2$. Because the difference in
angular momentum between the inner and outer parts of the CE neutron star is comparable
to the entire stellar angular momentum, the fraction of that difference which would be
dissipated before neutrino cooling would also be of order $\mathcal{E}_\nu/Mc^2$. This ratio is certainly
less than $10^{-1}$ and may be much less.

Another concern is the possibility of convective overturn (Ardeljan et al. 1996),
which would mix (Taylor-Proudman) cylinders rotating with different angular speeds.
(Fluid movements perpendicular to $r_\perp$ are not relevant.) But on the short timescales
of interest here, where viscosity and thermal conduction are negligible, this should be
strongly suppressed by the great increase of angular momentum per unit mass with
increasing $r_\perp$: $\frac{\rho_\perp}{\rho} (\Omega^2 \rho r_\perp)$ is much greater than any plausible convective force density when
$\Delta \Omega \approx \Omega \approx 10^4$/second and $kT \lesssim 10$ MeV.

During WD collapse there is also a small transfer of angular momentum between
different collapsing regions by magnetic fields which couple them. The WD’s (polar)
magnetic field which connects differently spinning rings during the collapse would take a
time
\[
\tau_A \approx \frac{(4\pi)^{1/2} R_{WD} \left( \frac{\bar{\rho}_{WD}}{\rho} \right)^{1/2}}{B_{WD}} \approx 10^6 \left( \frac{\bar{\rho}_{WD}}{\rho} \right)^{1/2} \text{ seconds}
\]
to transfer angular momentum between them, where $R_{WD}$, $\bar{\rho}_{WD}$ and $B_{WD}$ are the WD
radius, density and magnetic field at the beginning of the collapse, and $\rho$ is the (transient)
density at any stage of the collapse. This $\tau_A$ is far too long for the magnetic threading
during collapse to be a concern in modifying differential rotation.

This leaves the one mechanism for short time scale dissipation of differential rotation
which is fundamental to our model for the CEs of GRB sources. Because the differentially
rotating cylinders of the newly formed neutron star are coupled by the polar magnetic field
($B_\rho$) in the stellar interior, that field will begin to wind up a toroidal one, $B_\phi$. We turn
next to the stability, magnitude, and termination of that wind-up.
4. Stability of Toroidal Field Wind-up

The initial $\Omega(r_\perp)$ in neutron stars formed in this particular AIC genesis is one which grows strongly with increasing $r_\perp$:

$$\frac{\partial}{\partial r_\perp} \Omega > 0 .$$

This rotating fluid is certainly very linearly stable to axisymmetric hydrodynamic perturbations, according to the Rayleigh criterion that the angular momentum $r_\perp^2 \Omega$ increases outwards. Such a neutron star is also not unstable (or, at least, no instabilities have been discovered) to the so-called Tayler instabilities (Tayler 1973; see also Spruit 1999), which do exist in the special case when $\partial \Omega / \partial r_\perp = 0$, i.e., when the star is rotating rigidly. However, important recent work in MHD stability theory has shown that powerful instabilities may exist in differentially rotating systems when they contain even relatively weak magnetic fields (Velikhov 1959, Chandrasekhar 1961, Balbus & Hawley 1991). Could the differential rotation of a neutron star satisfying equation (11) be unstable to any of these magneto-rotational instabilities? Demonstrations of related MHD instabilities in differentially rotating objects have included non-axisymmetric perturbations, compressibility, and toroidal and poloidal fields (see, for example, Balbus & Hawley 1992 and Ogilvie & Pringle 1996). Magneto-rotational instabilities have been found when angular velocity decreases with $r_\perp$ but no instabilities have been exhibited for flows satisfying equation (11). Indeed that inequality adds to stabilizing forces in those cases for which the opposite inequality causes instability. Therefore it seems plausible that strong differential rotation satisfying equation (11) will be stable even when the differentially spinning object is threaded by a weak magnetic field: it will wind up an initial poloidal field, which threads it, into a toroidal field until that field becomes unstable because of buoyancy effects, or if magnetic forces grow to exceed gravitational ones.

For the magnetic toroid to become buoyantly unstable, the buoyant forces must
overcome whatever anti-buoyant stratification may exist in the star. (For a cold neutron star, the stratification is the compositional one from varying proton/neutron ratios, which can adjust via \( n \leftrightarrow p + e \) with neutrino emission being too slow to be efficient here.) The wound-up toroidal magnetic field \( B_\phi \) would be stable until the buoyancy force density

\[
F_b \approx \frac{B_\phi^2}{8\pi c_s^2 g} ,
\]

where \( c_s \) the speed of sound of the embedding medium, exceeds any anti-buoyancy force density. Wind-up would ultimately increase \( B_\phi \) until it reaches a critical value \( B_b \) at which the buoyancy force is enough to balance the neutron star’s interior anti-buoyancy. This has been estimated to give \( B_b \) of order \( 10^{17} \) G (KR). (Our estimates in this paper do not depend upon knowing this \( B_b \) accurately. It is certainly less than the equipartition value for a neutron star whose gravitational binding energy \( \approx 10^{-1}M_{NS}c^2 \). \( B_\phi(equipartition) \approx 10^{18} \) G. The KR estimate of \( B_b \approx 2 \times 10^{17} \) G, based solely upon anti-buoyancy forces from compositional stratification in a cooled neutron star, varies only as the square root of that force and is not changed qualitatively by inclusion of other, mainly thermal, contributions).

Thereafter, a (probably slightly twisted) toroid should rapidly rise towards the stellar surface, moving in the direction aligned with both the spin-axis \( \vec{\Omega} \) and \( \partial \vec{\Omega}/\partial r_\perp \). The wind-up of the non-axisymmetric poloidal field into a strong toroidal component may introduce some twist into the overwhelming toroidal field. (For the released toroid to rise stably through the neutron star, some twist may be what keeps it from fragmentation by Kelvin-Helmholtz or Rayleigh-Taylor instabilities; see Tsinganos 1980). It is appropriate to emphasize that we do not have a detailed description of the wound-up toroidal bundle’s dynamical evolution after it is released by buoyancy. The buoyancy force can push up the fluid column above it (acting like a plug) to make a surface bulge which can spread horizontally, and/or the approximate axial symmetry of the wound-up toroid may be diminished by some magnetized fluid movement perpendicular to \( r_\perp \).
5. Surface Field Penetration: Transient Pulsar (Hyper-Magnetar) Formation

As this toroid rises, the toroidal magnetic flux continues to increase because it is still being wound up from the poloidal component by the continuing differential rotation. As a function of time, $t$, measured from the moment when the field first reaches the critical strength $B_b$,

$$B_\phi = B_b + t B_p \Delta \Omega,$$

(13)

where $\Delta \Omega$ is again the characteristic difference in angular velocity across the wind-up region. (In our model the initial $\Delta \Omega \approx \Omega$.) The buoyance force density, which depends on the square of the toroidal component ($B_\phi$) minus that of the anti-buoyancy force density which balances it when $B_\phi = B_b$, grows nearly linearly with $t$:

$$F_b \approx \frac{t B_b B_p \Delta \Omega}{4 \pi c_s^2} g.$$  

(14)

Then the buoyancy timescale for rising up to and penetrating through the stellar surface is

$$\tau_b \approx \left( \frac{24 \pi R c_s^2 \rho}{B_b B_p g \Delta \Omega} \right)^{1/3}.$$  

(15)

For the special AIC formed neutron star, the radius $R \approx 10^6$ cm, the speed of sound $c_s \approx 10^{10}$ cm/sec, $B_b \approx 10^{17}$ G, $B_\phi \approx 10^{12}$ G, $g \approx 10^{14}$ cm/sec$^2$, and $\Delta \Omega \approx 10^4$/second. Then the buoyancy, post break-free, time scale of equation (15) is of the order $10^{-2}$ seconds. At the beginning of the wind up, the non-axisymmetric $B_\phi$ is not negligible relative to $B_p$.

Again during the interval $0 < t < \tau_b \approx 10^{-2}$ seconds when the (positive) difference between the buoyancy and anti-buoyancy forces is small, non-axisymmetric forces from $B_\phi$ may not be entirely negligible compared to other axisymmetric ones acting on the toroids. One effect of this would be a tilt to the rising wound-up toroids accomplished by slightly different forces and fluid movements in directions aligned with $\vec{\Omega}$ (and $\partial \vec{\Omega}/\partial r_\perp$). Because the toroid will not be exactly axisymmetric, a part of it will ultimately poke through the surface of the star. The field that penetrates the stellar surface will not escape because of the huge
conducting mass threaded by the rest of the toroid still below the surface to which it is still strongly attached. This strongly conducting mass remains gravitationally bound to the star.

Therefore \( \tau_b \) of equation (15) is the turn-on time for the star becoming a Usov type (hyper-magnetar) pulsar with a magnetic dipole field less than, but probably comparable to \( B_0 \approx 10^{17} \) G. (Because \( \Omega R/c \approx 1/3 \), more than just the dipole component of this pulsar may be important in analyzing its properties.)

6. Transient Pulsar Termination: Extinction of Surface Multipoles by the Surface Shear from Differential Rotation

Because of the continuing differential rotation of the star and stellar surface, a surface dipole (and all other multipole) cannot survive long beyond the characteristic differential rotation periods involved. If \( \tau_s \) is the time it take for the differential rotation to shear out the surface dipole (multipole), the characteristic value of the transient surface field would be

\[
B_{surf} \approx B_b \times \frac{\tau_s}{\tau_b} \quad \text{if} \quad \tau_s < \tau_b,
\]

\[
B_{surf} \approx B_b \quad \text{if} \quad \tau_s > \tau_b.
\]

(16)

Figure 2 shows a simplified example of effects of surface shear motion suppression of surface field moments beginning from a north-polar cap of radius \( a \) at a distance \( d \) from the spin-axis \( \vec{\Omega} \) together with a similar south-polar cap displaced by the same \( d \). Because of the different angular velocities of the different parts in both polar caps (increasing with \( r_\perp \)), the caps will be deformed into tighter and tighter interwoven spirals extending from \( r_\perp = d - a \) to \( d + a \). After many relative wind-ups between \( r_\perp = d - a \) and \( r_\perp = d + a \), the surface field will be entirely smeared out and cancelled. If the cap radii or distances differed
slightly, very little of the surface field would survive as rings around the spin-axis leading to hugely reduced power in the pulsar wind. [If $\epsilon$ is a measure of the small difference in the two polar cap radii ($a$) or their distances from the spin-axis ($d$), then the asymptotic power output after extended shearing is reduced by a factor of order $(\frac{\epsilon}{a})^2 \approx 10^{-2}$ relative to that from the initial configuration.]

The timescale for this surface dipole suppression to be essentially completed is

$$\tau_s \approx \text{several} \times \frac{2\pi}{\frac{\partial\Omega}{\partial r_{\perp}} a} \approx \text{several} \times \frac{2\pi}{\Omega} \left(\frac{R}{a}\right) \approx 10^{-2}\text{ seconds}, \quad (17)$$

where we assume $\frac{\partial\Omega}{\partial r_{\perp}} \approx \Omega/R \approx 2\pi/10^{-3}R$ seconds, and several$\times R/a \approx 10$. This estimated $\tau_s$ should be characteristic for the suppression of all important surface multipoles of typical surface field geometries.

Therefore a $10^{17}$ G toroid rises to the surface of the star in $\tau_b \approx 10^{-2}$ seconds, partly penetrates that surface and expands outside the star. It survives for a time $\tau_s$ which is about $10^{-2}$ seconds. So long as this field sticks out of the surface of this rapidly spinning neutron star, the star will have the canonical spin-down power and wind emission of a pulsar with dipole field $B_d \lesssim 10^{17}$ G and $P \approx 10^{-3}$ seconds.

During this time, the field will be not only be smearing out into a ring but also the north and south poles of the field will be brought into closer and closer contact with each other. This can result in some reconnection. However, reconnection is not the dominant process of field destruction and facilitates field destruction only after the field is already smeared out.

7. Sub-burst structures: Energies, Intervals and CE durations

During that brief interval while the millisecond pulsar’s $B_d \approx 10^{17}$ G dipole field exists, its pulsar wind (consisting of electromagnetic energy, $e^\pm$, and some baryons) carries away a
sub-burst energy
\[ \mathcal{E}_{sb} \approx \frac{B_d^2 R^6 \Omega^4}{c^3} \times \tau_s \approx 10^{52} \text{ ergs} \]  
(18)
(where it is assumed that the dipole \( B_d \approx B_b \)). This is about that required for sub-bursts in observed GRB events. After the wound-up toroidal field breaks away (and penetrates the surface), wind-up by still existing \( B_p \) could continue to give yet another such wind-up of \( B_\phi \) to \( B_b \), and another transient pulsar, the rewind-up interval, i.e., the time between sub-bursts,
\[ \tau_{sb} \approx \left( \frac{2\pi}{\Delta \Omega} \right) \frac{B_b}{B_p} \approx \frac{10 \text{ seconds}}{(B_p)_{12}}, \]  
(19)
where \( (B_p)_{12} \) is the poloidal field strength in units of \( 10^{12} \) Gauss.

In addition and perhaps even more important, depending upon the details of \( B_p \), there would often be significant simultaneous winding up at other rates in several different cylindrical regions within the star. Then sub-bursts from the transient pulsar formation which is the result of the wind-ups would occur from them at different times and could vary enormously with a typical separation of about the \( \tau_{sb} \) of equation (19) but often much longer or shorter than that.

These sequences of toroidal field wind-up, breaking free, surface penetration of magnetic field \( B_d \), transient hyper-magnetar pulsar wind, and its suppression can continue only as long as \( \Delta \Omega \) remains large enough to sustain yet another wind-up of \( B_\phi \) to \( B_b \). Typically the energy in the remaining differential rotation within the neutron star would then have to be greater than about \((1/10)B_\phi^2 R^3 \approx 10^{51} \) ergs, typically \(10^{-2} - 10^{-1}\) times the equipartition energy.

The ultimate energy source for the sub-bursts of pulsar wind emission is the spin-energy of the entire star, the differential rotation serving only as the key to open the pulsar wind emission gate for brief intervals. Because this total \((E_{max})\) has a maximum of about \( I\bar{\Omega}^2/2 \approx 10^{53} \) ergs, it could sustain repeated strong emission activity for a characteristic
time (cf. equation (18))

\[ \tau \approx \frac{E_{\text{max}}}{E_{sb}} \times \tau_{sb} \approx \frac{10^2 \text{ seconds}}{(B_p)_{12}}. \] (20)

Some CE's may have a \(\Delta \Omega\) which could give rise only to a single sub-burst with a \(\tau_b \approx 10^{-2}\) second rise time, but the turn-off time \(\tau_s\) of such a CE should be stretched considerably for small \(\Delta \Omega\). It is difficult to compare sub-burst turn-off times directly with observations of the emission of \(\gamma\)-rays powered by the pulsar wind sub-burst because these observed \(\gamma\)-rays are created in relativistic expanding regions so very far from the CE.

8. Baryon loading and beaming

The transient emission (sub-bursts) from a CE is not what is directly observed in GRBs. The emission radius of observed \(\gamma\)-rays must not be less than about \(10^{15}\) cm if \(\gamma + \gamma \rightarrow e^+ + e^-\) is not to absorb those \(\gamma\)-rays far above the pair creation threshold. At these large radii, almost all the CE emitted burst energy has all been transferred by expansion into kinetic energy of its co-moving baryon load. To account for the short time scale of many sub-bursts \((\approx 1\) second\), the observed emitting region must be expanding relativistically with such a large Lorentz \(\gamma\) that the rest mass of baryons \(\lesssim 10^{-4} M_\odot\) for each \(10^{53}\) ergs in bursts.

If, as indicated in the previous section, the sub-burst emission is powered by wind from a pulsar with \(\Omega \approx 10^4/\text{second}\) and transient dipole \(B \lesssim 10^{17}\) G, the maximum possible baryon outflow from the pulsar should be the maximum for a canonical pulsar with these parameters. The flow rate of nuclei with charge \(Z e\) out from the stellar surface \(\langle \dot{N}_Z \rangle\) should then not exceed the Goldreich-Julian limit which would quench that outflow:

\[ \dot{N}_z \lesssim \frac{\Omega^2 B R^3}{e c} \approx 10^{-15} B_{17} M_\odot/\text{second}. \] (21)

This baryon load is negligible in itself and also compared to what the pulsar wind, and
especially the first sub-burst, would sweep up from matter around the WD beyond the pulsar.

As noted in Section 6, there may also be small contributions to CE sub-bursts from some reconnection of magnetic loops which extend up from the stellar surface. While these loops are essentially free of baryon loading above the surface (beyond the negligible Goldreich-Julian one), how much they might pull out and up with them during reconnection is far less clear. Of course, each emission sub-burst from the pulsar need not itself be a source of power for ultimate $\gamma$-ray emission. Some might become beam dumps for slightly faster, later, much higher Lorentz $\gamma$ pulsar-wind sub-burst emissions with much less baryon loading.

If most of the CE emission is in transient pulsar winds from a spinning ephemeral surface dipole (or higher multipoles) as described above, that dipole is mainly orthogonal to the neutron star spin $\vec{\Omega}$. In the simplest models for pulsar wind emission, with only electromagnetic power in the wind from the star, the emission would be proportional to $\cos^2 \theta$ with $\theta$ the emission angle with respect to the spin axis. Beaming in the spin direction would then be a modest 3 times the emission’s angular average. Higher multipoles ($\Omega R/c \approx 1/3$) could significantly increase this beaming.

9. Variability among GRB events

Details of emissions from these proposed GRB source CEs should be sensitive to initial properties of the imploding ancestral WDs. There is an important dependence on the ancestral WD’s $\dot{M}$ and, especially, details of its magnetic field.

(a) If the accreting WD’s dipole component $B$ is much less than $10^6$ G, its steady state spin period from equation (4) becomes so small that it would not quickly and simply
form a nuclear density neutron star. Centrifugal forces would stop much of that collapse before it had evolved that far (forming what T. Gold called a “fizzler”). Ultimate formation of a neutron star would be achieved only after angular momentum had been removed [perhaps mainly into a surrounding disk and/or, as $P \to 10^{-3}$ seconds (the maximum spin rate of an axisymmetric neutron star), through the transient formation of a Jacobi ellipsoid and subsequent powerful gravitational radiation]. There is no obvious reason for the final distribution of differential rotation after such a genesis being the same as that of the proposed “canonical” CE from the AIC of a magnetic WD with $B \approx 10^6$ G. There are expected to be many more WDs with smaller $B$ than those with $B \approx 10^6$ G.

(b) If the accreting WD’s dipole $B$ greatly exceeds $10^6$ G, the steady state $P$ of equation (4) increases. If any of these more slowly spinning WDs were to collapse to a GRB source CE, that CE’s spin energy could not support burst events with total energy near the maximum $10^{53}$ ergs. However, if $\dot{M}$ could greatly exceed the Eddington limit (indicated in Fig. 1) from a sufficiently massive Roche Lobe overflow from the companion, $P(\text{NS}) \approx 10^{-3}$ seconds might be achieved even from a $B \approx 10^9$ G WD, as assumed by Usov in his CE model.

(c) The strongest sensitivity of CE emission pulse structures would probably be to details of $B_p$, the initial polar field in the newly formed differentially rotating neutron star, because of possible magnification of small initial $B_p$ differences to large ones in the wound-up $B_\phi$. For example, $B_p$ details determine the number density of toroids which begin simultaneous wind-up in different cylindrical regions around the spin-axis; these wound-up toroids overcome anti-buoyancy constraints and break free at different times. Locally, $B_p$ would be expected to change somewhat during these releases and any rewind-ups so CE emission pulse shapes would not be repetitive during a GRB event.
10. Discussion

The required properties of GRB source CEs (summarized in the Introduction) are total energy stored and emitted (a), peak power and fluctuations within a given burst event (b), CE lifetimes (c), maximum baryon loading in the CE emission (d), CE birthrate (e) and very large variability among different CEs (f). None of these seem an embarrassing problem for the proposed model CE genesis, structure and dynamics outlined in the Introduction and described in Sections 2-9. Indeed each seems a rather expected consequence. However, a crucial point which should be considered further is the absence (so far) of any demonstrated instability in the wind-up of the toroidal field for of order $10^4$ turns (in about 10 seconds) by the much more energetic initial differential rotations in the neutron star.

A second related, but less crucial, question is the robustness of our presumption that during and after such toroidal wind up and release the initial much smaller poloidal field component of the differentially rotating neutron star is not hugely increased. If this does not turn out to be an adequate approximation, the often observed sub-burst multiplicity could still come from toroidal field wind-up and break-away in different cylindrical regions with different wind-up times rather than from long time delays for rewinding $B_\phi$. Of course, because of the very great variability within the family of GRB events, neither mechanisms may hold in all, or perhaps not even in most cases, but at least one of them should certainly not be uncommon.

Finally there is our unproven assumption that large toroidal field bundles wound-up by differential rotation can overcome anti-buoyancy restraints and break free as a unit (or almost so). If, instead, buoyant toroidal field continually dribbled up and out to support a steady state in which increasing $B_\phi$ from wind-up is balanced by a that loss, there would be no strong fluctuations in CE output. Instead a CE would be an Usov-like pulsar with emission decreasing monotonically after the first emission maximum is reached. This is
a generic problem for many kinds of CE models. Why does the CE depart so far from a steady equilibrium that stored energy is released in huge sub-bursts (which are often separated by very many characteristic engine periods) rather than in smoother continuous steady way? Here too such a question needs further investigation.

11. Acknowledgments

We are happy to thank J. Applegate, E. Costa, C. Knigge, J. Pringle, M. Rees, E. Spiegel, H. Spruit, and J. Stone for informative conversations, and the Institute of Astronomy (Cambridge) and the Aspen Center for Physics for their hospitality while much of this work was begun.
REFERENCES


Balbus, S., & Hawley, J. 1998, Rev. Mod. Phys. 70, 1


Usov, V.V. 1992 Nature 357, 472

Velikhov, E.P. 1959, Sov. Phys. JETP 9, 995

This manuscript was prepared with the AAS LiTeX macros v4.0.